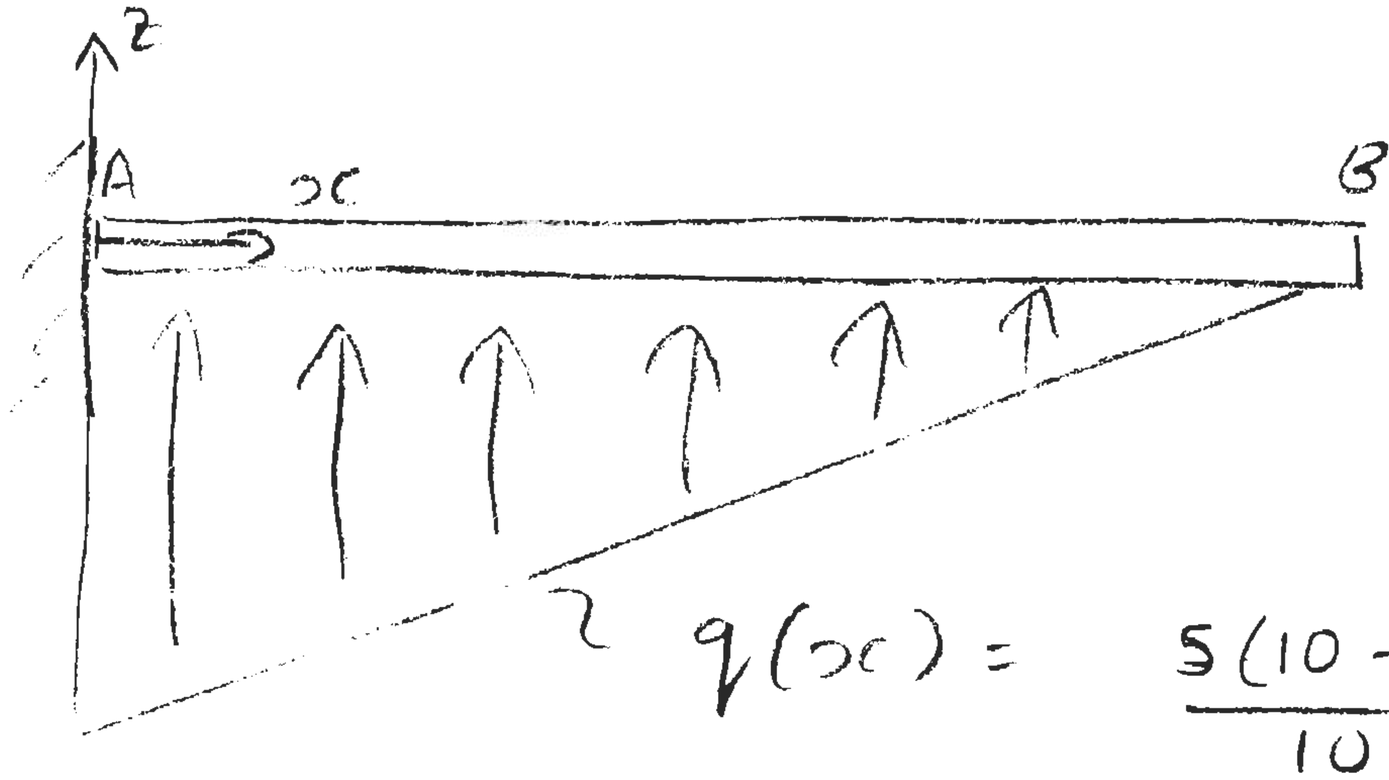
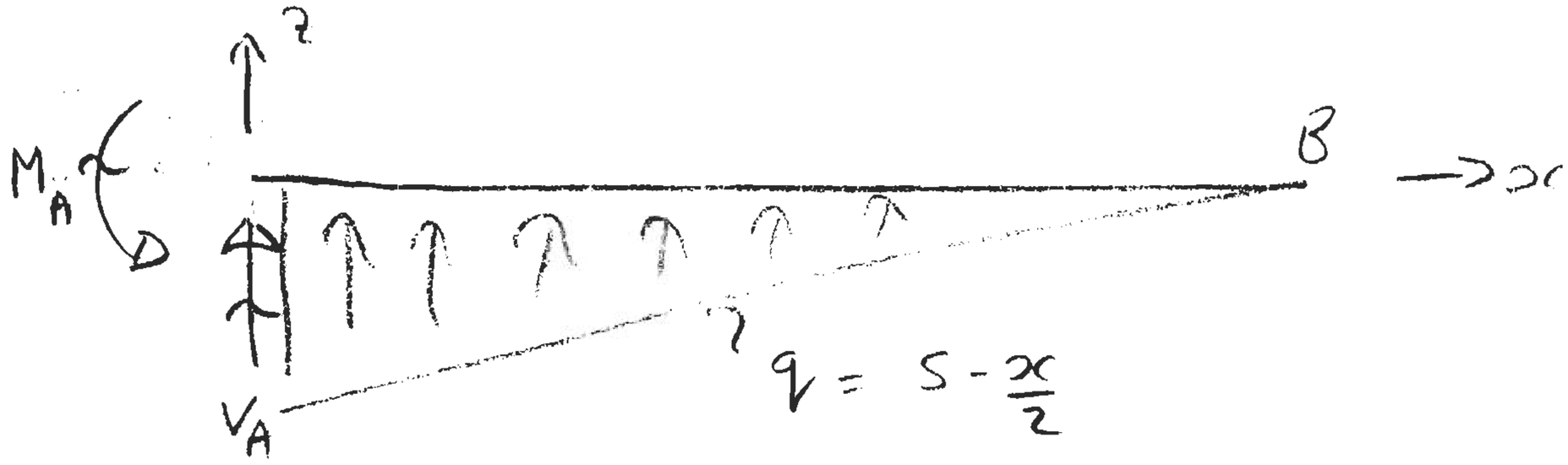
M 4



$$q(x) = \frac{5(10-x)}{10} = 5 - \frac{x}{2} \text{ N/m}$$



$$q = 5 - \frac{x}{2}$$

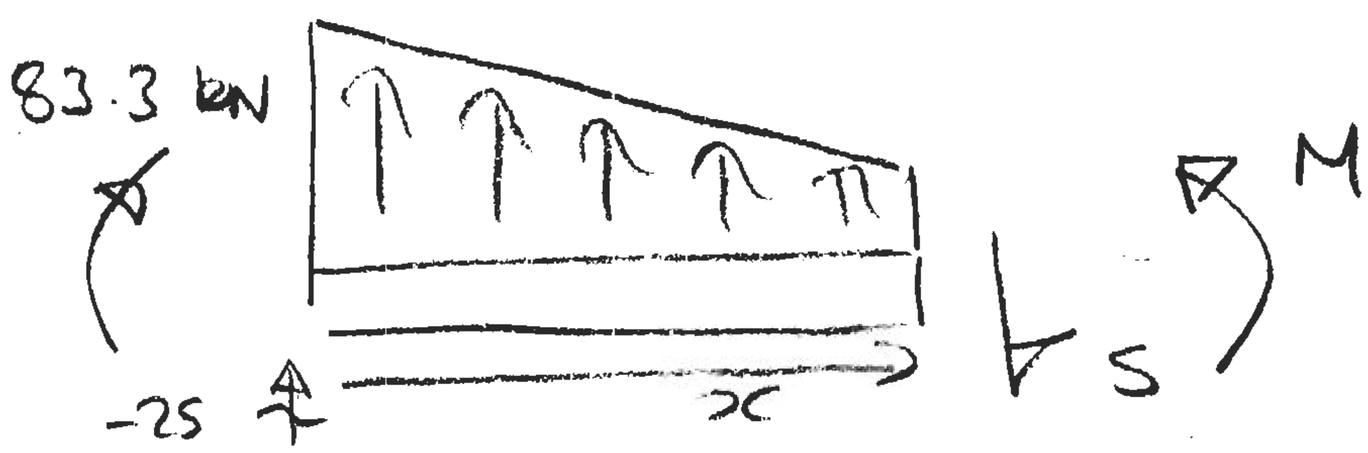
$$\sum F_z \uparrow = 0 \qquad V_A + \int_0^{10} 5 - \frac{x}{2}\, dx = 0$$

$$V_A = -\left[5x - \frac{x^2}{4}\right]_0^{10} = -25 \text{ kN}$$

$$\left(\sum M_A = 0\right) \qquad M_A + \int_0^{10} \left(5 - \frac{x}{2}\right) x\, dx = 0$$

$$M_A = -\left[\frac{5x^2}{2} - \frac{x^3}{6}\right]_0^{10} = -83.3 \text{ kNm}$$

Moment distribution



$$\Sigma F_z \uparrow = 0 :$$

$$-25 - S' + \int_0^x 5 - \frac{x}{2}\, dx = 0$$

$$S' = \left[5x - \frac{x^2}{4}\right]_0^x - 25 = 5x - \frac{x^2}{4} - 25 \Leftarrow$$

$$\Sigma \left(\circlearrowright M_A = 0 : M - 83.3 + \int (5 - \frac{x}{2}) x\, dx - S'x = 0\right.$$

$$M - 83.3 + \frac{5x^2}{2} - \frac{x^3}{6} - 5x^2 + \frac{x^3}{4} + 25x = 0$$

$$M = \frac{-1}{12} x^3 + \frac{5}{2} x^2 - 25x + 83.3 \Leftarrow$$

Max bending stress at root $x = 0$ $z = \pm \frac{100\,mm}{2}$

$M_{max} = 83.3\ kNm$ @ $x = 0$

$$I = \frac{1}{12} b h^3 \qquad \sigma_{max} = \frac{6 M_{max} \frac{h}{2}}{\frac{1}{12} b h^3} = \frac{6 M_{max}}{b h^2}$$

$$= \frac{6 \times 83.3 \times 10^3}{50 \times 10^{-3} \times (100 \times 10^{-3})^2} = 1\ GPa\ !!\ (high)$$

M4 tip deflection, from moment curvature relation

$$M = EI\frac{d^2w}{dx^2}$$

Integrate moment twice →

$$EI\frac{dw}{dx} = -\frac{1}{12}\frac{x^4}{4} + \frac{5}{2}\frac{x^3}{3} - \frac{25x^2}{2} + 83.3x + A$$

$$\frac{dw}{dx} = 0 \quad @ \; x = 0 \Rightarrow A = 0$$

$$EIw = -\frac{1}{12}\frac{x^5}{20} + \frac{5}{2}\frac{x^4}{12} - \frac{25x^3}{6} + \frac{83.3x^2}{2} + B$$

$$w = 0 \quad @ \; x = 0 \Rightarrow B = 0$$

$$EIw = -\frac{x^5}{240} + \frac{5x^4}{24} - \frac{25x^3}{6} + \frac{83.3}{2}x^2$$

at tip $x = 0$

$$\delta = \frac{1}{EI}\left[\frac{-10^5}{240} + \frac{5.10^4}{24} - \frac{25\times10^3}{6} + \frac{83.10^2}{2}\right] = \frac{1650\times10^3}{EI}$$

$E = 70\times10^9$, $I = \frac{1}{12}\times50\times10^{-3}\times(100\times10^{-3})^3 = 4.17\times10^{-6}$

$$\delta = \frac{1650 \times 10^3}{70 \times 10^9 \times 4.17 \times 10^{-6}} = 5.66 \text{ m.}$$

clearly rather large - not small deflection