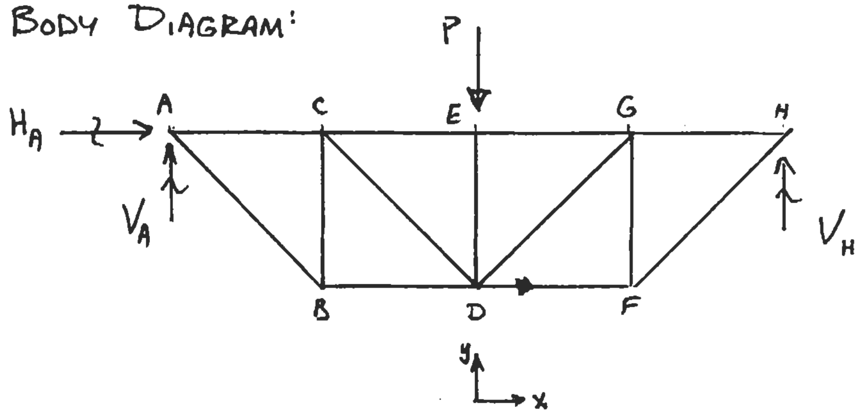
M~~X~~8

FREE BODY DIAGRAM:



APPLY EQUILIBRIUM TO FIND REACTIONS

$$\xrightarrow{+} \Sigma F_x = 0 \qquad \boxed{H_A = 0\ N}$$

$$+\uparrow \Sigma F_y = 0 \qquad V_A + V_H - P = 0$$

$$+\circlearrowleft \Sigma M_A = 0 \qquad V_H(4L) - P(2L) = 0$$

$$\boxed{\begin{aligned} V_H &= \frac{P}{2} \\ \Rightarrow V_A &= \frac{P}{2} \end{aligned}}$$

TO DETERMINE THE DEFLECTION OF D, WE NEED TO EMPLOY COMPATIBILITY + CONSTITUTIVE LAWS.

OUR CONSTITUTIVE LAW FOR BAR DEFORMATION IS:

$$\delta_{ij} = \frac{F_{ij} L_{ij}}{AE}$$

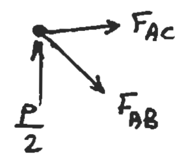
SO WE'LL NEED TO SOLVE FOR THE BAR FORCES IN ORDER TO DETERMINE THEIR EXTENSIONS, AND HENCE THE TRUSS DEFLECTION.

BECAUSE OF SYMMETRY, I ONLY NEED TO FIND HALF OF THE BAR FORCES. ALL OF THE PAIRS MIRRORED IN THE D-E AXIS WILL HAVE THE SAME BAR FORCE:

$$F_{AC} = F_{GH} \qquad F_{BD} = F_{DF}$$
$$F_{AB} = F_{FH} \qquad F_{CD} = F_{DG}$$
$$F_{BC} = F_{FG} \qquad F_{CE} = F_{EG}$$

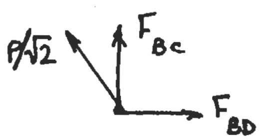
SOLVE FOR INDEPENDENT BAR FORCES:

MOJ @ A:



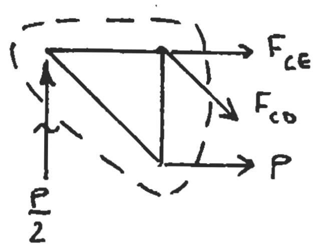
$$\Sigma F_y = \frac{P}{2} - F_{AB}\cos 45 = 0$$
$$F_{AB} = P/\sqrt{2}$$
$$\Sigma F_x = F_{AC} + F_{AB}\sin 45 = 0$$
$$F_{AC} = -\frac{P}{2}$$

MOJ @ B:



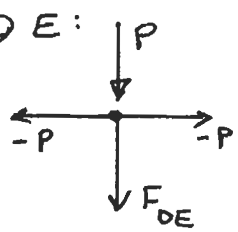
$$\Sigma F_y = 0 = F_{BC} + (P/\sqrt{2})\cos 45$$
$$F_{BC} = -P/2$$
$$\Sigma F_x = 0 = F_{BD} - (P/\sqrt{2})\sin 45$$
$$F_{BD} = P/2$$

MOS:



$$\Sigma F_y = 0 = \frac{P}{2} - F_{CD}\cos 45$$
$$F_{CD} = \frac{P}{\sqrt{2}}$$
$$\circlearrowleft M_D = 0 = -F_{CE}\cancel{L} - \frac{P}{\cancel{2}}(\cancel{2L}) = 0$$
$$F_{CE} = -P$$

MOJ @ E:

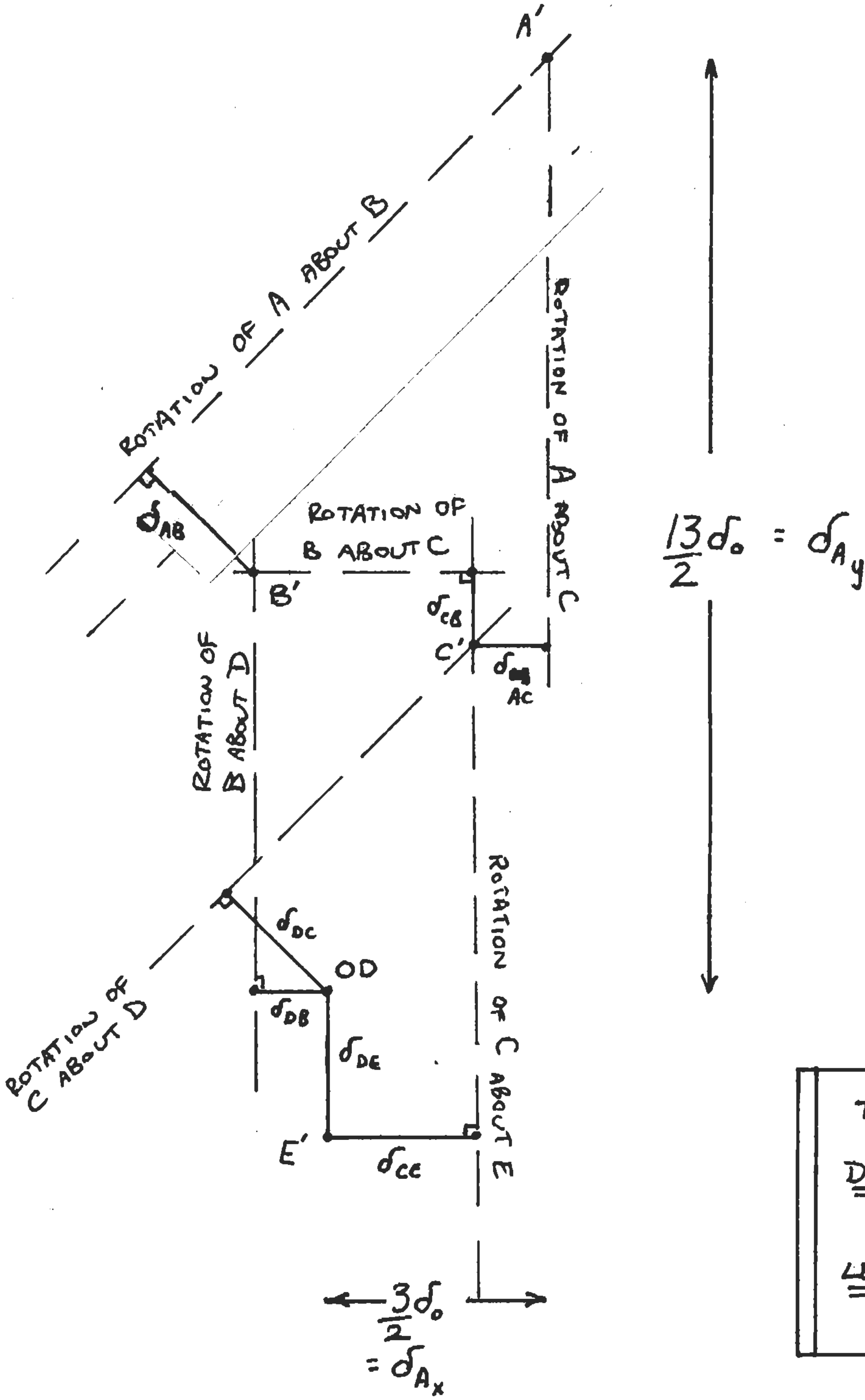


$$\Sigma F_y = 0$$
$$-P - F_{DE} = 0$$
$$F_{DE} = -P$$

BAR	FORCE $\left(\frac{F_{ij}}{P}\right)$	LENGTH $\left(\frac{L_{ij}}{L}\right)$	DEFORMATION $\delta_{ij}/\frac{PL}{AE}$
AB	$+1/\sqrt{2}$	$\sqrt{2}$	$+1$
AC	$-1/2$	1	$-1/2$
CB	$-1/2$	1	$-1/2$
CE	-1	1	-1
CD	$+\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$+1$
BD	$+\frac{1}{2}$	1	$+\frac{1}{2}$
ED	-1	1	-1
EG	-1	1	-1
DG	$+\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$+1$
DF	$+\frac{1}{2}$	1	$+\frac{1}{2}$
GF	$-1/2$	1	$-\frac{1}{2}$
GH	$-1/2$	1	$-\frac{1}{2}$
FH	$+1/\sqrt{2}$	$\sqrt{2}$	$+1$

NOW WE CAN GO AHEAD AND PLOT OUR TRUSS DEFLECTION DIAGRAM.

$\delta_o = \frac{PL}{AE}$



IF MY HINGE POINT A' ENDS UP DISPLACED FROM MY ORIGIN BY δ_{A_x} AND δ_{A_y}, THEN MY ORIGIN OD IS DISPLACED FROM A' BY $-\delta_{A_x}$ AND $-\delta_{A_y}$.

IF I NOW CONSIDER THE FIXED FRAME OA, WHERE A AND A' ARE THE SAME, I CAN FIND THE DEFLECTION OF D' IN THE FIXED FRAME, WHICH IS JUST ITS DISPLACEMENT FROM A', NAMELY

$$-\delta_{A_x}\hat{\imath} - \delta_{A_y}\hat{\jmath}.$$

THE JOINT D WILL TRANSLATE <u>DOWN</u> BY $\frac{13}{2}\frac{PL}{AE}$, AND <u>LEFT</u> BY $\frac{3}{2}\frac{PL}{AE}$

ESTIMATE OF TRUSS DEFLECTIONS

BARS IN EXPERIMENTAL TRUSS MADE OF STEEL
- HOLLOW WITH 22 mm OUTER DIAMETER AND
1.5 mm WALL THICKNESS (IGNORE END FITTINGS)

$$A \approx 2\pi r t \approx 10.5\times10^{-3} \times 2 \times \pi \times 1.5\times10^{-3}$$

$$\approx 100 \text{ mm}^2$$

$L = 0.5\text{ m}$

$E = 210 \text{ GPa}$

CENTER POINT DEFLECTION

$$\frac{\delta_D}{P} = \left(\frac{0.5}{100\times10^{-6} \times 210\times10^{9}}\right)\left(-\frac{13}{2}\hat{j} - \frac{3}{2}\hat{\imath}\right)$$

$$\boxed{\frac{\delta_D}{P} = -\frac{3.1}{2}\times10^{-7}\,\hat{j} - \frac{7.1}{2}\times10^{-8}\,\hat{\imath} \quad \frac{m}{N}}$$