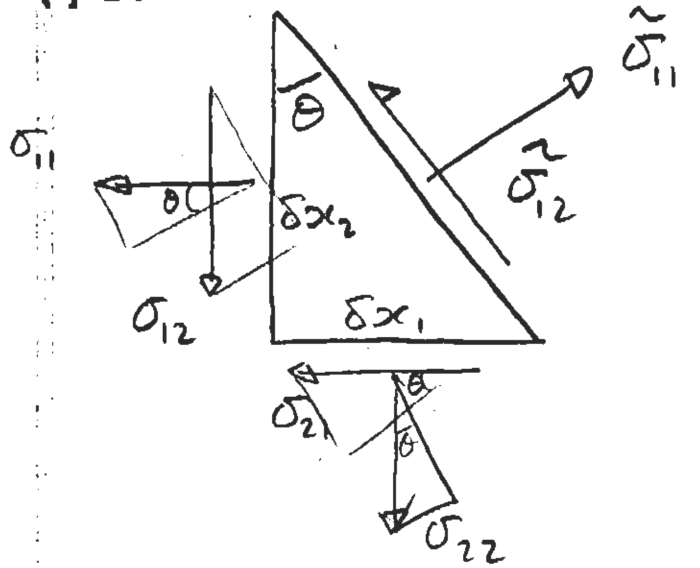
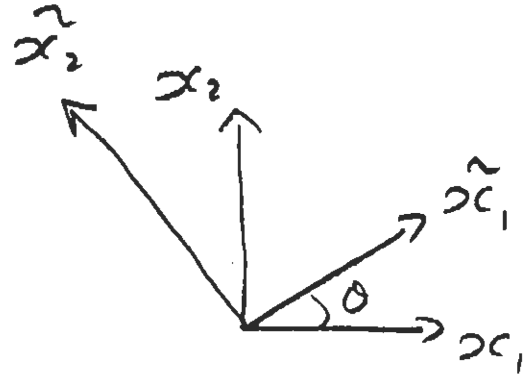
[redacted] M12





consider thickness δx_3

work in terms of $\delta\tilde{x}_1 \Rightarrow \delta x_1 = \delta\tilde{x}_1 \sin\theta$

$$\delta x_2 = \delta\tilde{x}_1 \cos\theta$$

$$\sum \vec{F}_1 = 0: \quad \tilde{\sigma}_{11}\,\delta\tilde{x}_1\,\delta x_3 - \sigma_{11}\,\delta\tilde{x}_1 \cos\theta\,\delta x_3 \cdot \cos\theta$$

$$- \sigma_{12}\,\delta\tilde{x}_1 \cos\theta \cdot \delta x_3 \cdot \sin\theta - \sigma_{22} \cdot \delta\tilde{x}_1 \sin\theta \cdot \delta x_3 \sin\theta$$

$$- \sigma_{21} \cdot \delta\tilde{x}_1 \sin\theta \cdot \delta x_3 \cos\theta = 0$$

δx_3's cancel, $\quad \sigma_{21} = \sigma_{12}$

$$\Rightarrow \tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta \cdot \sigma_{12}$$

and similarly for $\tilde{\sigma}_{12}$

$$\frac{d\tilde{\sigma}_{11}}{d\theta} = -2\sigma_{11}\cos\theta\sin\theta + 2\sigma_{22}\sin\theta\cos\theta$$

$$+ (2\cos^2\theta - 2\sin^2\theta)\,\sigma_{12} = 0$$

simplify $\cos\theta\sin\theta = \frac{1}{2}(\sin 2\theta)$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\frac{1}{2}(\cancel{1} + \cos 2\theta) - \frac{1}{2}(\cancel{1} - \cos 2\theta)$$

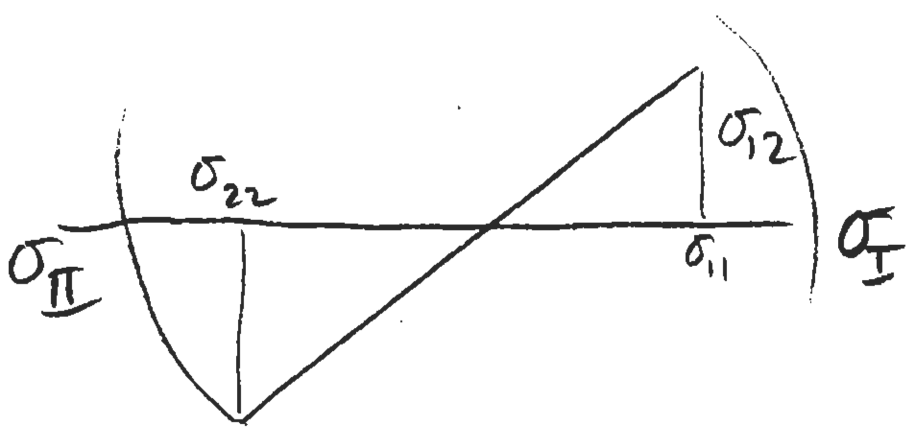
$$= \cos 2\theta$$

$$\Rightarrow \quad \sigma_{22}\sin 2\theta - \sigma_{11}\sin 2\theta + 2\cos 2\theta\,\sigma_{12} = 0$$

$$(\sigma_{22} - \sigma_{11})\tan 2\theta + 2\sigma_{12} = 0$$

$$\tan 2\theta = \frac{2\sigma_{12}}{(\sigma_{22} - \sigma_{11})}$$

cf. Mohr's circle



$$\sum \overset{\nwarrow}{F}_{\tilde{x}} = 0$$

$$\sigma_{12}\,\delta\tilde{x}_1\,\delta x_3 + \tilde{\sigma}_{11}\,\delta x_1 \cos\theta\,\delta x_3 \sin\theta$$

$$-\sigma_{12}\,\delta\tilde{x}_1 \cos\theta\,\delta x_3 \cos\theta - \sigma_{22}\,\delta\tilde{x}_1 \sin\theta \cdot \delta x_3 \cos\theta$$

$$+\sigma_{21}\,\delta\tilde{x}_1 \sin\theta\,\delta x_3 \sin\theta = 0$$

$$\Rightarrow \tilde{\sigma}_{12} = -\cos\theta\sin\theta\,\sigma_{11} + \cos\theta\sin\theta\,\sigma_{22}$$

$$+(\cos^2\theta - \sin^2\theta)\,\sigma_{12} \quad \Leftarrow$$