M19.

a) This is a transversely isotropic material
So it will require 5 elastic constants

b) Specimen (1) is loaded in the longitudinal fiber direction ε_L =

$$E_L = \frac{\sigma_L}{\varepsilon_L} = \frac{14\times10^3}{(10\times10^{-3})^2} \times \frac{1}{500\times10^{-6}} = 2.8\times10^{11} = 280\,GPa$$

$$\nu_{LT} = -\frac{\varepsilon_T}{\varepsilon_L} = \frac{-(-120)}{500} = 0.24$$

$$E_T = \frac{\sigma_T}{\varepsilon_T} = \frac{14\times10^3}{(10\times10^{-3})^2} \times \frac{1}{700\times10^{-6}} = 2.0\times10^{11} = 200\,GPa \Subset$$

$$\nu_{TL} = -\frac{\varepsilon_{\text{gauge c}}}{\varepsilon_{\text{gauge d}}} = \frac{-(-125)}{700} = 0.18 \Subset$$

$$\nu_{TT} = -\frac{\varepsilon_t}{\varepsilon_d} = -\frac{210}{700} = 0.3$$

Hence $G_{TT} = \frac{E_{TT}}{2(1+\nu_{TT})}$ = 77 GPa $\Subset$

c) from longitudinal modulus:

$$E_L = V_f E_f + (1 - V_f) E_m$$

$$E_L - V_f = V_f (E_f - E_m) + E_m$$

$$\frac{E_L - E_m}{E_f - E_m} = V_f = \frac{280 - 110}{450 - 110} = 0.5 \Leftarrow$$

for transverse modulus, lower bound estimate

$$E_T = \frac{1}{\frac{V_f}{E_f} + \frac{1 - V_f}{E_m}}$$

$$\frac{E_T V_f}{E_f} + \frac{E_T (1 - V_f)}{E_m} = 1$$

$$E_T E_m V_f + E_T E_f (1 - V_f) = 1$$

$$V_f (E_T E_m - E_T E_F) = -E_T E_f$$

$$V_f = \frac{-E_T E_F}{E_T E_m - E_T E_f} = \frac{-200 \times 450}{200 \times 110 - 200 \times}$$

$$= \frac{E_f}{E_f - E_m} = \frac{450}{450 - 110} = \frac{450}{340} = 1.32$$

This is greater than 0.5 so not inconsistent!