Solution to T1 by Waitz. (Unified Thermodynamics)
Below is a schematic diagram of a helium-pressurized, bipropellant rocket engine system (for the fourth stage of the Peacekeeper ballistic missile). Describe the conversions of energy (internal, potential, kinetic and chemical) and exchanges of heat and work for this system <u>and</u> that of the rocket itself. Note at which points in the process heat is transferred to/from the surroundings. (LO#1, LO#2, LO#3)

It is understood at this point in Unified that you have only had one or two lectures on thermodynamics, and no propulsion and fluid mechanics lectures. So what is expected is that you recognize conversions between various forms of energy when they occur and know then difference between transfers of energy called "work" and transfers of energy called "heat".

For the gases in the system, changes in potential energy will typically be small compared to the changes in internal energy, kinetic energy and chemical energy due to their low density. Internal energy is stored in the high pressure helium. The helium expands, pushing against (i.e., applying a force over a distance or doing work on) the propellants. As the helium expands, its internal energy is reduced. Energy may also be exchanged with the surroundings through heat transfer if the high pressure helium tank and the tubing and valves are not thermally-insulated. The work from the helium probably does not raise the internal energy (pressure and temperature) of the propellants very much since there is a pressure regulator in the system, However, it does cause the propellants to move (i.e., giving the propellants kinetic energy). The propellants flow into a combustion chamber where they react, converting chemical energy stored in the propellants into internal energy (high temperature and pressure) in the gases formed as products of the reaction. Here again, energy may be exchanged with the surroundings through heat transfer depending on how well thermally-insulated the combustion chamber and nozzle are. These high temperature and high pressure gases are then accelerated through an exhaust nozzle (exchanging internal energy for kinetic energy). As they pass through the exhaust nozzle they push on the walls of the nozzle doing work on the rocket. The kinetic and/or potential energy of the rocket increase as a result. Any energy left in the exhaust (internal and kinetic) is lost to the atmosphere which sees a very slight increase in internal energy (e.g. temperature) as a result of the whole process.

Solution to U1 by Waitz. (Range Equation)

a) Assuming steady-level flight and no fuel reserves, estimate the range of a B-777 using the information given in the lecture notes (and/or on Boeing's web page). How well does this compare to the estimates Boeing publishes on their web page?

BASIS FOR COMPARISON: BOEING 777-200/200IGW
Max. take-off mass ≈ 275,000 kg
Typ. operating empty mass ≈ 144,000 kg
Max. fuel capacity ≈ 171,000 liters (137,000 kg kerosine)
Cargo volume ≈ 160,000 liters
Takeoff thrust ≈ 760 kN
Cruise thrust ≈ 160 kN
Design range ≈ 9600 to 13800 km
Passengers ≈ 330
Length ≈ 63.7m, wingspan = 60.9m
L/D ≈ 18, η_o ≈ 0.36
Cost ≈ $140 million

Using:

$$\text{Range} = \frac{h}{g} \; \frac{L}{D} \; \eta_{overall} \; \ln \frac{W_{initial}}{W_{final}}$$

You get about 18000km using the ratio of the operating empty mass and the max takeoff mass (1.9). The estimate of 18000km is more than 30% too high, but I did neglect the weight of the passengers and their cargo, food (such as it is), and reserve fuel. When these items are taken into account the estimate is within 10% of the published values.

b) Now assuming that L/D, propulsion system efficiency and final weight are unchanged, estimate the range of a B-777 if the same volume of liquid hydrogen were to be used instead of Jet-A.

To do this I wrote $W_{final}=W_{initial}-W_{fuel} = W_{initial}-\rho_{fuel} \, V_{fuel}$. The ratio of the two densities is 0.0875. So the initial weight is only 156,000 kg (144,000kg + 0.0875x137,000kg), and the weight ratio drops to 1.08. Of course the heating value is increased by a factor of 2.8, but it hardly makes up for the reduction in the amount of energy that is carried due to hydrogen's low density. My estimate for the range is 6100km, a reduction by a factor of three from the case with Jet-A.

c) Derive an equation for the range of a battery-powered aircraft in steady-level flight. Express the range in terms of L/D, propulsion system efficiency, battery mass and heating value, and aircraft weight. Estimate the range of a B-777 if the fuel was taken out and replaced with its equivalent weight in batteries.

The key with a battery-powered aircraft is that its mass does not change as it burns the energy. This makes the range equation more straightforward.

$m_b \cdot h = $ *energy available in the battery* (J)

$\dfrac{T \cdot u_o}{\eta_{overall}} = $ *rate of energy usage to overcome drag* (J/s)

time of flight $= \dfrac{m_b \cdot h}{\dfrac{T \cdot u_o}{\eta_{overall}}}$ $\quad (s)$

Range of flight $= u_o \left(\dfrac{m_b \cdot h}{\dfrac{T \cdot u_o}{\eta_{overall}}} \right)$ $\quad (m)$

or

Range of flight $= \dfrac{m_b \cdot h \cdot \eta_{overall}}{T} = \dfrac{m_b \cdot h \cdot \eta_{overall}}{W}\left(\dfrac{L}{D}\right)$ $\quad (m)$

With m_b = 137,000kg, h=2.5MJ/kg, W=(275,000kg)(9.8m/s^2)=2695kN, I calculate the range to be: 820km. As you can see, the low energy density of the battery is a disaster for range—it is reduced by a factor of more than 20 relative to the Jet-A powered model.

"FUEL"	Heating Value (MJ/kg)	Density (kg/m^3)
Jet-A	42.8	800
Liquid Hydrogen	120	70
Batteries	2.5	8000