

February 27, 2004

UNIFIED QUIZ 1MS**(Beam Handout, (equations of elasticity handout available on request) no books, no notes; calculators allowed)**

- Put your name on the top page, and on each subsequent page of the exam if you disassemble the pages.
- Read all questions carefully.
- Do all work on that question on that page. Use back of that page if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the actual answers.*
- Be sure to show the units as well (if necessary). Final answers are not correct without the units.
- State any assumptions you are making
- Report significant digits only.
- Box your final answers.

EXAM SCORING

#1 (24%)	
#2 (25%)	
#3 (26%)	
#4 (25%)	
FINAL SCORE	

PROBLEM #1 (24%)

The questions that follow should require only a few sentences as answers, or very simple calculations. Provide sufficient detail to support your answer, but be brief.

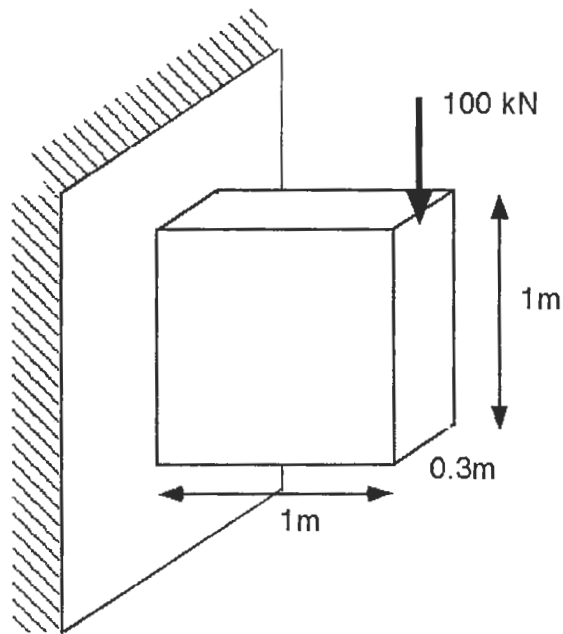
- a.) Structural steel girders and aluminum wing spars are often made with an "I" cross section rather than a solid rectangular cross-section, Why is this?

The stiffness and strength of a beam are governed by the second moment of area, I , $(\int z^2 dA)$ via $M = EI \frac{d^2w}{dx^2}$ and $\sigma_{xx} = -\frac{Mz}{I}$.

Stronger, stiffer beams for a given material choice will have a higher value of I .

An I section achieves this by moving material away from the (unloaded) neutral axis to form the flanges (spar caps).

- b Estimate the maximum tensile stress in the structure below, loaded by a load of 100 kN.



This is NOT a beam ($L \approx h$)

Still need to provide an estimate, only have

$$\sigma_{xx} = -\frac{Mz}{I} = \frac{M h/2}{\frac{1}{12}bh^3} = \frac{6PL}{bh^2} = \frac{6 \times 100 \times 10^3 \times 1}{0.3 \times 1^2} = 2 \times 10^6 \text{ Pa} \Leftarrow$$

$$\left(\text{Shear stress} \sim \frac{P}{bh} = 0.33 \times 10^6 \text{ Pa} \right)$$

- c Aircraft structures are designed according to requirements of high strength and stiffness and low mass. Steel, aluminum and titanium alloys all have virtually identical stiffness to density ratios (E/ρ) and strength to density ratios (σ/ρ). And yet aluminum is the material predominantly used for commercial transport airframes. Why?

For stiffness and strength in bending, which is the main design driver for wings and fuselages, need to maximize E/ρ and σ_f/ρ .

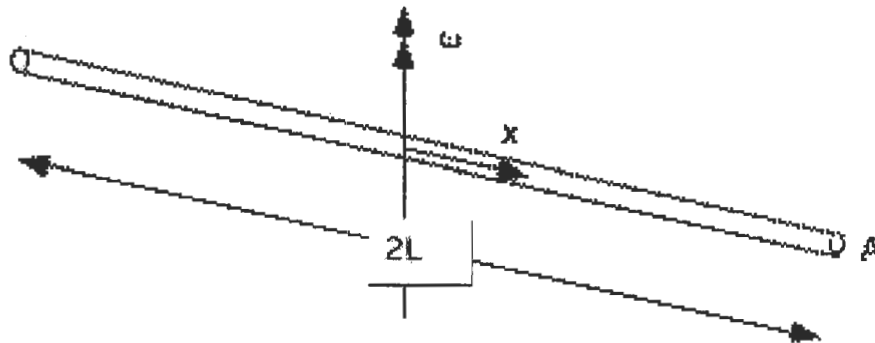
If Al, Ti, steel all have same E/ρ & σ_f/ρ then the material with the lowest density, ρ , will perform best in bending. This is Al.

d Simple beam theory is derived based on the assumption "plane sections remain planar and perpendicular to the midplane of the beam" This only strictly applies to beams under pure bending (i.e with no shear forces). In general beams will carry shear forces. Justify the applicability of simple beam theory to cases where shear forces are present.

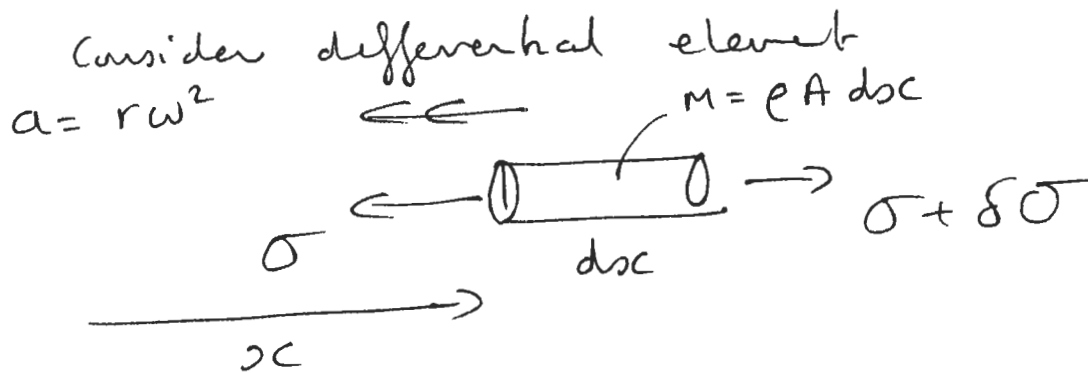
Shear stresses (and shear strains) are an order of magnitude smaller than the bending, σ_{xx} , stresses and strains. Therefore the solutions for pure bending apply even when there is some shear. As beams become shorter and thicker the shear stresses will become more significant.

PROBLEM #2 (25%)

A uniform bar of length $2L$ and cross sectional area A (with a solid circular cross-section), made of a material of density ρ , Young's modulus E and yield stress, σ_y , rotates at constant angular velocity ω about an axis through its center, perpendicular to its length (see figure below). You should assume that no transverse forces or moments or torques act on the bar.



- a) Determine and sketch $F(x)$, the internal axial force distribution along the bar. Hint. The centrifugal force, F , required to keep a point mass m rotating in a circle radius r at a speed ω is $F = mr\omega^2$



$$(\sigma + \delta\sigma)A - \sigma A = -\rho A dx \cdot r\omega^2$$

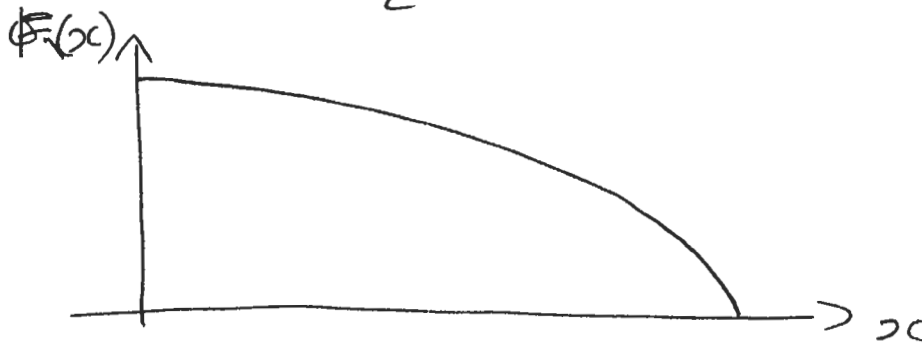
$$\frac{d\sigma}{dx} = -\rho x\omega^2$$

$$\sigma = -\int \rho\omega^2 x dx = -\frac{\rho\omega^2 x^2}{2} + A$$

at $x=L$ $\sigma=0$: $A = \frac{\rho L^2 \omega^2}{2}$
free end

$$\therefore \sigma = \frac{\rho L^2 \omega^2}{2} \left(1 - \left(\frac{x}{L}\right)^2\right)$$

$$\text{or } F(x) = \frac{\rho A L^2 \omega^2}{2} \left(1 - \left(\frac{x}{L}\right)^2\right)$$



b) Calculate the extension of the bar.

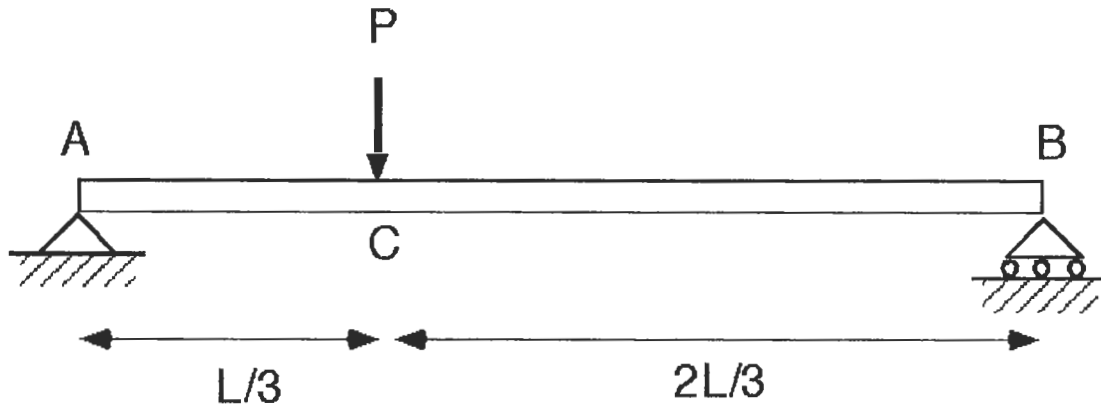
Strain $\epsilon_{xx} = \frac{\sigma_{xx}}{E}$, displacement = $\frac{1}{E} \int_0^L \frac{\rho L^2 \omega^2}{2} \left(1 - \frac{x^2}{L^2}\right) dx$

$$U(x=L) = \frac{\rho L^2 \omega^2}{2E} \left[x - \frac{x^3}{3L^2} \right]_0^L$$

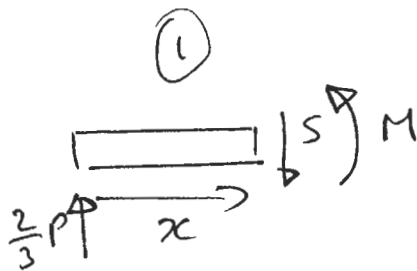
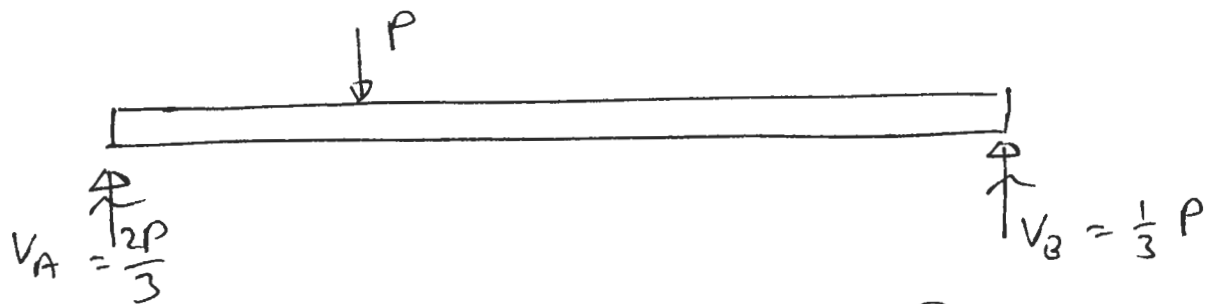
$$U(x=L) = \frac{\rho L^2 \omega^2}{2E} \left[L - \frac{L^3}{3L^2} \right] = \frac{\rho L^3 \omega^2}{3} \quad \Leftarrow$$

PROBLEM #3 (26%)

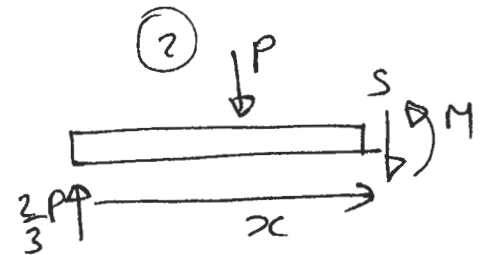
A simply supported beam is loaded by a point load P , $1/3$ of the way along its length as shown below:



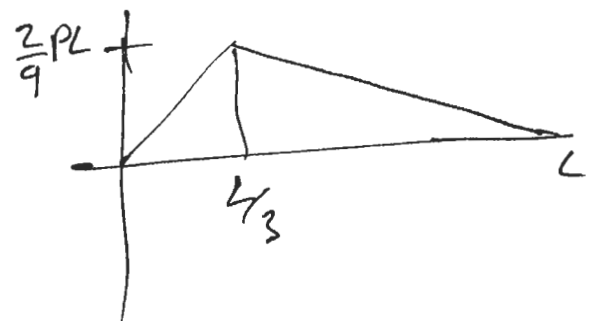
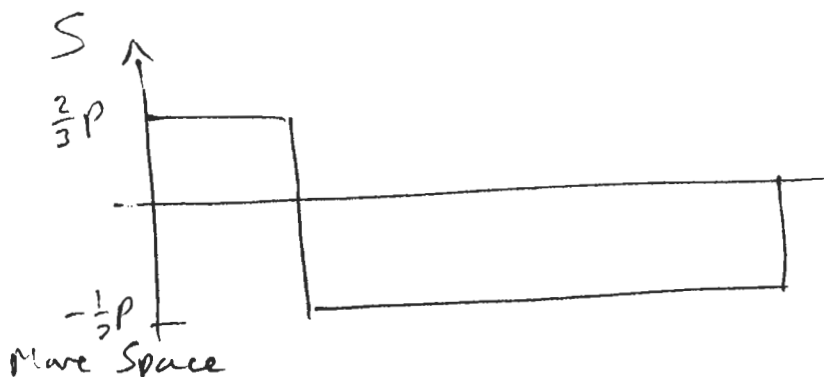
a) Calculate and sketch the bending moment and shear force distributions as a function of position along the beam. Note the maximum values of each and their locations.



$$S = \frac{2}{3}P, \quad M = \frac{2}{3}Px$$



$$S = \frac{2}{3}P - P = -\frac{1}{3}P \quad M = \frac{2}{3}Px + P\left(\frac{L}{3} - x\right)$$



- b) The beam has a rectangular cross-section, height h , breadth b . It is made of a material with a Young's modulus E and a yield stress, σ_y . Outline the process you would use to calculate the deflection of point C, the loading point. Write down all relevant equations and quantities where known, but do not solve the resulting equations.

Calculate 2nd moment of area $I = bh^3$

from moment - curvature $M = EI \frac{d^2w}{dx^2}$

Integrate moment-curvature relationship twice for each segment of beam. (1) & (2)

Obtain two constants of integration for each segment (four in total)

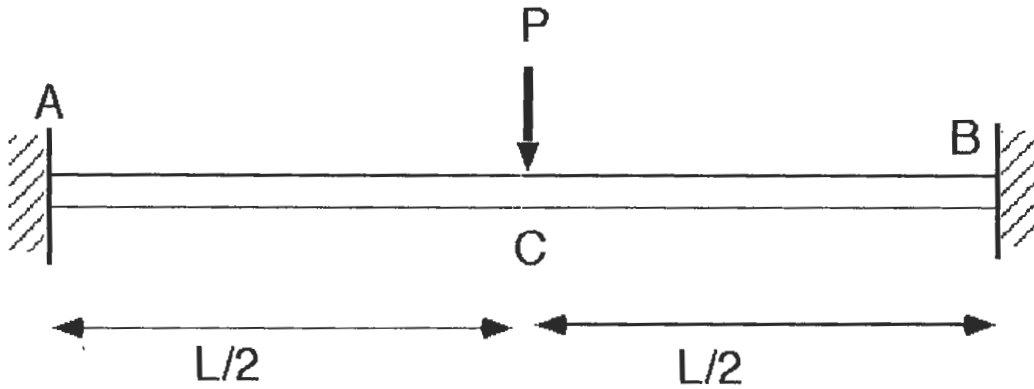
Solve by setting $w = 0$ @ $x = 0, L$

and by matching slope and deflection for each segment at $x = \frac{L}{3}$

Insert $x = \frac{L}{3}$ back into the expression for $w(x)$ and calculate $w(\frac{L}{3})$

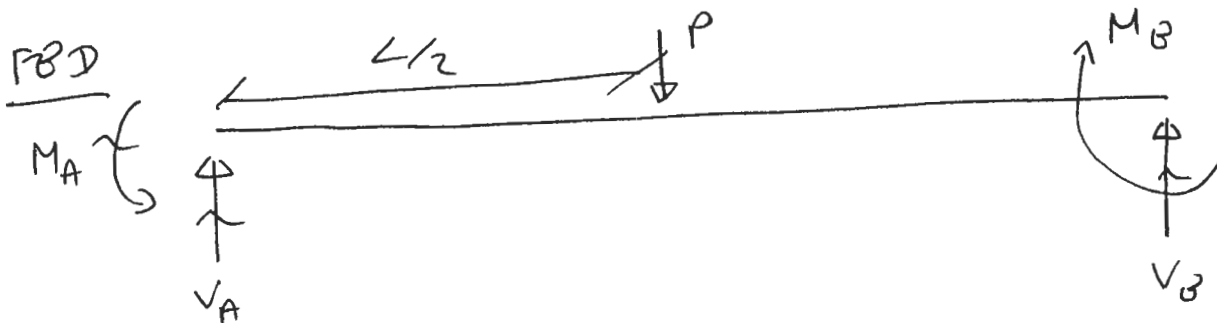
PROBLEM #4 (25%)

A beam of length L and flexural rigidity EI is clamped at each end. The beam has a point load of magnitude P applied at the midpoint of the beam.



Outline a solution approach to calculate the deflection of point C of the midpoint of the beam. Write down the relevant (final) equations you will need to use, describe how you would use them, but do not solve them.

This is a statically indeterminate beam. Could use superposition. Might be easiest to proceed with simultaneous application of equilibrium, compatibility & constitutive behavior



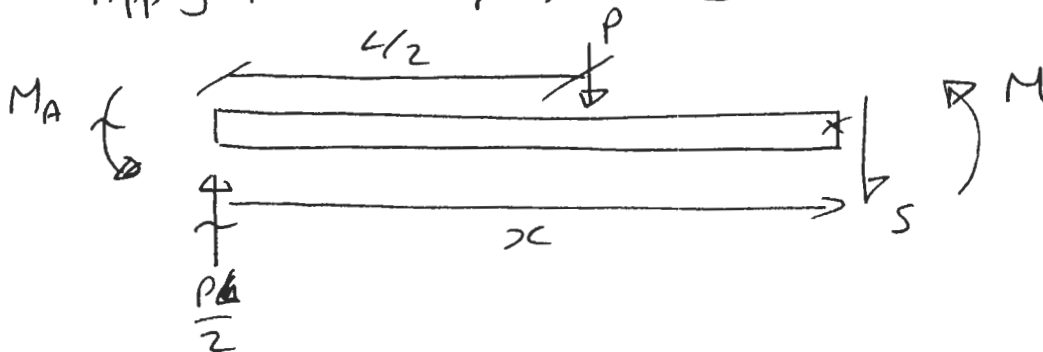
$$\sum F_z \uparrow = 0 \quad V_A + V_B - P = 0 \quad (\text{e by symmetry } V_A = V_B = \frac{1}{2}P) \quad (1)$$

$$\left(\sum M_A = 0 : M_A - \frac{PL}{2} + M_B + \frac{P_B L}{2} = 0 \right) \quad (2)$$

$$\therefore M_A = M_B \quad \Leftarrow$$

PROBLEM #4 (25%)

Apply method of sections



Use Macaulay's method / integration

$$\left(\sum M_x = 0 : M + M_A - \frac{P x}{2} + P \{x - L\} = 0 \Rightarrow M = -M_A + \frac{P x}{2} - P \{x - L\} \right) \textcircled{3}$$

Apply $EI \frac{d^2 w}{dx^2}$

$$EI \frac{dw}{dx} = -M_A x + \frac{P x^2}{4} - \frac{P \{x - L\}^2}{2} + A \textcircled{4}$$

$$EI w = -\frac{M_A x^2}{2} + \frac{P x^3}{12} - \frac{P \{x - L\}^3}{6} + A x + B \textcircled{5}$$

apply B.C's. $w = 0$ @ $x = 0, x = L$

$$\frac{dw}{dx} = 0 \text{ @ } x = 0, x = L$$

Solve for A, B & $M_A \rightarrow = M_B$

Calculate $w(x = \frac{L}{2})$ via $\textcircled{5}$.