16.06 Principles of Automatic Control

Problem Set 1

Issued: September 7, 2011
Due: September 14, 2011

Instructions: Do each problem on separate sheets of paper, and staple the sheets for each problem together. Write your name on each problem.

Problem 1 — Review of Laplace Transforms

Laplace Transforms: If necessary, read Appendix A of FPE for a review of Laplace transforms.

1. Find the Laplace transform of the function

$$g(t) = te^{-2t}\sigma(t)$$

   where $\sigma(t)$ is the unit step function. Note: FPE uses the notations $1(t)$ instead of $\sigma(t)$.

2. Find the inverse LT of the transform

$$G(s) = \frac{6s^2 - 2s - 2}{s(s^2 - 1)}, \qquad \mathrm{Re}[s] > 1$$

3. Use the Final Value Theorem to find

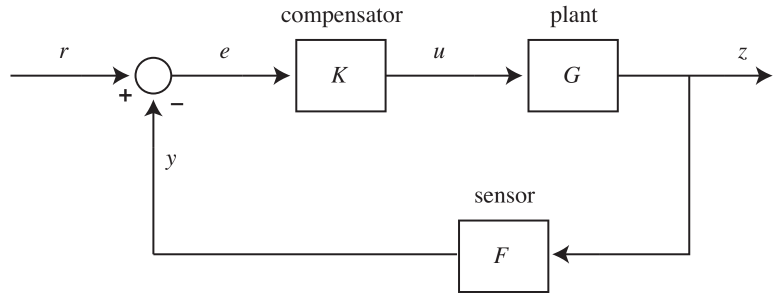
$$\lim_{t \to \infty} y(t)$$

   if $y(t)$ has transform

$$Y(s) = \frac{s^2 + 4s + 6}{s(s+1)(s+3)}, \qquad \mathrm{Re}[s] > 0$$

Problem 2

Consider the feedback system block diagram shown below:



1. Find the transfer function from r to z, given by

$$H(s) = \frac{Z(s)}{R(s)}$$

2. We are often interested in how sensitive a control system is to variations in components of the control system. For example, the plant may be expected to have transfer function G, but in fact have transfer function $G + \delta G$. These variations are often expressed not as an absolute variation, say, δG, but as a relative or percentage change, $\frac{\delta G}{G}$. The sensitivity function S_G describes the percent change in the closed-loop transfer function H for a given percent change in the transfer function G. Find the sensitivity

$$S_G = \frac{\delta H/H}{\delta G/G} = \frac{\partial H}{\partial G}\frac{G}{H}$$

Make sure to simplify your expression as much as possible. In order for the closed-loop transfer function to be relatively insensitive to variations in G, what property should the loop gain KGF have?

3. Find the sensitivity

$$S_K = \frac{\delta H/H}{\delta K/K} = \frac{\partial H}{\partial K}\frac{K}{H}$$

to changes in the compensator, again simplifying as much as possible. How does it compare to S_G?

4. Find the sensitivity

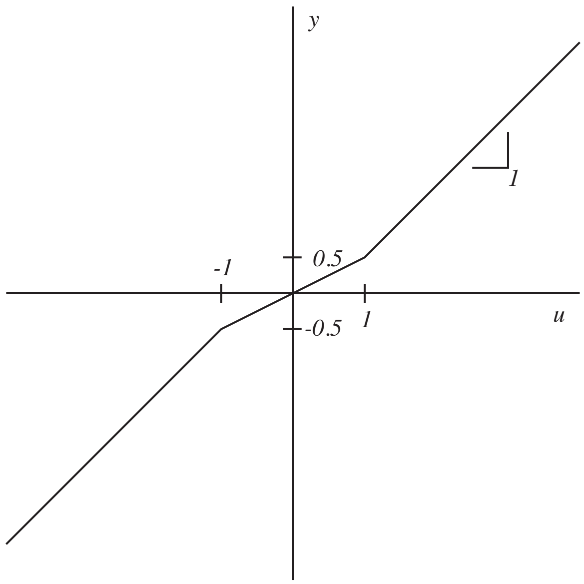
$$S_F = \frac{\delta H/H}{\delta F/F} = \frac{\partial H}{\partial F}\frac{F}{H}$$

to changes in the sensor gain and simplify. Is the sensitivity greater for small loop gain or large loop gain?

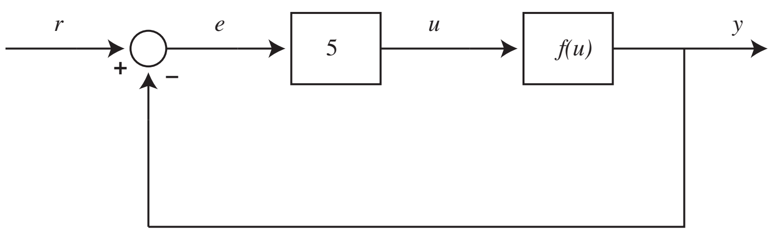
5. If you were designing a control system that is supposed to have a specific closed-loop transfer function, would it be more important to have a precise actuator (one that has small variations from its expected transfer function) or a precise sensor?

Problem 3

Feedback can be used to improve the linearity of systems with nonlinearities. For example, consider an audio power amplifier, with input u and output y, which has a static respones (*i.e.*, the response does not have any dynamics) given by $y = f(u)$, where the nonlinear function f is given as in the plot below:



A unity feedback control system is used to improve the linearity of the system, as shown in the block diagram below.



Plot the closed-loop response, that is, plot y as a function of r. Describe in words the degree to which feedback did or did not improve the linearity of the system.

Problem 4

Do Problem 2.9 from FPE.

Problem 5

Do Problem 2.5 from FPE.

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