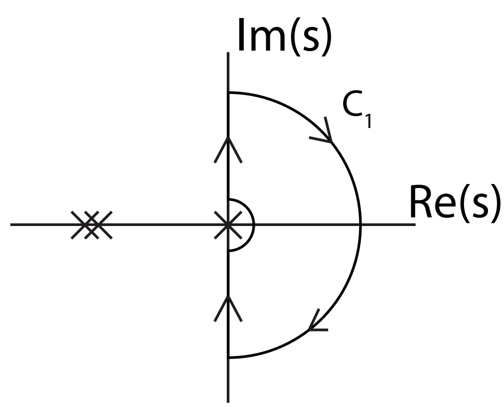
16.06 Principles of Automatic Control
Lecture 22

Nyquist Plot for $G(s)$ with $j\omega-$axis poles

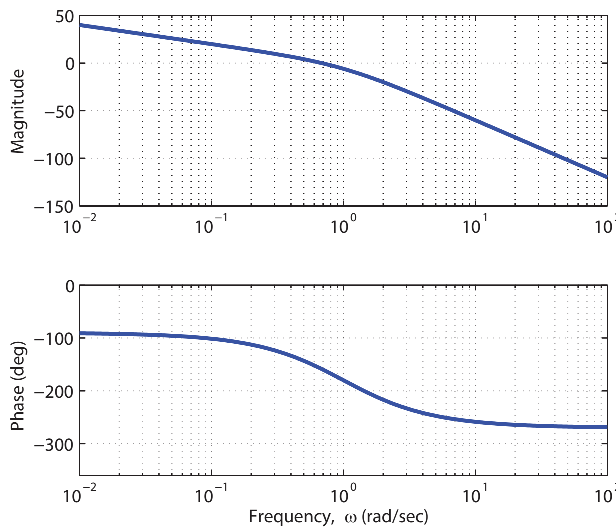
Consider

$$G(s) = \frac{1}{s(s+1)^2}$$

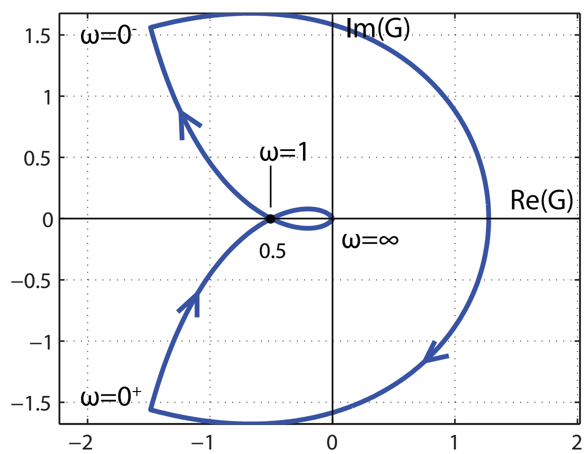
Because of pole at $s = 0$, must deform "D contour" (C_1).



Bode:



Nyquist:



Note that deformation in contour (small semicircle in C_1) maps to large semicircle in $G(C_1)$. Since there are no open loop poles inside C_1, the number of closed loop poles is

$$
\begin{aligned}
&2, \quad \text{if} \quad -0.5 < -1/k < 0 \qquad && (k > 2) \\
&1, \quad \text{if} \quad 0 < -1/k < \infty \qquad && (k < 0)
\end{aligned}
$$

This result is of course in agreement with Routh, root locus.

A note on drawing the Nyquist diagram:

As $\omega \to 0^+$, note that the Nyquist diagram is asymptotic to the vertical line $Re(s) = -2$. Since the phase at zero frequency goes to $-90°$, it seems that the diagram should be asymptotic to the imaginary axis. Why isn't it?

Express $G(s)$ as:

$$G(s) = \frac{1}{s(s+1)^2} = \frac{1}{s} \cdot \frac{1}{s^2 + 2s + 1}$$

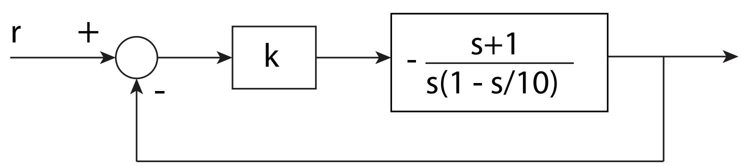
For small s, can express as series around $s = 0$:

$$\begin{aligned} G(s) \approx& \frac{1}{s}(1 - 2s + O(s^2)) \\ =& \frac{1}{j\omega} - 2 + O(\omega) \end{aligned}$$

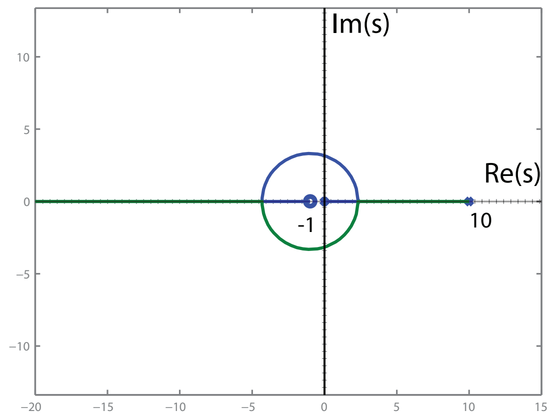
So the diagram is asymptotic to $\frac{1}{j\omega} - 2$.

Nyquist Plot of Open Loop Unstable System

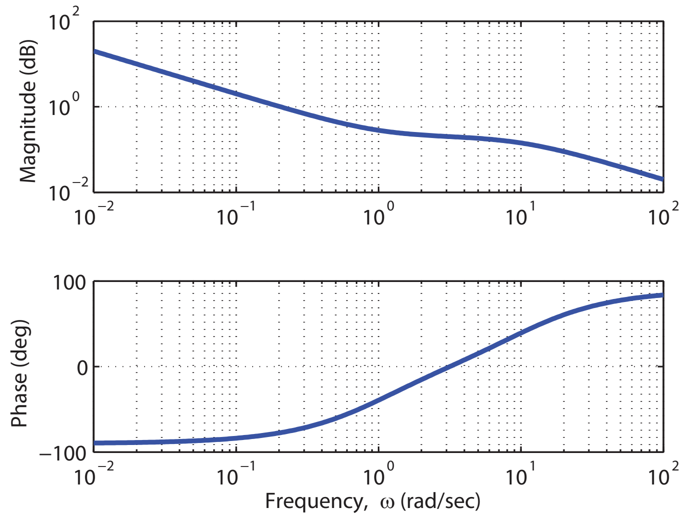
Now consider the proportional control of an unstable system:



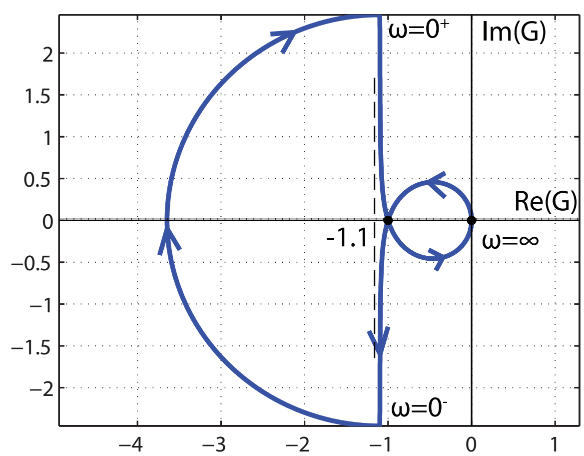
The root locus:



Bode:



Nyquist diagram:



Note that arc at ∞ is clockwise, because deformation at $s = 0$ around pole is counter-clockwise.

Since there is one open loop pole in right hand plane, need one counter-clockwise encirclement for stability.

$$\begin{aligned} Z &= N + P \\ 0 &= -1 + 1 \end{aligned}$$

where 0 means no closed loop poles,
-1 means counter-clockwise encirclement,
$+1$means right-half-plane open-loop or pole.

So system is stable for:

$$\begin{aligned} -1 <& -1/k < 0 \\ \Rightarrow k >& 1 \end{aligned}$$

Also, note that for $-1/k < -1 \quad (0 < k < 1), \quad N = 1$, so the number of unstable closed-loop poles is:

$$\begin{aligned} Z =& N + P \\ =& 1 + 1 \\ =& 2 \end{aligned}$$