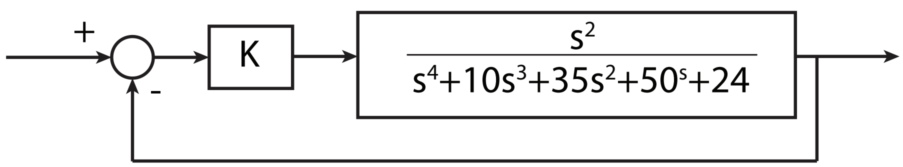
16.06 Principles of Automatic Control
Recitation 3

Routh Array



Part 1.

For the feedback system above, find the values of K that make system stable/unstable.

For unstable system. determine # of RHP poles.

$$H(s) = \frac{Ks^2}{s^4 + 10s^3 + (35 + K)s^2 + 50s + 24}$$

Routh Array:

s^4 :	1	$35 + K$	24
s^3 :	10	50	0
s^2 :	$30 + K$	24	0
s^1 :	$\frac{1260+50K}{K+30}$	0	0
s^0 :	24	0	0

In order for the system to be stable, we need all elements in the first column to be positive, but the elements in the first column will change depending on K. Start by finding critical values for K :

$K + 3- > 0 \rightarrow K > -30$

$\frac{1260+50K}{30+K} > 0 \rightarrow K > -25.2$

Now consider the sign on the first elements of the first column for different ranges of K:

$K < -30$	$-25.2 < K < -30$	$K > -25.2$
+	+	+
+	+	+
−	+	+
+	−	+
+	+	+
2 sign changes	2 sign changes	no sign changes
2 RHP poles	2 RHP poles	no RHP poles
unstable	unstable	stable

Part 2.

$$s^3 - 3s + 2$$

Routh array:

$$\begin{array}{lll} s^3: & 1 & -3 \\ s^2: & 0 & 2 \\ s & & \\ 1 & & \end{array}$$

Here, in the second row, we have a "0" in the first column. Since we can't divide by "0", we replace that entry by a small constant ϵ and proceed.

$$\begin{array}{lcc} s^3: & 1 & -3 \\ s^2: & \epsilon & 2 \\ s: & -3-2/\epsilon & 0 \\ & 2 & 0 \end{array} \rightarrow \begin{array}{cc} \epsilon > 0 & \epsilon < 0 \\ + & + \\ + & - \\ - & + \\ + & + \end{array} \rightarrow \text{2 sign changes} \rightarrow \quad \text{2 RHP poles}$$

One can factor the polynomial and check:

$s^3 - 3s + 2 = (s-1)^2(s+2) \rightarrow$ poles at:$s = 1, 1, -2$.

Indeed, we have 2 RHP poles.

Part 3.

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50$$

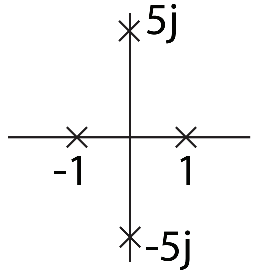
s^5 :	1	24	-25					
s^4 :	2	48	-50		$+$			
s^3 :	0	0	0		$+$			
s^3 :	8	96	0	$\rightarrow$	$+$	$\rightarrow$	One sign change	$\rightarrow$ 1 RHP pole.
s^2 :	24	-50			$+$			
s :	112.7	0			$+$			
1 :	-50				$-$			

For row 3, we get the entire row of zeros. Take previous characteristic equation (s^4): auxiliary poles.

$P(s) = 2s^4 + 48s^2 - 50$, differentiate this:

$\frac{\partial P}{\partial s} = 8s^3 + 96s$. We use this results' coefficients in the s^3 row.

Roots of auxiliary polynomial are shown below:



$s^2 = 1 \rightarrow s = \pm 1$

$s^2 = 25 \rightarrow s = \pm 5j$

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