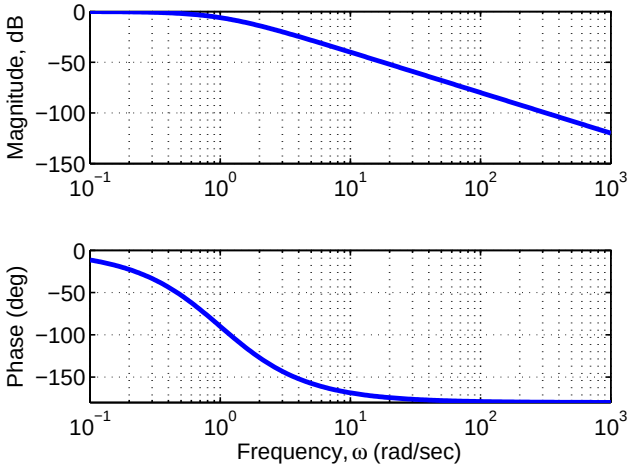
16.06 Principles of Automatic Control
Recitation 8

Given $G(s) = \frac{1}{(s+1)^2}$, design a PD controller so that $\omega_c = 10\,\text{rad/sec}$, and PM $= 50^\circ$.



$K(s) = K\,(1 + s/a)$

Have to decide values for a, k. To choose a, we know that we want PM $= 50$, but have to consider phase from poles and zeros at $\omega_c = 10$.

$-180^\circ + 50^\circ = -130^\circ = \tan^{-1}\frac{\omega_a/a}{1} - 2\tan^{-1}\frac{\omega_c}{1}$

$\Rightarrow a = 12.5$

Now we need to choose K.

$$\begin{aligned} \left|K(j\omega_c)G(j\omega_c)\right| &= 1 \\ \frac{K\sqrt{1^2 + 10^2/12.5^2}}{\left(\sqrt{10^2+1^2}\right)^2} &= 1 \end{aligned}$$

$$K = \frac{1}{0.0127}$$
$$K = 78.87$$

$$\boxed{K(s) = 78.87\,(1 + \tfrac{s}{12.5})}$$

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