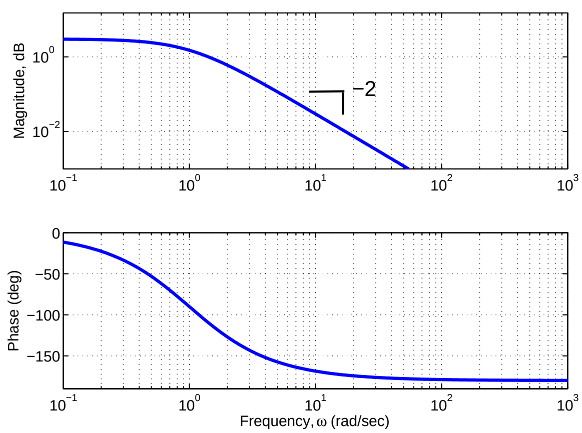
16.06 Principles of Automatic Control
Recitation 9

Design a lead compensator for $G(s) = \frac{3}{(s+1)^2}$ such that:

$\omega_c = 50\,\mathrm{rad/sec}$

$\mathrm{PM} = 50^\circ$

$K(s) = K\frac{1+s/a}{1+s/b}$
Bode Plot:



We want $\omega_c = 10\,\mathrm{rad/sec}$ and $\mathrm{PM} = 50^\circ$. Our lead compensator will add phase to the system, but we need to specify how much phase to add.

Current phase at ω_c is
$-2\tan^{-1}\left(\sqrt{\frac{b}{a}}\right) - 90^\circ = 38.6^\circ$

$$\boxed{\sqrt{\frac{b}{a}} = 2.08 \ldots (1)}$$

Strategically place pole and zero symmetrically about crossover (in log scale):

$$\boxed{\omega_c = \sqrt{ab} = 10 \ldots (2)}$$

where $\sqrt{ab}$ is the geometric mean.

Now solve (1) and (2) for a and b:

$$a = 4.8$$
$$b = 20.8$$

Now, to find the gain, use the condition that $|K(j\omega_c)G(j\omega_c)| = 1$:

$$\frac{K \cdot 3\sqrt{1^2 + \left(\frac{10}{4.8}\right)^2}}{\underbrace{\left(1^2 + 10^2\right)}\sqrt{1^2 + \left(\frac{10}{20.8}\right)^2}} = 1$$

$\hookrightarrow$ double pole

$$K \approx 16.1$$

$$\boxed{K(s) = 16.1 \cdot \frac{(1+\frac{s}{4.8})}{(1+\frac{s}{20.8})}}$$

Now, we add the requirement of having $K_p = 200$.

We need to design an additional lag compensator, but this will add about 6° phase lag at ω_c.

So we need to redesign our lead compensator to account for this 6°phase lag:

$\phi_{\max} = 38.6^\circ + 6^\circ = 44.6^\circ = 2\tan^{-1}\left(\sqrt{\frac{b}{a}}\right) - 90^\circ$

$\therefore \sqrt{\frac{b}{a}} = 2.39 \rightarrow$ see equation (2) above: $\omega_c = \sqrt{ab} = 10$.

$b = 23.9$

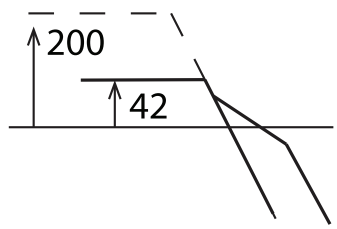
$a = 4.18$

and we find out new $K = 14.07$.

$$K_p = \lim_{s\to 0}\left(14.07\,\frac{1+\frac{s}{4.18}}{1+\frac{s}{23.9}}\right)\cdot\frac{3}{(s+1)^2} = 42.2$$

Need our lag ratio to be

$$\frac{K_{p\ \text{desired}}}{K_{p\ \text{current}}} = \frac{200}{42.2} = 4.74$$



We can place the zero of the lag compensator $\sim 1^\circ$ below:

$$\omega_c = 10 \rightarrow \frac{s+1}{s+?} \rightarrow \text{set pole to satisfy the lag ratio}$$
$$\text{of 4.74; i.e. at 0.21.}$$

$$\therefore K(s) = 14.07\,\frac{1+s/4.18}{1+s/23.9}\cdot\frac{s+1}{s+0.21}$$
$$\text{PM} = 51.5^\circ$$
$$\omega_c = 10.0$$

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