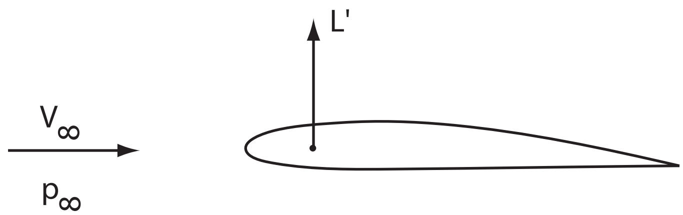
Kutta Condition

Thought Experiment[1]

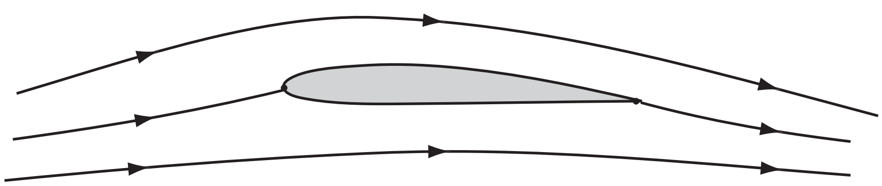
Suppose we model the flow around an airfoil using a potential flow approach.



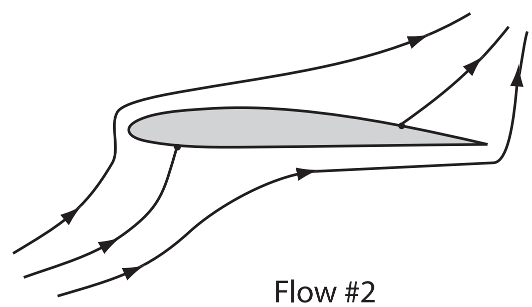
We know the following:	
$L' = \rho V_\infty \Gamma$	$\vec{u} = \nabla \phi$
$D' = 0$	$\vec{\omega} = 0$
	$\vec{u} \bullet \vec{n} = 0$
Bernoulli applies	

<u>Question</u>: How many potential flow solutions are possible?
<u>Answer</u>: Infinitely many!

For example:



Flow #1



Flow #2

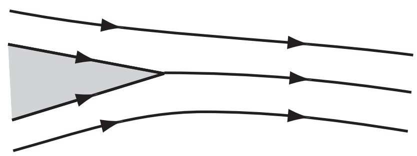
Both of these flows have circulation which are not all equal

$$\Gamma_1 \neq \Gamma_2 \Rightarrow L_1' \neq L_2'$$

[1] Anderson, Sec. 4.5

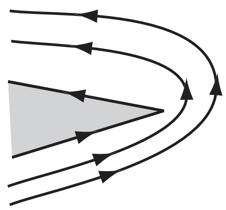
Another difference can be observed at the trailing edge:

Flow #1



Flow leaves t.e. smoothly.
Velocity is not infinite.

Flow #2



Flow turning around a sharp
corner has infinite velocity
at corner for potential flow.

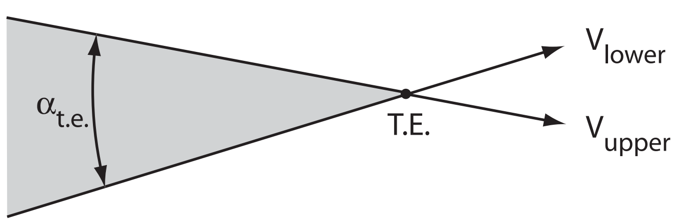
As a result of this and the physical evidence, Kutta hypothesized:

In a physical flow (i.e. having viscous effects), the flow will smoothly leave a sharp trailing edge. -*Kutta Condition*

⇒ Flow #1 is physically correct!

Let's look at Flow #1 a little more closely:

<u>Finite angle T.E.</u> $(\alpha_{te} > 0)$



Upper and lower surface velocities must still be tangent to their respective surfaces.

This implies 2 different velocities at TE.

Only realistic option:

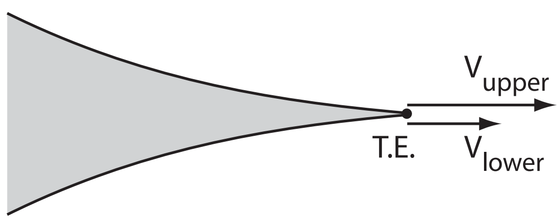
$$\boxed{V_{lower} = V_{upper} = 0 \text{ for finite angle T.E.}}$$

Note: from Bernoulli, this implies

$$p_{t.e.} = p_\infty + \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho \underbrace{V_{t.e.}^2}_{=0}$$

$$\Rightarrow \boxed{\begin{array}{l} p_{t.e.} = p_\infty + \frac{1}{2}\rho V_\infty^2 \\ TE \text{ is a stagnation point} \\ \text{with } p_{t.e.} \equiv \text{ total pressure} \end{array}}$$

<u>Cusped TE</u> $(\alpha_{te} = 0)$



In this case, velocities from upper and lower surface are aligned.

In order for the pressure at the TE to be unique:

$$\boxed{V_{upper} = V_{lower}}$$