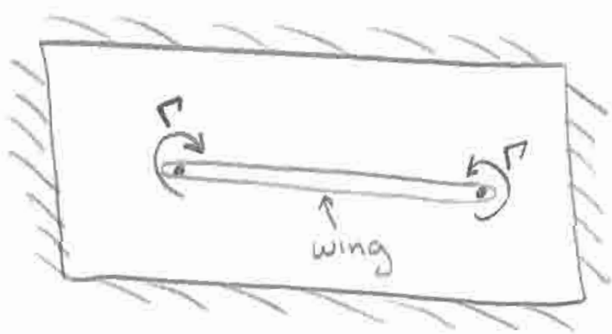
Three-Dimensional Wall Effects

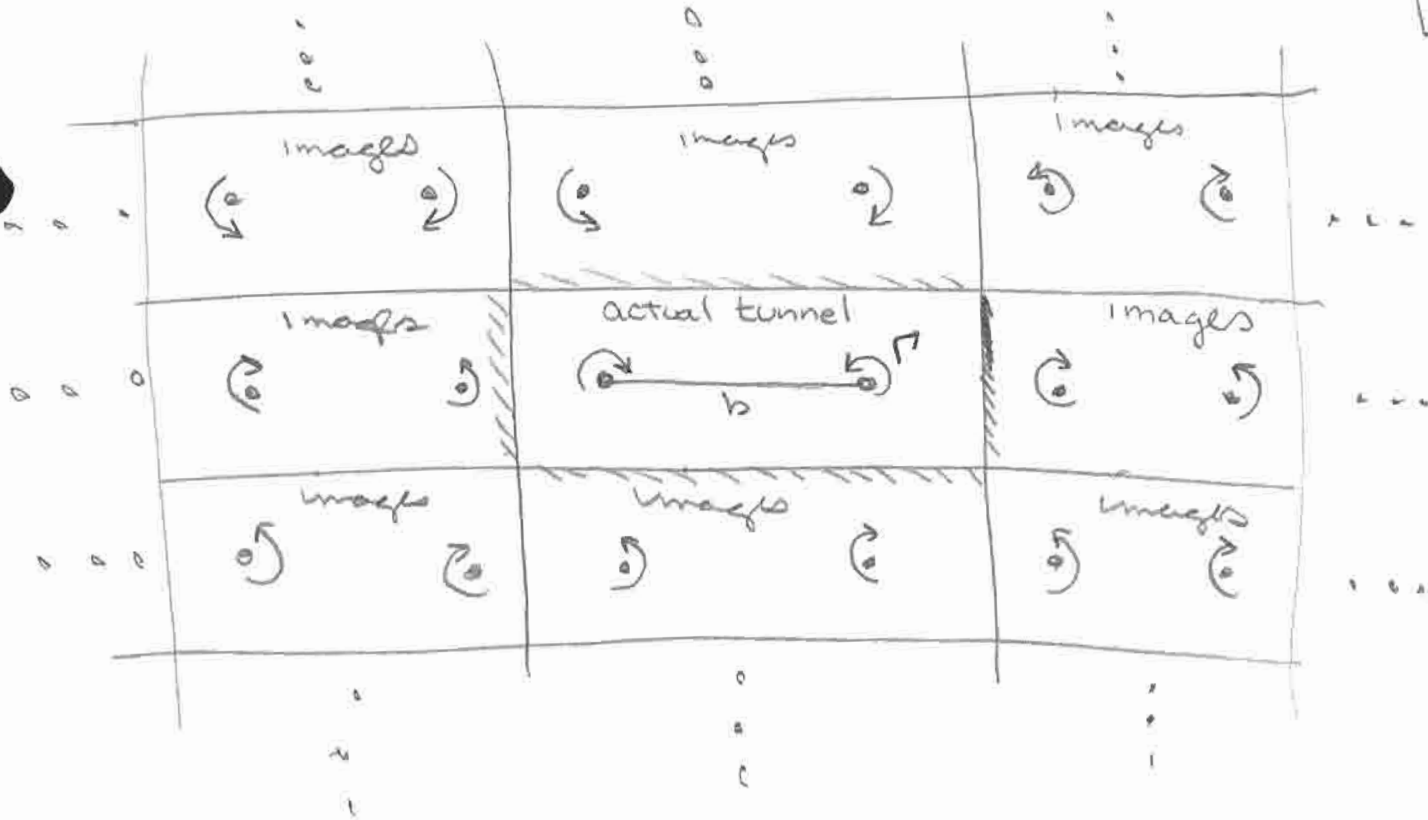
- In a freestream, recall that a lifting body can be modeled by a horseshoe vortex:

- Consider a rectangular cross-section tunnel:

Flow is into page



- The image system for this looks like:



The effect of these images is:

For fixed lift, such that Γ is constant,

* an upwash exists due to images ⇒ α is effectively larger

$$\underbrace{\alpha_\infty}_{\text{effective freestream AOA}} \cong \underbrace{\alpha_{tunnel}}_{\text{AOA of model in tunnel}} + \underbrace{\Delta\alpha_i}_{\text{correction due to upwash induced by images}}$$

* Similarly, this creates decrease in induced drag relative to freestream flight:

Recall,

$$C_{D_i} \propto C_L \alpha_i$$

$$\Rightarrow \Delta C_{D_i} = C_L \Delta\alpha_i$$

$$\Rightarrow C_{D_{i_\infty}} = C_{D_{i_{tunnel}}} + \Delta C_{D_i}$$

Or, since we are interested in the total drag:

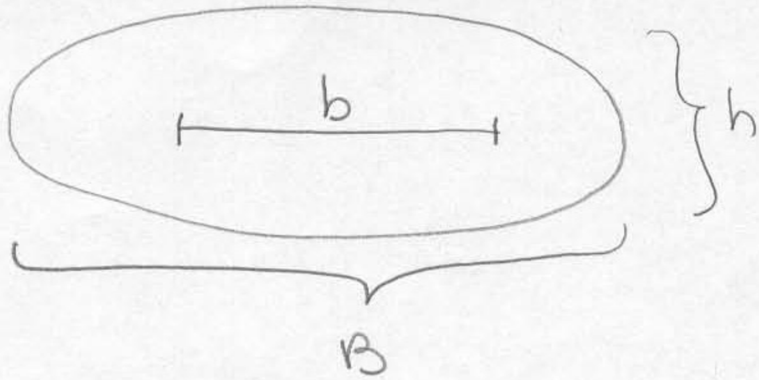
$$\boxed{C_{D_\infty} = C_{D_{tunnel}} + \Delta C_{D_i}}$$

Specific formulas derived in detailed analysis give that:

$$\Delta\alpha_i = \delta\left(\frac{S}{C}\right)C_L$$

where S = reference area
C = tunnel cross-sectional area
δ = factor which depends on tunnel & model geometry.

Wright Brothers is an elliptic cross-section with dimensions 10 ft wide by 7 ft high.

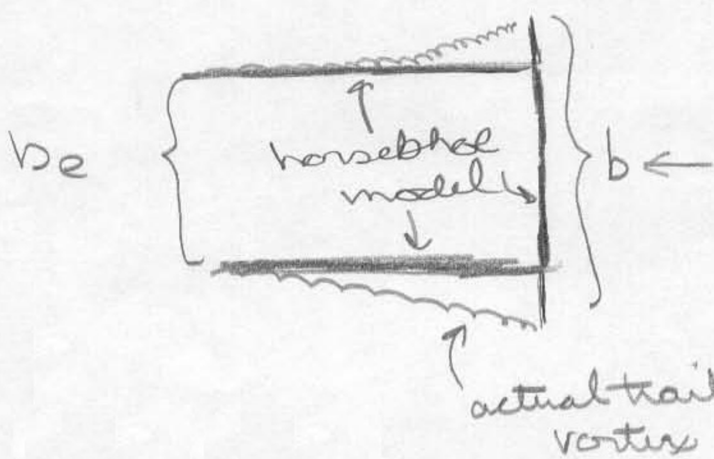


Define: $\lambda \equiv \frac{h}{B}$

$k \equiv \frac{b_e}{B}$

$b_e \equiv$ effective span $\approx 0.9b$

caused because wing tip trailing vortices are inboard of tips a short distance downstream



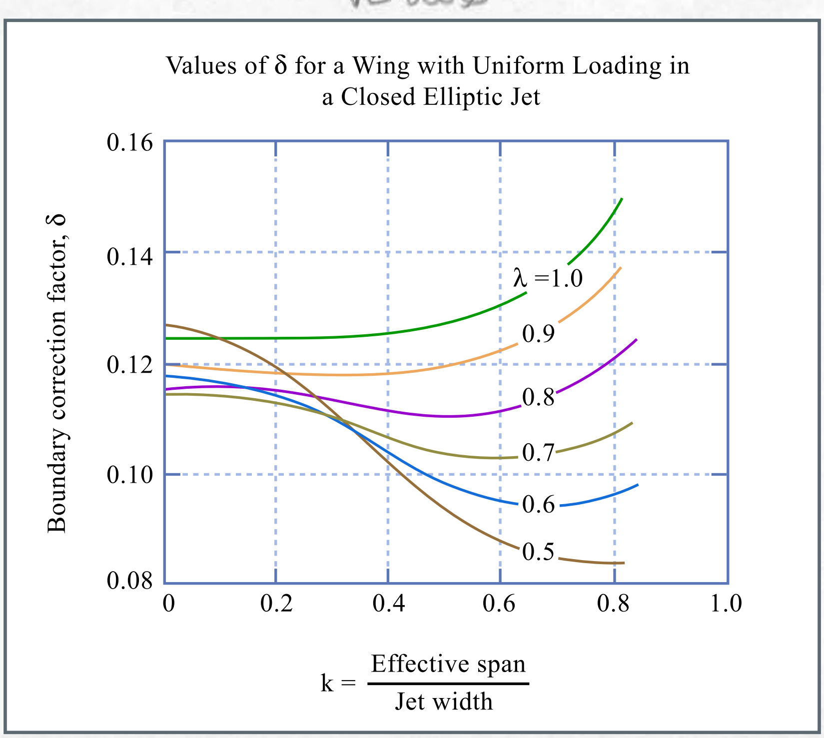


Figure by MIT OCW.