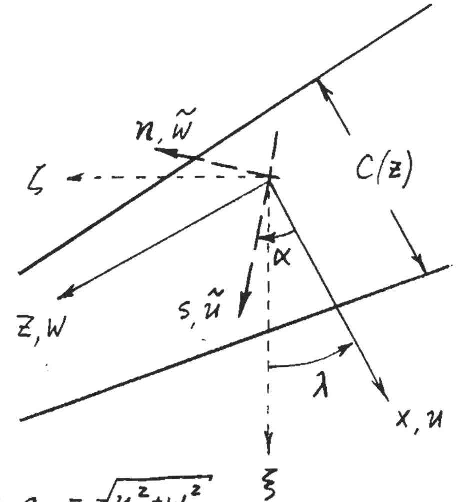
3D BL Equations – Sweep / Taper



$$\frac{\partial}{\partial x}\left(\rho q_e^2 \theta_{xx}\right) + \frac{\partial}{\partial z}\left(\rho q_e^2 \theta_{xz}\right) + \rho q_e \delta_x^* \frac{\partial u_e}{\partial x} + \rho q_e \delta_z^* \frac{\partial u_e}{\partial z} = \tau_x$$

$$\frac{\partial}{\partial x}\left(\rho q_e^2 \theta_{zx}\right) + \frac{\partial}{\partial z}\left(\rho q_e^2 \theta_{zz}\right) + \rho q_e \delta_x^* \frac{\partial w_e}{\partial x} + \rho q_e \delta_z^* \frac{\partial w_e}{\partial z} = \tau_z$$

$$\frac{\partial}{\partial x}\left(\rho q_e^3 \theta_x^*\right) + \frac{\partial}{\partial z}\left(\rho q_e^3 \theta_z^*\right) + \rho q_e \delta_x^{**} \frac{\partial q_e^2}{\partial x} + \rho q_e \delta_z^{**} \frac{\partial q_e^2}{\partial z} = 2D$$

Local streamwise coordinates (s, n), $(\tilde{u}, \tilde{w})$

$$u = \tilde{u}\cos\alpha - \tilde{w}\sin\alpha \qquad \cos\alpha = \frac{u_e}{q_e} \qquad \tilde{u}_e = q_e = \sqrt{u_e^2 + w_e^2}$$

$$w = \tilde{u}\sin\alpha + \tilde{w}\cos\alpha \qquad \sin\alpha = \frac{w_e}{q_e} \qquad \tilde{w}_e = 0$$

$$\rho q_e^2 \theta_{xx} = \rho u_e^2 \theta_{11} + \rho w_e^2 \theta_{22} - \rho u_e w_e (\theta_{12} + \theta_{21}) \qquad \rho q_e^2 \theta_{xz} = \rho u_e^2 \theta_{12} - \rho w_e^2 \theta_{21} + \rho u_e w_e (\theta_{11} - \theta_{22})$$

$$\rho q_e^2 \theta_{zx} = \rho u_e^2 \theta_{21} - \rho w_e^2 \theta_{12} + \rho u_e w_e (\theta_{11} - \theta_{22}) \qquad \rho q_e^2 \theta_{zz} = \rho u_e^2 \theta_{22} + \rho w_e^2 \theta_{11} + \rho u_e w_e (\theta_{12} + \theta_{21})$$

$$\rho q_e \delta_x^* = \rho u_e \delta_1^* - \rho w_e \delta_2^* \qquad \rho q_e \delta_z^* = \rho u_e \delta_2^* + \rho w_e \delta_1^*$$

$$\rho q_e \delta_x^{**} = \rho u_e \delta_1^{**} - \rho w_e \delta_2^{**} \qquad \rho q_e \delta_z^{**} = \rho u_e \delta_2^{**} + \rho w_e \delta_1^{**}$$

$$\rho q_e^3 \theta_x^* = q_e^2 \left[\rho u_e \theta_1^* - \rho w_e \theta_2^*\right] \; ; \; \theta_1^* = E_{11} + E_{21} \qquad \rho q_e^3 \theta_z^* = q_e^2 \left[\rho u_e \theta_2^* + \rho w_e \theta_1^*\right] \; ; \; \theta_2^* = E_{12} + E_{22}$$

$$\tau_x = \frac{u_e}{q_e}\tau_1 - \frac{w_e}{q_e}\tau_2 \qquad \tau_z = \frac{u_e}{q_e}\tau_2 + \frac{w_e}{q_e}\tau_1$$

$$D = \int \tau_1(y)\, d\tilde{u} + \int \tau_2(y)\, d\tilde{w}$$

Assumed Sweep / Taper Relations: $\frac{\partial u_e}{\partial z} = \frac{\partial w_e}{\partial z} = \frac{\partial q_e}{\partial z} = 0 \qquad \theta \sim c^k \rightarrow \frac{\partial \theta}{\partial z} = \theta \frac{k}{c}\frac{dc}{dz}$

$$\frac{\partial}{\partial x}\left(\rho q_e^2 \theta_{xx}\right) + \rho q_e^2 \theta_{xz} K + \rho q_e \delta_x^* \frac{\partial u_e}{\partial x} = \tau_x \qquad ; \; K = \frac{k}{c}\frac{dc}{dz}$$

$$\frac{\partial}{\partial x}\left(\rho q_e^2 \theta_{zx}\right) + \rho q_e^2 \theta_{zz} K = \tau_z \qquad \text{since } \frac{\partial w_e}{\partial x} = \frac{\partial u_e}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\rho q_e^3 \theta_x^*\right) + \rho q_e^3 \theta_z^* K + \rho q_e \delta_x^{**} \frac{\partial q_e^2}{\partial x} = 2D$$

Integration coordinates ξ, ζ:

$$\xi = x\cos\lambda + z\sin\lambda \qquad \frac{\partial()}{\partial x} = \frac{\partial()}{\partial \xi}\cos\lambda - \frac{\partial()}{\partial \zeta}\sin\lambda$$

$$\zeta = -x\sin\lambda + z\cos\lambda \qquad \frac{\partial()}{\partial z} = \frac{\partial()}{\partial \xi}\sin\lambda + \frac{\partial()}{\partial \zeta}\cos\lambda = () \cdot K$$

$$\frac{\partial()}{\partial x} = \frac{1}{\cos\lambda}\frac{\partial()}{\partial \xi} - \tan\lambda\, ()K$$

$$\frac{1}{\cos\lambda}\frac{\partial}{\partial \xi}\left(\rho q_e^2 \theta_{xx}\right) - \tan\lambda\, \rho q_e^2 \theta_{xx} K + \rho q_e^2 \theta_{xz} K + \rho q_e \delta_x^* \frac{1}{\cos\lambda}\frac{\partial u_e}{\partial \xi} = \tau_x$$

$$\frac{1}{\cos\lambda}\frac{\partial}{\partial \xi}\left(\rho q_e^2 \theta_{zx}\right) - \tan\lambda\, \rho q_e^2 \theta_{zx} K + \rho q_e^2 \theta_{zz} K = \tau_z$$

$$\frac{1}{\cos\lambda}\frac{\partial}{\partial \xi}\left(\rho q_e^3 \theta_x^*\right) - \tan\lambda\, \rho q_e^3 \theta_x^* K + \rho q_e^3 \theta_z^* K + \rho q_e \delta_x^{**} \frac{1}{\cos\lambda}\frac{\partial q_e^2}{\partial \xi} = 2D$$