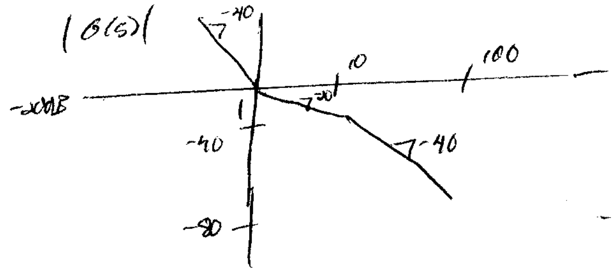
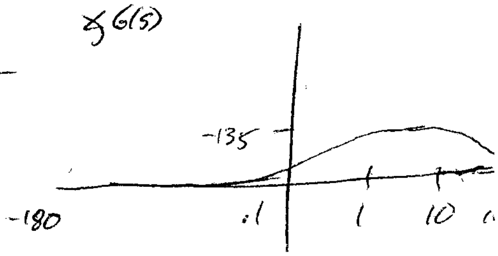
16.30

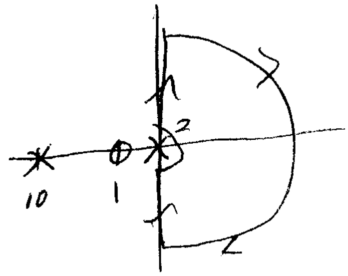
PSET 2

#1

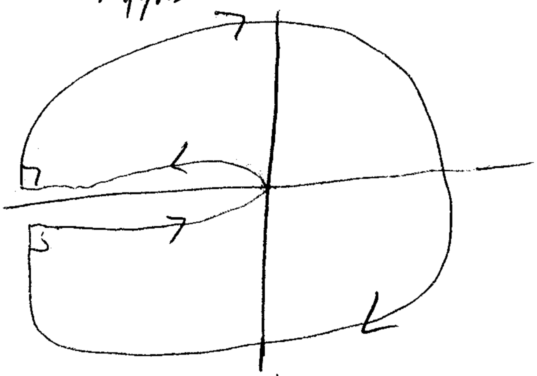
A.) $G(s)=\dfrac{K/_{10}(s+1)}{s^2(s/_{10}+1)}$







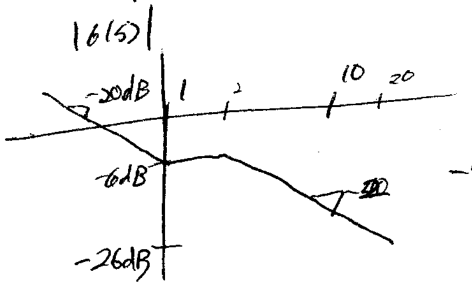
Nyquist

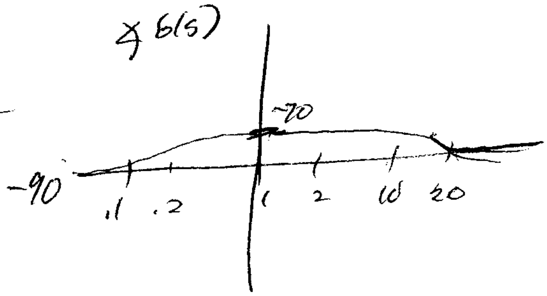


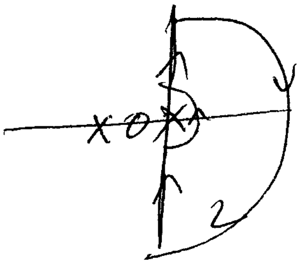
	N	P	Z	
$-\infty < -1/K < 0$	0	0	0	$0<K<\infty$
$0 < -1/K < \infty$	$\emptyset$	0	1	$-\infty<K<0$

$0<K<\infty$ stable

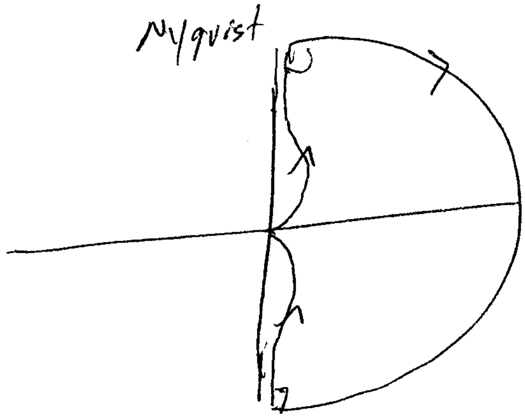
B.) $G(s)=\dfrac{K/_2(s+1)}{s(s/_2+1)}$







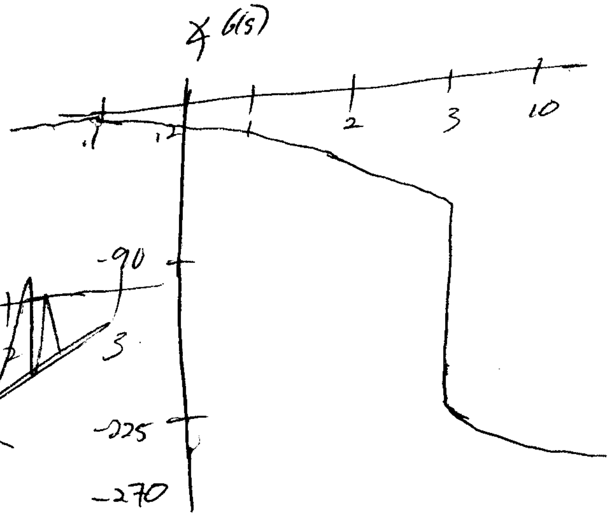
Nyquist

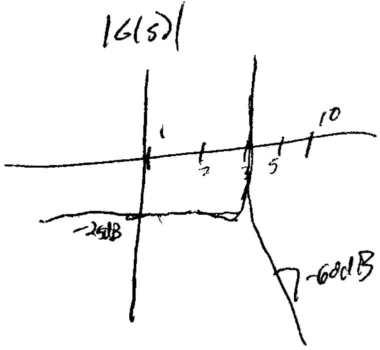


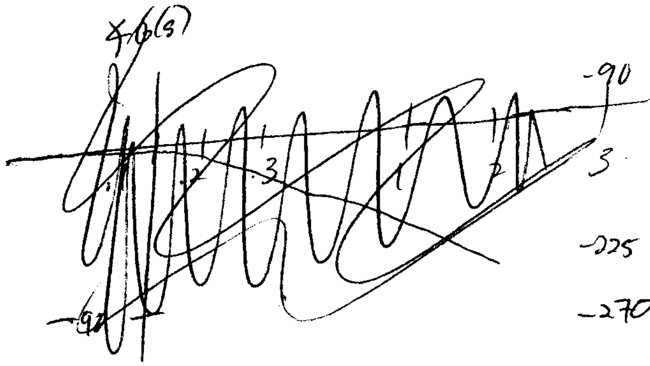
	N	P	Z
$-\infty < -1/K < \infty$	0	0	0
$0 < -1/K < \infty$	1	0	1

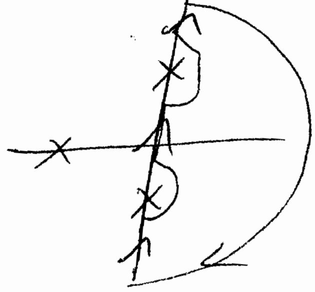
$0<K<\infty$ stable

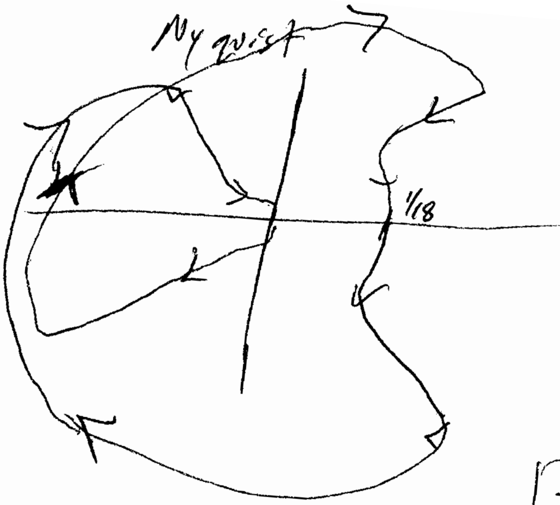
c.) $G(s)=\dfrac{K}{(s+2)(s^2+9)}=\dfrac{K/_{18}}{(s/_2+1)(s^2/_9+1)}$











	N	P	Z
$-\infty < -1/K < 0$	2	0	2
$0 < -1/K < 1/18$	1	0	1
$1/18 < -1/K < \infty$	0	0	0

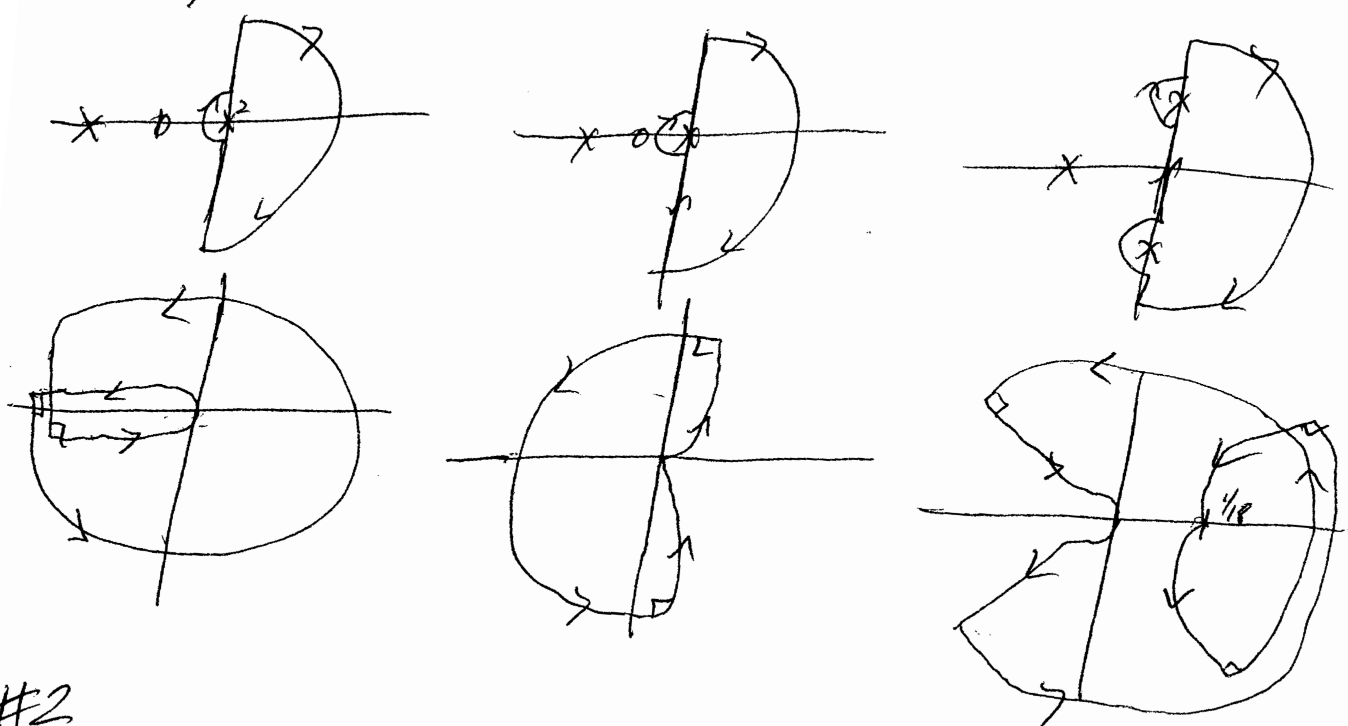
$0 < K < \infty$ unstable

$-\infty < K < -18$ unstable

$\boxed{-18 < K < 0 \text{ stable}}$

D.)

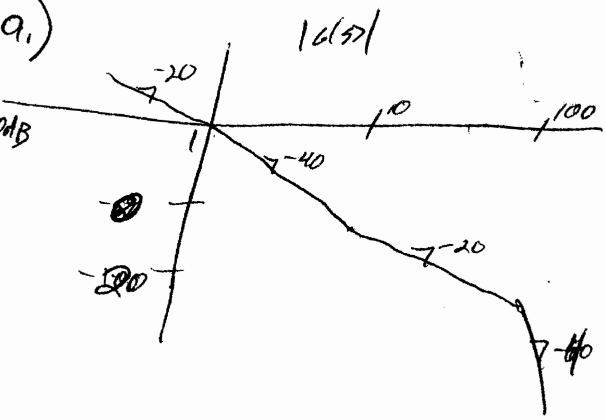
a.)

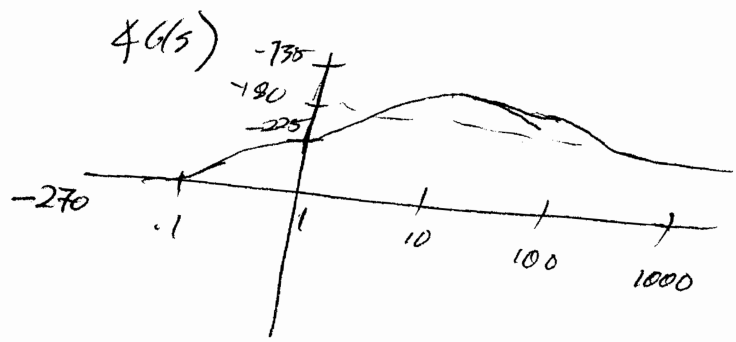


#2

$$G(s) = \frac{100\left(\frac{s}{10}+1\right)}{s(s-1)\left(\frac{s}{100}+1\right)} = \frac{-100\left(\frac{s}{10}+1\right)}{s(1-s)\left(\frac{s}{100}+1\right)}$$

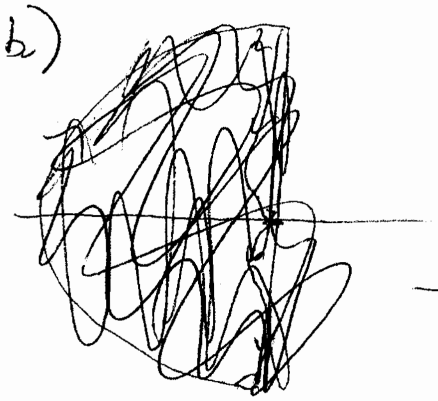
a.)



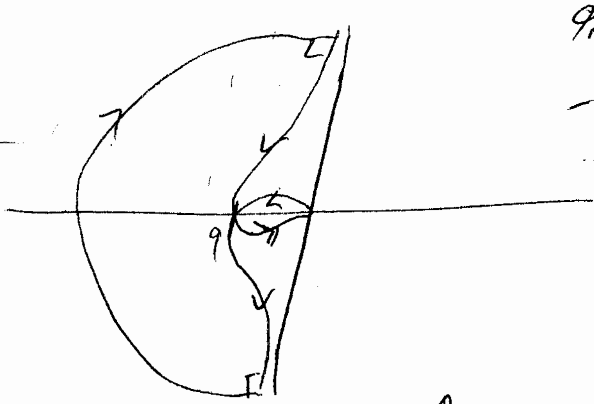


MATLAB should look something like this.

b.)



c.) Stable



One way, look at bode plots crossover at $\approx \omega = 10$

$\phi_m \cong 45^\circ$

OR

	N	P	Z	
$-\infty < -1/K < -A$	1	1	$\cancel{2}$	$0 < K < 1/A$
$-A < -1/K < 0$	$\cancel{1}$	1	0	$1/A < K < \infty$
$0 < -1/K < \infty$	0	1	1	$-\infty < K < 0$

$A = 9$

Stable for $9 < K < \infty$

It contains $K=1$, so stable.

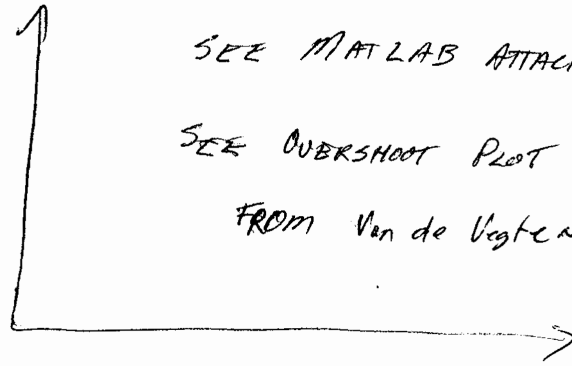
Using Formula

#3 $\zeta = 0.1 \quad \phi_m = 11.421$

$\zeta = 0.4 \quad \phi_m = 43.118$

$\zeta = 0. \quad \phi_m = 65.156$

b.)



SEE MATLAB ATTACHMENT

SEE OVERSHOOT PLOT

FROM Van de Vegte pg 129

c.) $\phi_m = 180 + \angle G$

$$\phi_m = 180 - \tan^{-1}\left(\frac{2\zeta\omega_n}{-\omega_c}\right) = 180 - \left(180 - \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_c}\right)\right)$$

$|G_c| = 1$ when $\omega_c^4 + (2\zeta\omega_n\omega_c)^2 - \omega_n^4 = 0$

$$\frac{\omega_c}{\omega_n} = \left(\sqrt{4\zeta^4+1} - 2\zeta^2\right)^{1/2}$$

Plug-in...

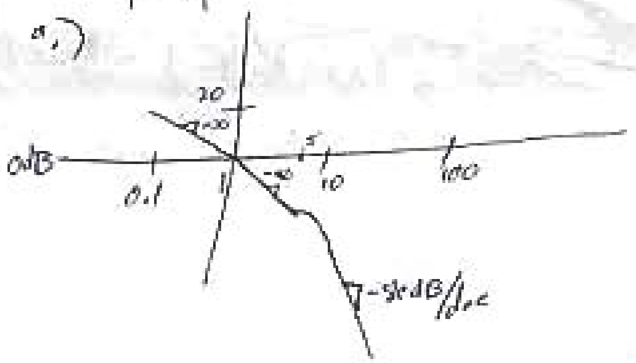
$$\phi_m = \tan^{-1}\left(\frac{2\zeta}{\left(\sqrt{4\zeta^4+1} - 2\zeta^2\right)^{1/2}}\right) \checkmark$$

THIS PROOF IS IN VAN DE VEGTE.

#4 $G(s) = \dfrac{K}{s(s+1)\left(\frac{s^2}{25} + \frac{0.4s}{5} + 1\right)}$

$|G(s)|$

a.)

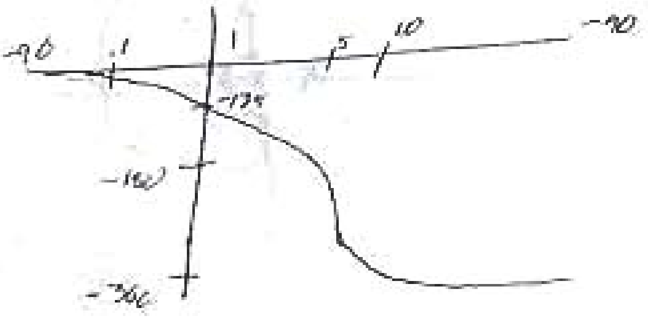


$2\zeta = 0.4$

$\zeta = 0.2$

$m_p = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = 2.5 = 8dB$

$\angle G(s)$



b.) $-135 = -90 - \tan^{-1}(\omega) - \tan^{-1}\left(\dfrac{\frac{0.4\omega}{5}}{1-\omega^2/25}\right)$

$\omega = 0.866$

$1 = \dfrac{K}{0.866\,(0.866^2+1)^{1/2}\left(\left(1-\frac{0.866^2}{25}\right)^2 + \left(\frac{0.4(0.866)}{5}\right)^2\right)^{1/2}}$

$\boxed{K = 1.114}$

$-180 = -90 - \tan^{-1}(\omega) - \tan^{-1}\left(\dfrac{0.08\omega}{1-\omega^2/25}\right)$

$\omega = 2.89$

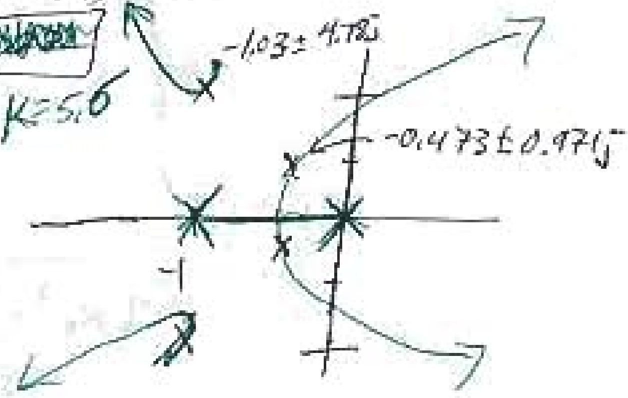
$K = 5.6$

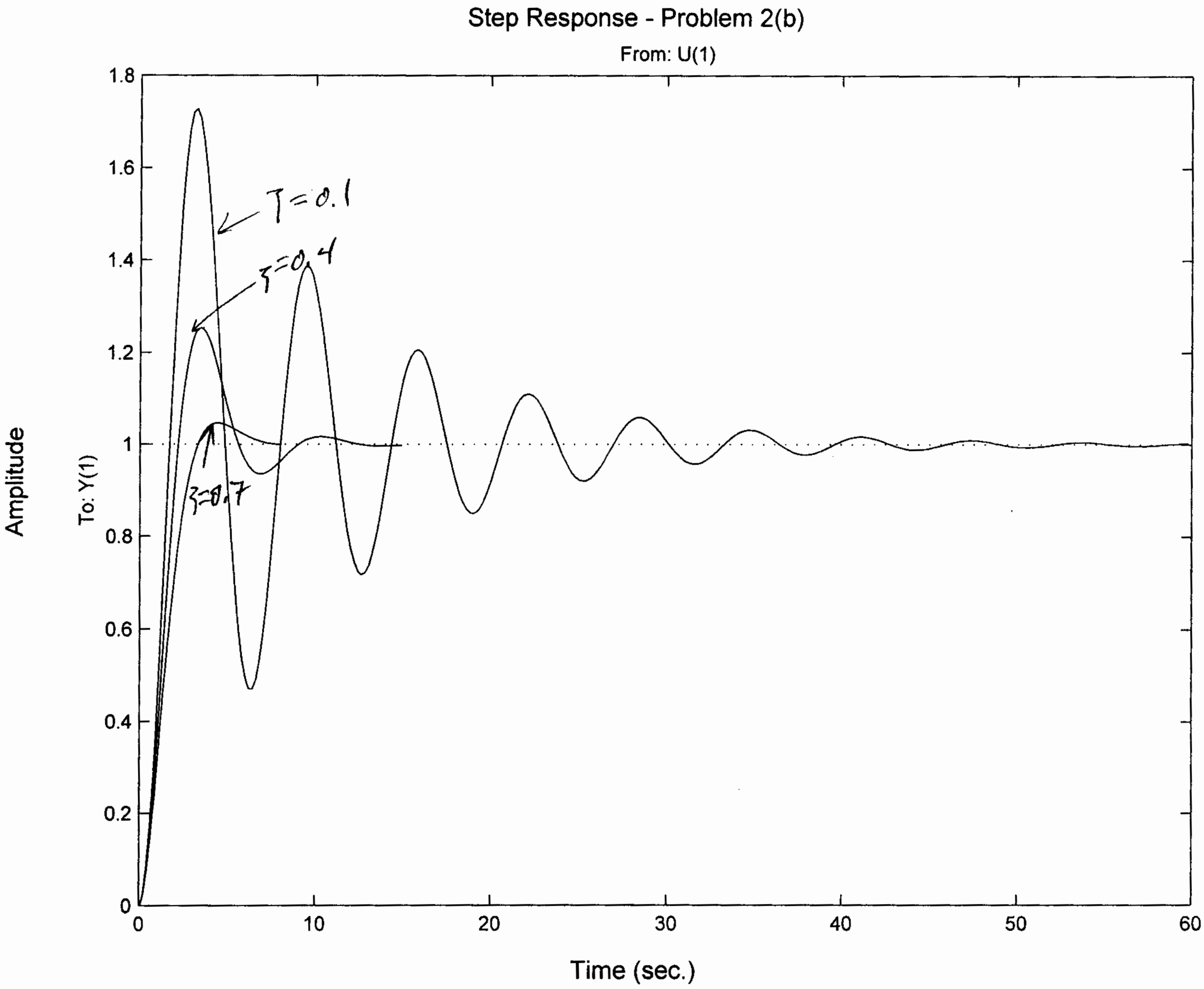
Plug ω into $|G(s)| = \dfrac{1.114}{\omega(\omega^2+1)^{1/2}\left(\left(1-\frac{\omega^2}{25}\right)^2 + \left(\frac{0.4\omega}{5}\right)^2\right)^{1/2}}$

$\dfrac{1}{|G(s)|_\omega} = \boxed{\begin{array}{l} K = 15dB \\ \;\;\; = 5.6 \end{array}}$

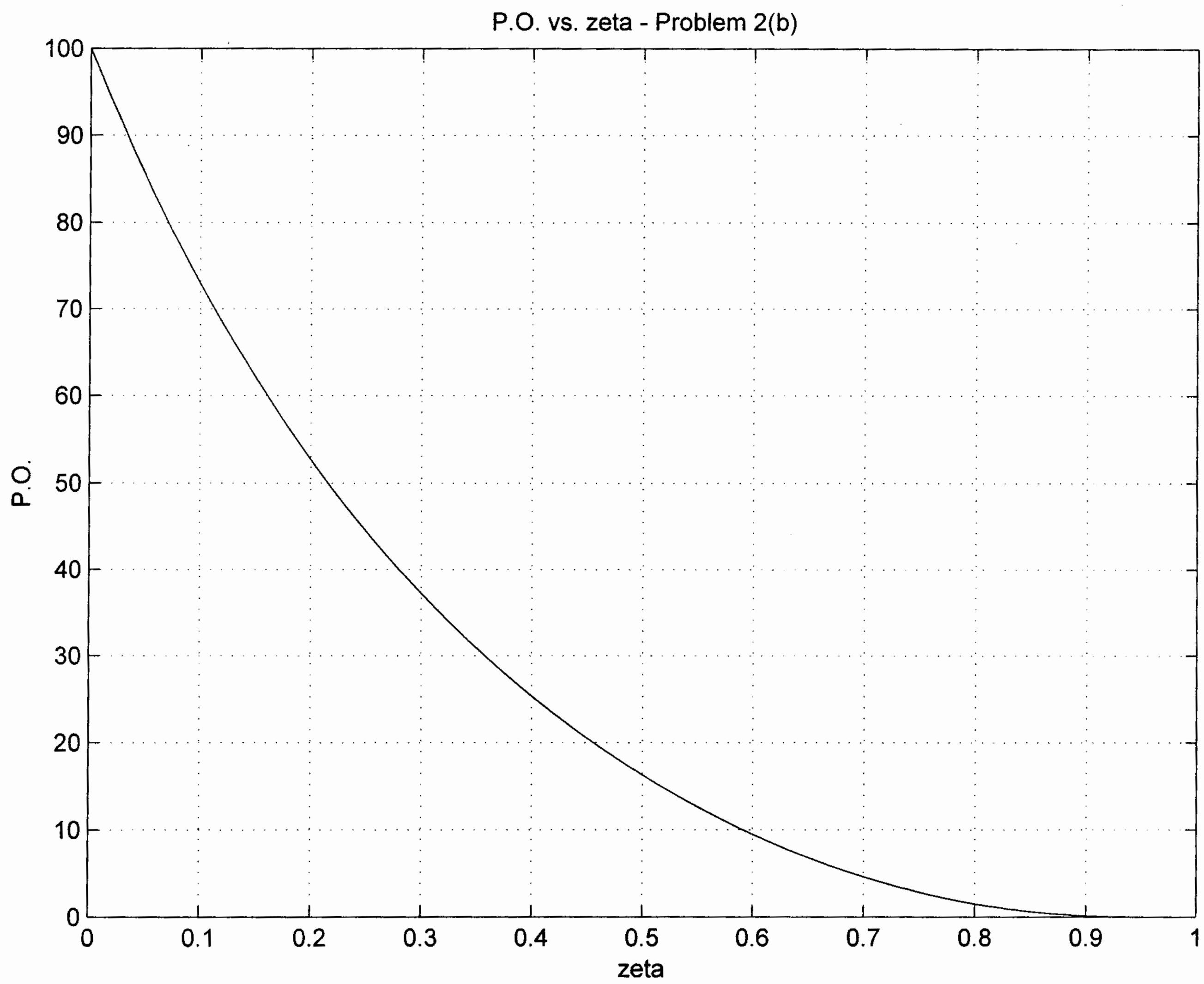
c.) $s^2 + 2s + 25 = 0$

$s = -1 \pm j4.9$





Step Response - Problem 2(b)
From: U(1)
Amplitude
To: Y(1)
Time (sec.)
ζ=0.1
ζ=0.4
ζ=0.7
0
10
20
30
40
50
60
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


P.O. vs. zeta - Problem 2(b)
P.O.
zeta
100
90
80
70
60
50
40
30
20
10
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1