16.30/31 Handout #1
Prof. J. P. How and Prof. E. Frazzoli September 10, 2010
T.A. B. Luders **Due:** September 17, 2010

16.30/31 Homework Assignment #1

Goals: Refresh skills for Matlab and classical analysis.

1. Sketch the root locus for the following systems, using the rules discussed in class and the lecture notes. (Concentrate on the real axis and the asymptotes/centroids.)
   (a) $G_cG_p(s) = \frac{K}{(s^2+6s+8)(s^2+2s+5)}$
   (b) $G_cG_p(s) = \frac{K(s^2-2s+2)}{s(s+2)}$
   (c) $G_cG_p(s) = \frac{K(s+3)}{s^4-4s^2}$
   (d) After completing these sketches, verify the results using Matlab. How closely do your sketches resemble the actual plots (we would not expect them to be exact)?

2. (adapted from FPE 4.10, page 215) Consider the DC motor control system with rate feedback shown in Figure 1(a).

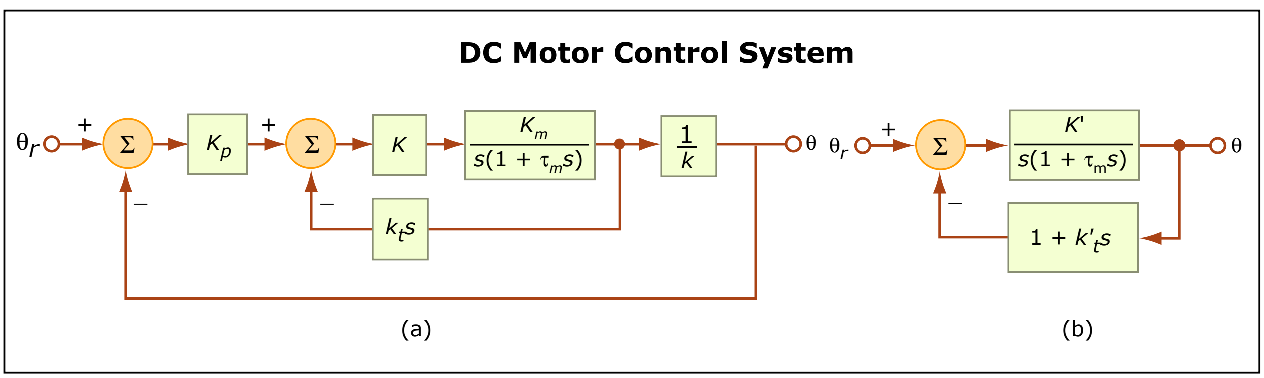

Figure 1: DC motor control system Image by MIT OpenCourseWare.

   (a) Find values for K' and k_t' so that the system of Figure 1(b) has the same transfer function as the system of Figure 1(a). Your answers should be in terms of K_p, K, k_t, K_m, and k.
   (b) Suppose $\tau_m = 0.25$ sec. For the case with no rate feedback (i.e., $k_t' = 0$), use root-locus techniques to select the proportional gain K' to achieve a closed-loop damping ratio of $\zeta = 0.2$.
   (c) Using the value of K' found in part (b), sketch the locus of closed-loop pole locations for $k_t' > 0$.
   (d) Determine the system type number with respect to tracking θ_r, and compute the corresponding error constant in terms of parameters K' and k_t'. What happens to the steady state error if K' is increased? If k_t' is increased?
   Hint: Your tracking error should take the form $e(t) = \theta_r(t) - \theta(t)$.

3. The attitude-control system of a space booster is shown in Figure 2. The attitude angle θ is controlled by commanding the engine angle δ, which is then the angle of the applied thrust, F_T. The vehicle velocity is denoted by v. A typical rigid-body transfer function for such a booster might take the form

$$G_p(s) = \frac{0.9}{s^2 - 0.03}$$

(This is *very* simplified, ignoring fuel slosh, aeroelasticity, motor dynamics, etc.) The rigid-body vehicle can be stabilized by the addition of rate feedback (Figure 2(b)).

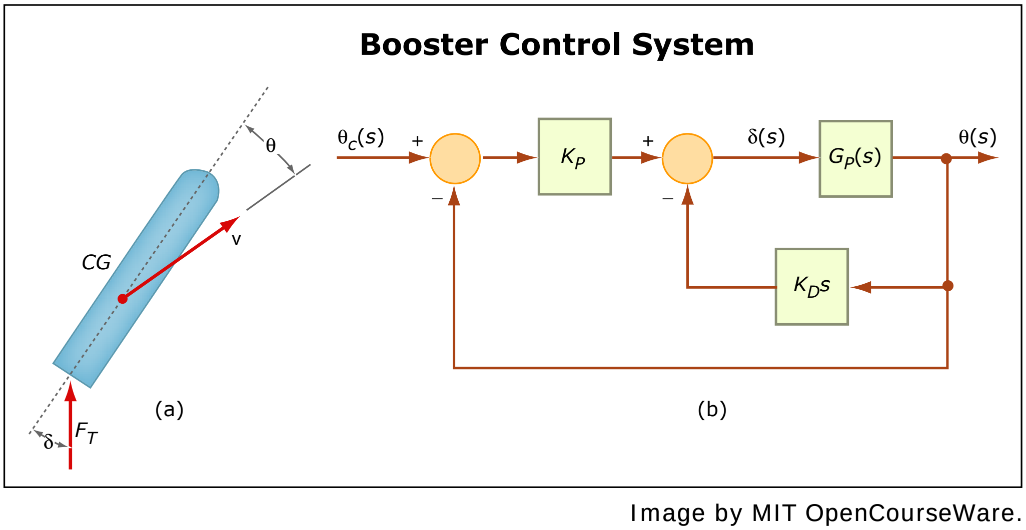

Figure 2: Booster control system

(a) With $K_D = 0$, plot the root locus and state the different types of responses possible (relate the response with the possible pole locations). Why is K_P alone not sufficient to stabilize the dynamics?

(b) Design the compensator shown (which is PD) to place the closed-loop poles at $s = -0.2 \pm j0.3$ (the resulting time constant is 5 sec).

(c) Plot the root locus of the compensated system, with K_p variable and K_D set to the value found in (b). Compare with your answer in (a).

(d) For some value of K_p, use Matlab to compute the closed-loop response to an impulse for θ_c.

4. (16.31 required/16.30 extra credit). Suppose that you are to design a unity gain feedback controller for a first order plant. The plant and controller respectively take the form

$$G_p(s) = \frac{s-2}{s}, \qquad G_c(s) = K\frac{s+p}{s+z},$$

where $K > 0$, $p \in \mathbb{R}$, and $z \in \mathbb{R}$ are parameters to be specified.

(a) Using root-locus methods, specify some p and z for which it is possible to make the closed-loop system *strictly stable*. Include a sketch of the closed-loop root locus, as well as the corresponding range of gains K for which the system is strictly stable.

(b) Suppose p and z are fixed to the values chosen in (a). Design K to meet the following specifications:

- The closed-loop system must be strictly stable.
- The damping ratio ζ must be between 0.4 and 0.6.
- Given these constraints, minimize the natural frequency ω_n.

16.30 / 16.31 Feedback Control Systems
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