16.410-13 Recitation 7 Problems

Problem 1: Propositional Logic

Julia says that Sarah and Fred say the truth. Sarah says that Peter is lying.
Peter says that Fred says the truth. Fred says that Peter lies or Julia lies.
Dick says, he lies.

- Write this sentence in propositional logic after defining appropriate atomic propositions.
- Determine those people who are lying and those who are telling the truth.
- What can you tell about Dick?

Examples 1: Producing Conjunctive Normal Form

Remember the following rule to put a formula into conjunctive normal form.

1. Eliminate *iff* and *implies* statements using the rules: "(A implies B) iff ((not A) or B)" and "(A iff B) iff ((A implies B) and (B implies A))".

   Example:
   P implies ((Q implies R) or (not S))
   P implies (((not Q) or R) or (not S))
   (not P) or (((not Q) or R) or (not S))

2. Push *negations* down using de Morgan's laws: "(not (A or B)) iff ((not A) and (not B))", "(not (A and B)) iff ((not A) or (not B))", "(not (not A)) iff A".

   Example:
   not ((not P) and (Q or (not (R and S))))
   (not (not P)) or (not (Q or (not (R and S))))
   P or ((not Q) or (not (not (R or S))))
   P or ((not Q) or ((not Q) or (R and S)))

3. Use distribution to convert to CNF: "(A or (B1 and ... and BN)) iff ((A or B1) and ... (A or BN))", "((B1 and ... and BN) or A) iff ((B1 or A) and ... and (BN or A))".

   Example:
   (P and Q) or (P and (not Q))
   ((P and Q) or P) and ((P and Q) or (not Q))
   ((P or P) and (Q or P)) and ((P or (not Q)) and (Q or (not P)))

Problem 2: Conjunctive Normal Form

Put the following statements into conjunctive normal form:

- (A iff B) and (B iff A)
- (A implies (not (B or C))) and (A iff B)

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