



IN CS, IT CAN BE HARD TO EXPLAIN  
THE DIFFERENCE BETWEEN THE EASY  
AND THE VIRTUALLY IMPOSSIBLE.

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# Image Classification via Deep Learning

Ryan Alexander, Ishwarya Ananthabhotla, Julian Brown,  
Henry Nassif, Nan Ma, Ali Soylemezoglu

# Overview

- What is Deep Learning?
- Image Processing
- CNN Architecture
- Training Process
- Image Classification Results
- Limitations

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# Deep Learning Refers to...

Machine Learning algorithms designed to

extract **high-level abstractions** from data

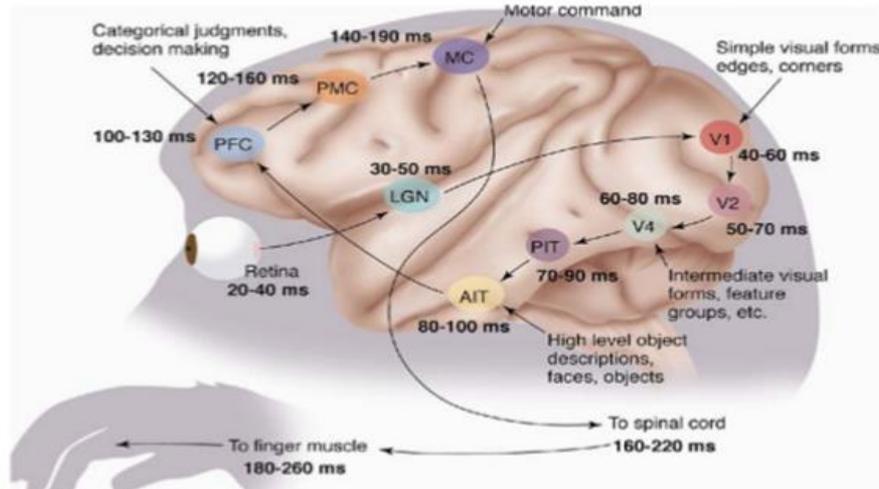
via **multi-layered processing** architectures

using **nonlinear transformations** at each layer

# Human Visual System

- Distributed Hierarchical processing in the primate cerebral cortex (1991)

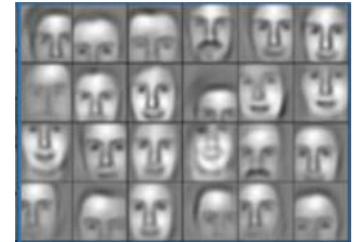
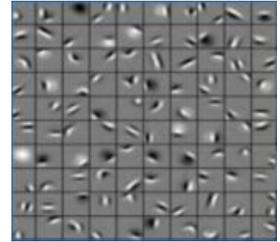
- The ventral (recognition) pathway in the visual cortex
  - Retina → LGN → V1 → V2 → V4 → PIT → AIT (80-100ms)



[picture from Simon Thorpe]

# How To Classify a Face?

- Identify where the face region is
  - Foreground Extraction
  - Edge Detection
- Classify features of the face
  - Identify and describe eyes, nose, mouth areas
- Look at face as a collection of those features



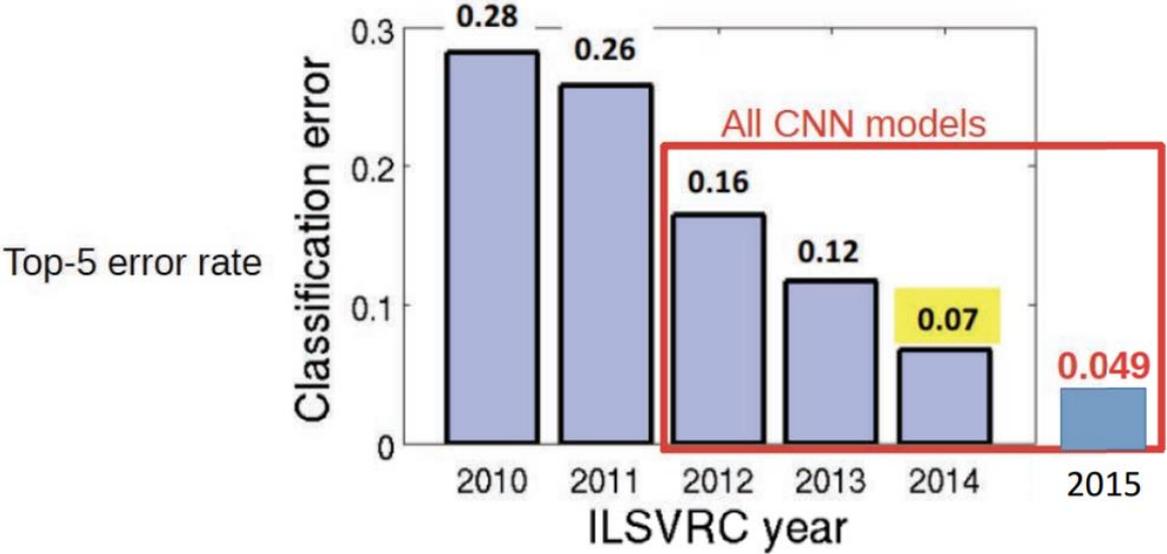
# Common Architectures

- Deep Convolutional Neural Networks (CNNs)
- Deep Belief Networks (DBNs)
- Recurrent Neural Network

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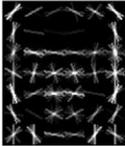
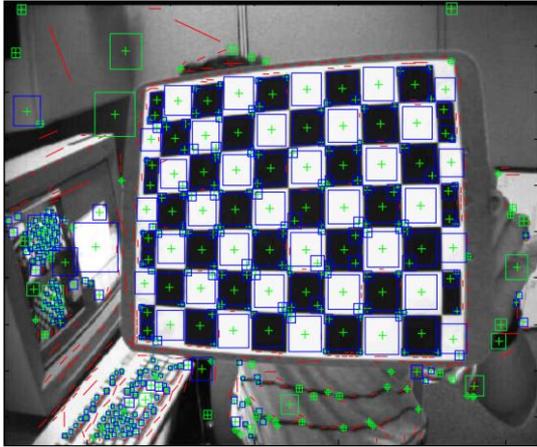
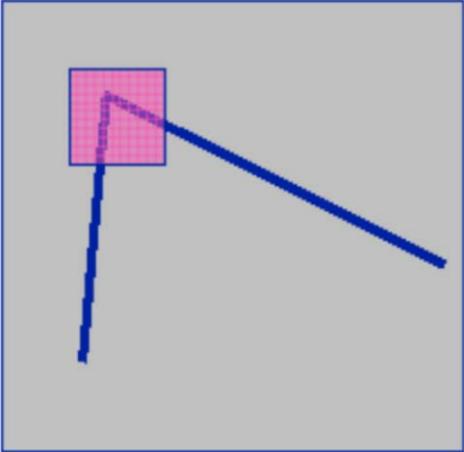
# ImageNet Competition Through Time



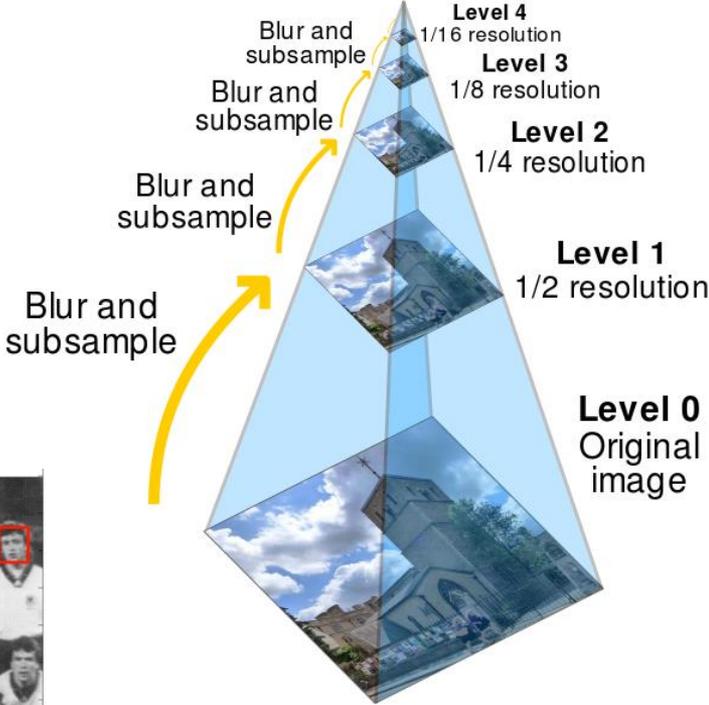
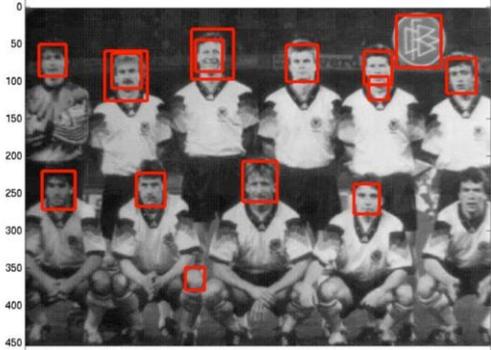
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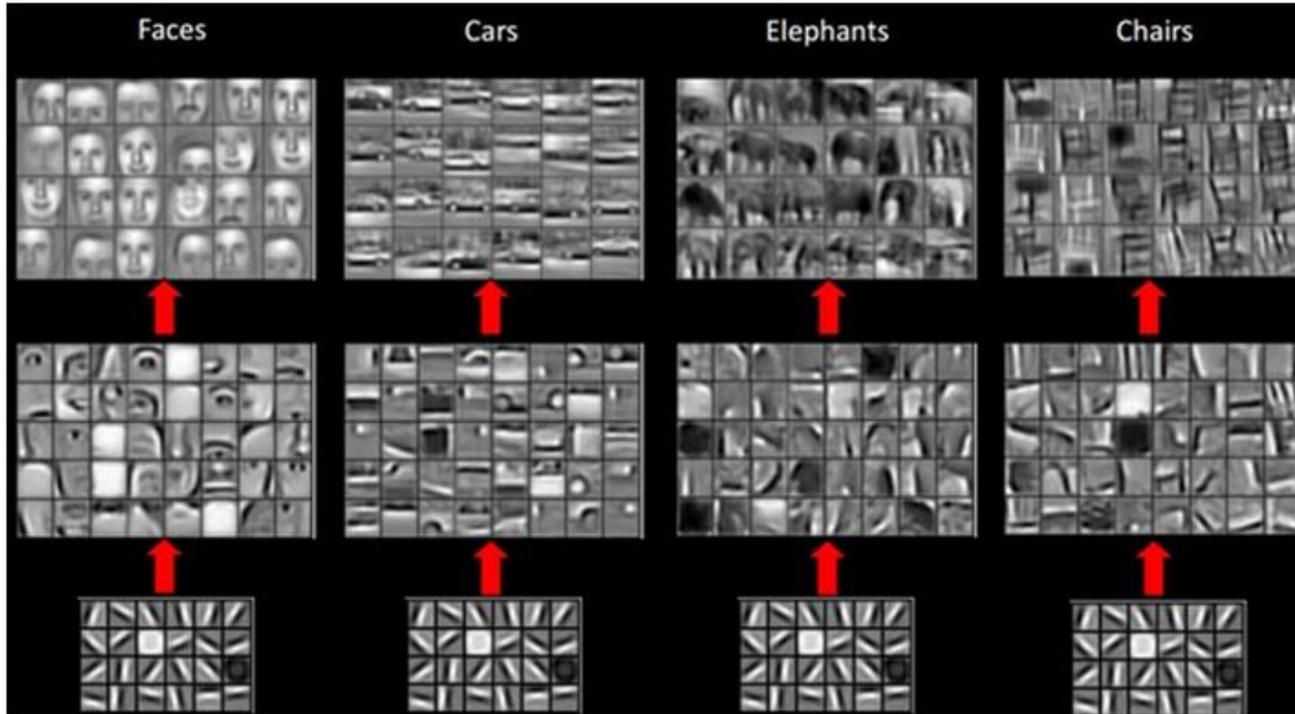
# Classic Classification -- Feature Engineering



template



# What if the techniques could be “learned”?



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# Step 1: Convolution - Definition

Informal Definition: Procedure where two sources of information are intertwined.

Formal Definition :

**Discrete :**

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1,y-n_2]$$

**Continuous :**

$$f(x,y) * g(x,y) = \int_{\tau_1=-\infty}^{\infty} \int_{\tau_2=-\infty}^{\infty} f(\tau_1,\tau_2) \cdot g(x-\tau_1,y-\tau_2) d\tau_1 d\tau_2$$

# Convolution - Example

Assume the following kernel/filter :

1	0	1
0	1	0
1	0	1

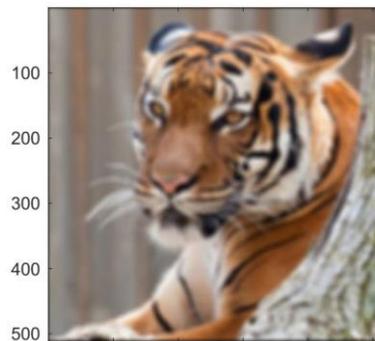
# Convolution

1 <sub>x1</sub>	1 <sub>x0</sub>	1 <sub>x1</sub>	0	0
0 <sub>x0</sub>	1 <sub>x1</sub>	1 <sub>x0</sub>	1	0
0 <sub>x1</sub>	0 <sub>x0</sub>	1 <sub>x1</sub>	1	1
0	0	1	1	0
0	1	1	0	0

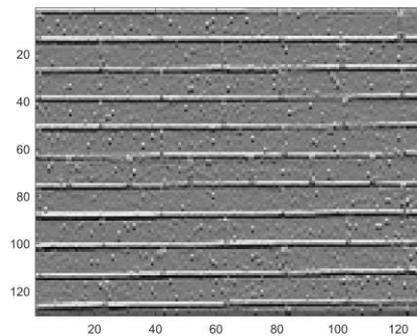
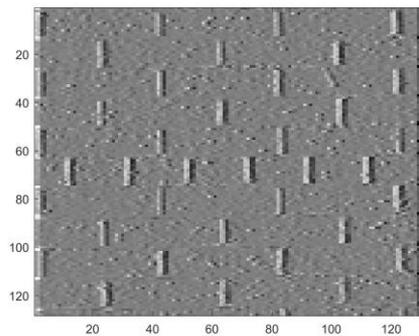
Image

4		

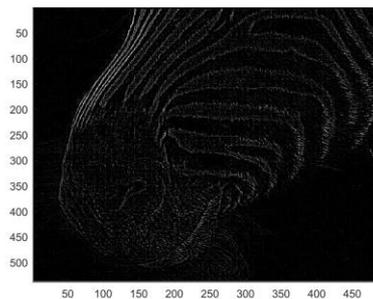
Convolved  
Feature



.0113	.0838	.0113
.0838	.6193	.0838
.0113	.0838	.0113



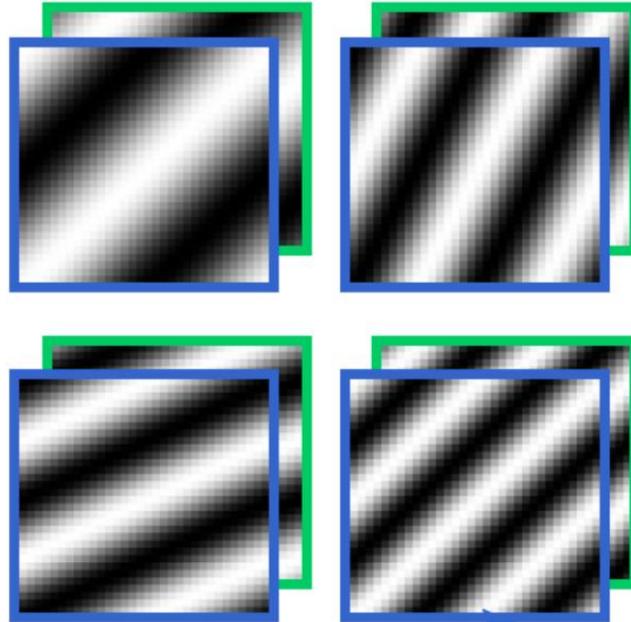
1	2	1
0	0	0
-1	-2	-1



1	1	1
1	-8	1
1	1	1

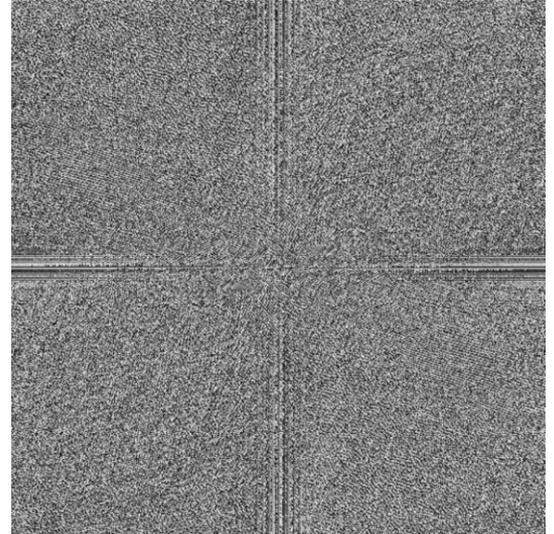
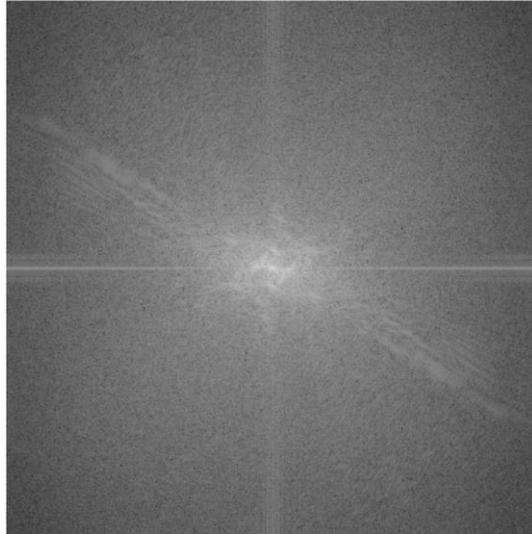
# More Information? Fourier Transform!

Sum of a set of sinusoidal gratings differing in spatial frequency, orientation, amplitude, phase



# Fourier Transform

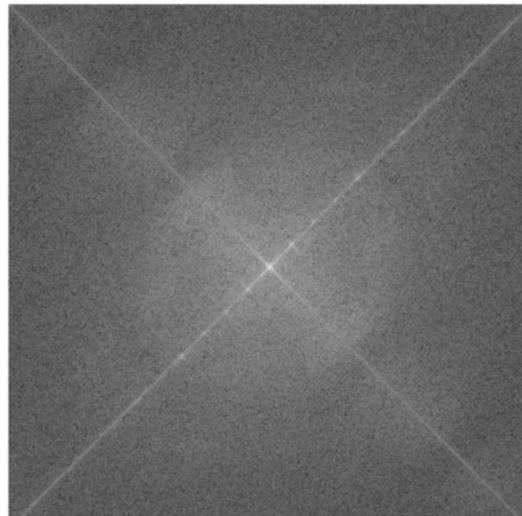
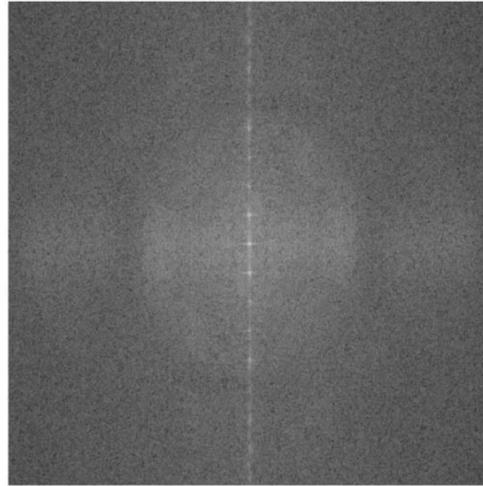
- Fourier Transform image itself is weird to visualize -- Phase and Magnitude!
- Magnitude -- orientation information at all spatial scales
- Phase -- contour information



## Sonnet for Lena

O dear Lena, your beauty is so vast  
It is hard sometimes to describe it fast.  
I thought the entire world I would impress  
If only your portrait I could compress.  
Alas! First when I tried to use VQ  
I found that your cheeks belong to only you.  
Your silky hair contains a thousand lines  
Hard to match with sums of discrete cosines.  
And for your lips, sensual and tactual  
Thirteen Crays found not the proper fractal.  
And while these setbacks are all quite severe  
I might have fixed them with hacks here or there  
But when filters took sparkle from your eyes  
I said, 'Damn all this. I'll just digitise.'

Thomas Cochran

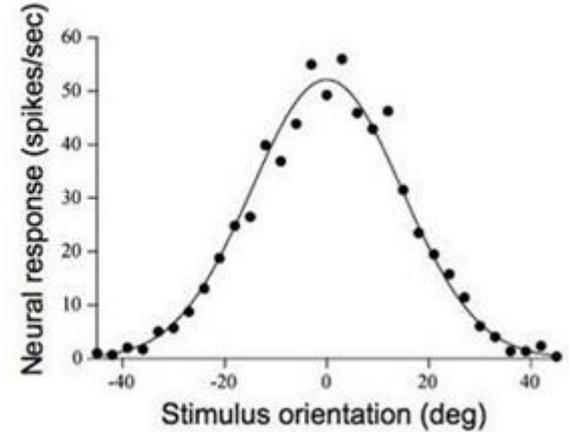
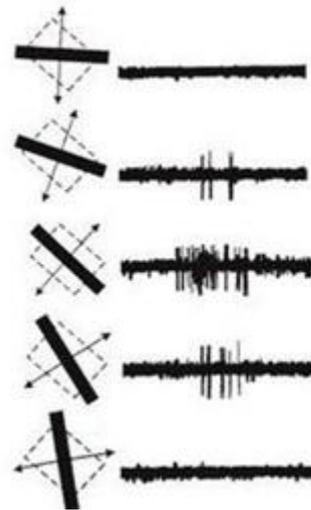
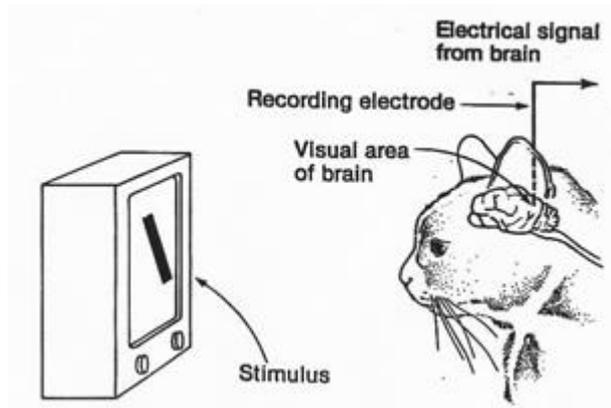


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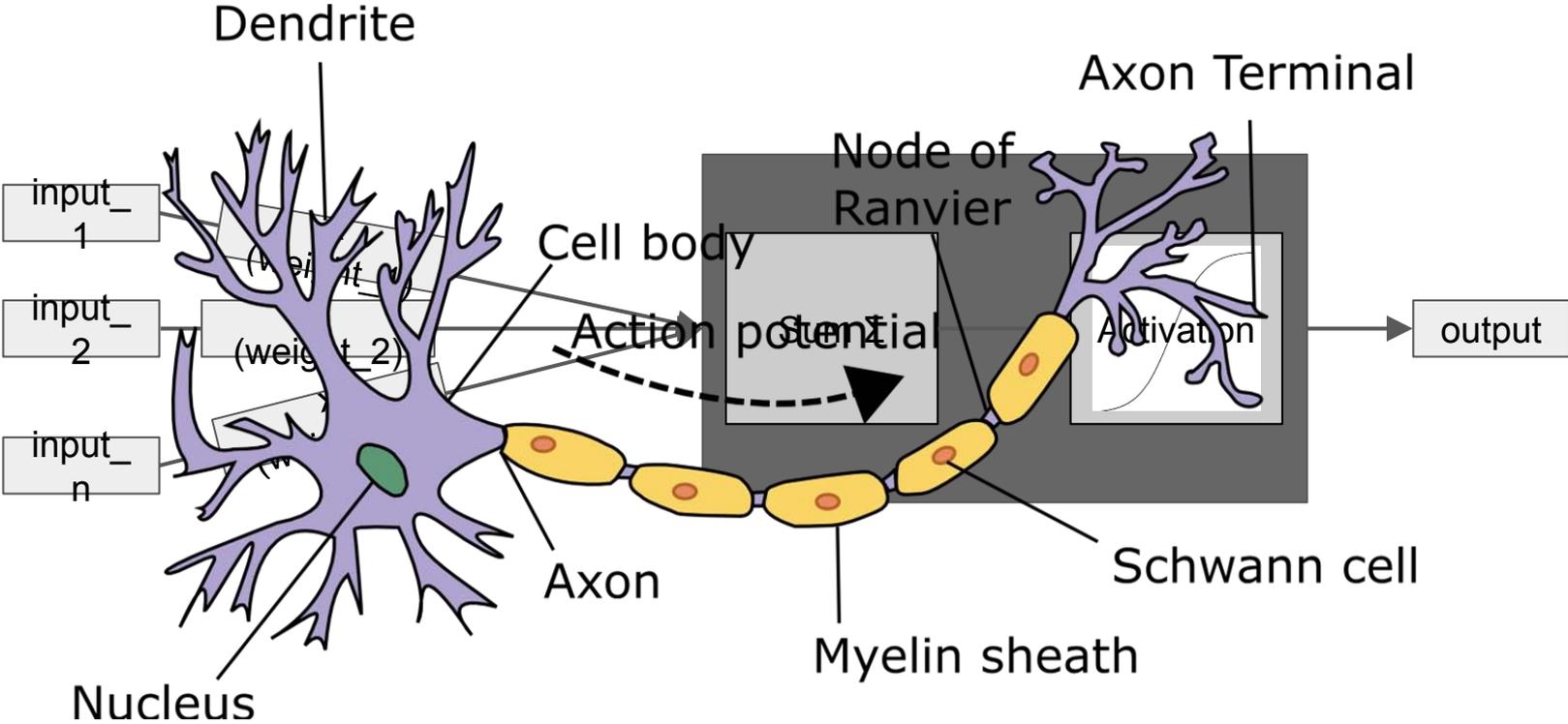
# Why Neural Net

Hubel & Wiesel (1959, 1962)

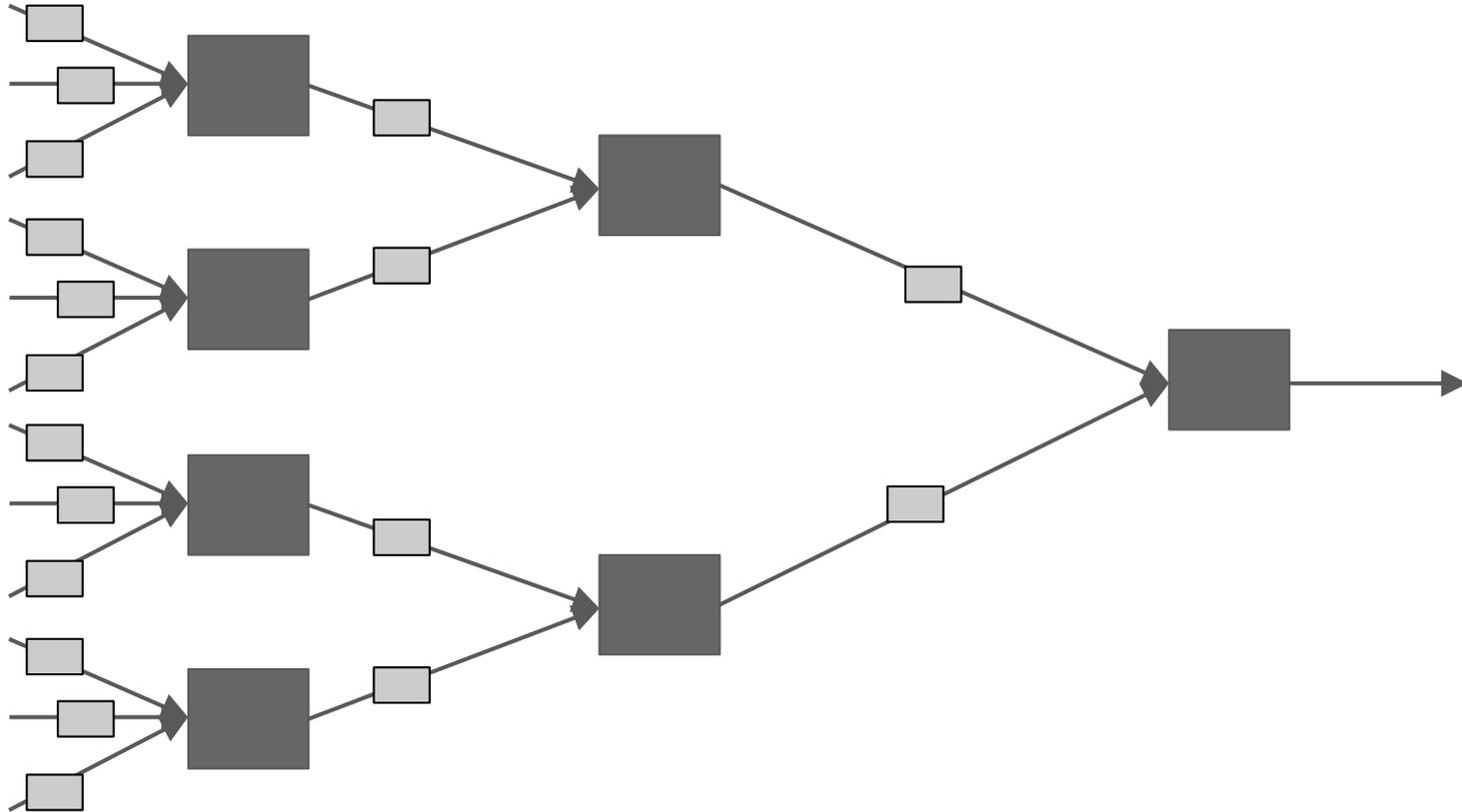


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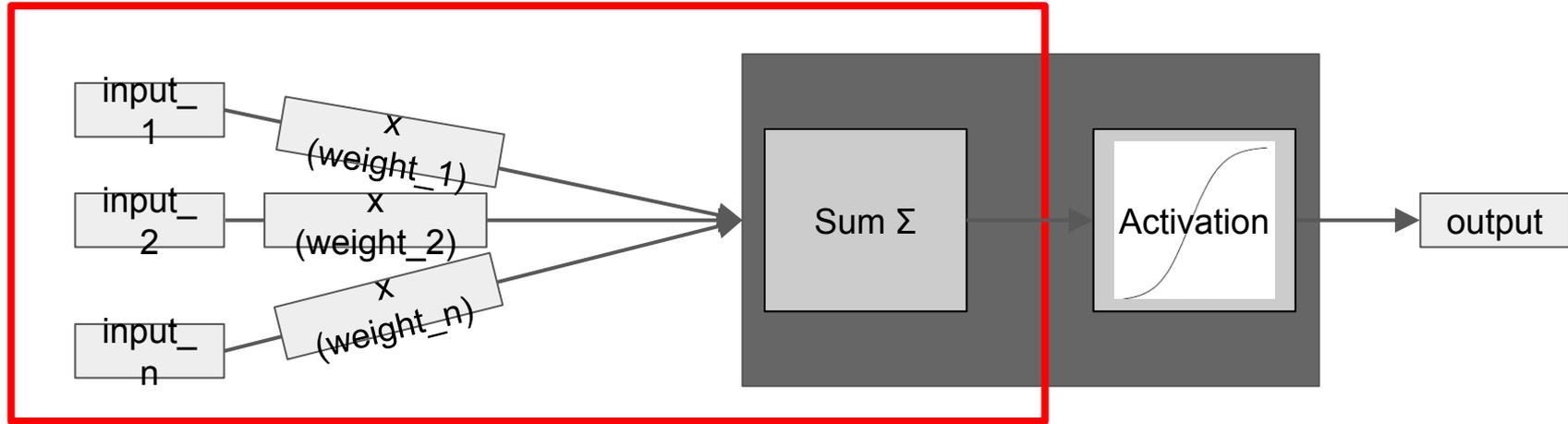
# The Structure of a Neuron



# Combining Neurons into Nets

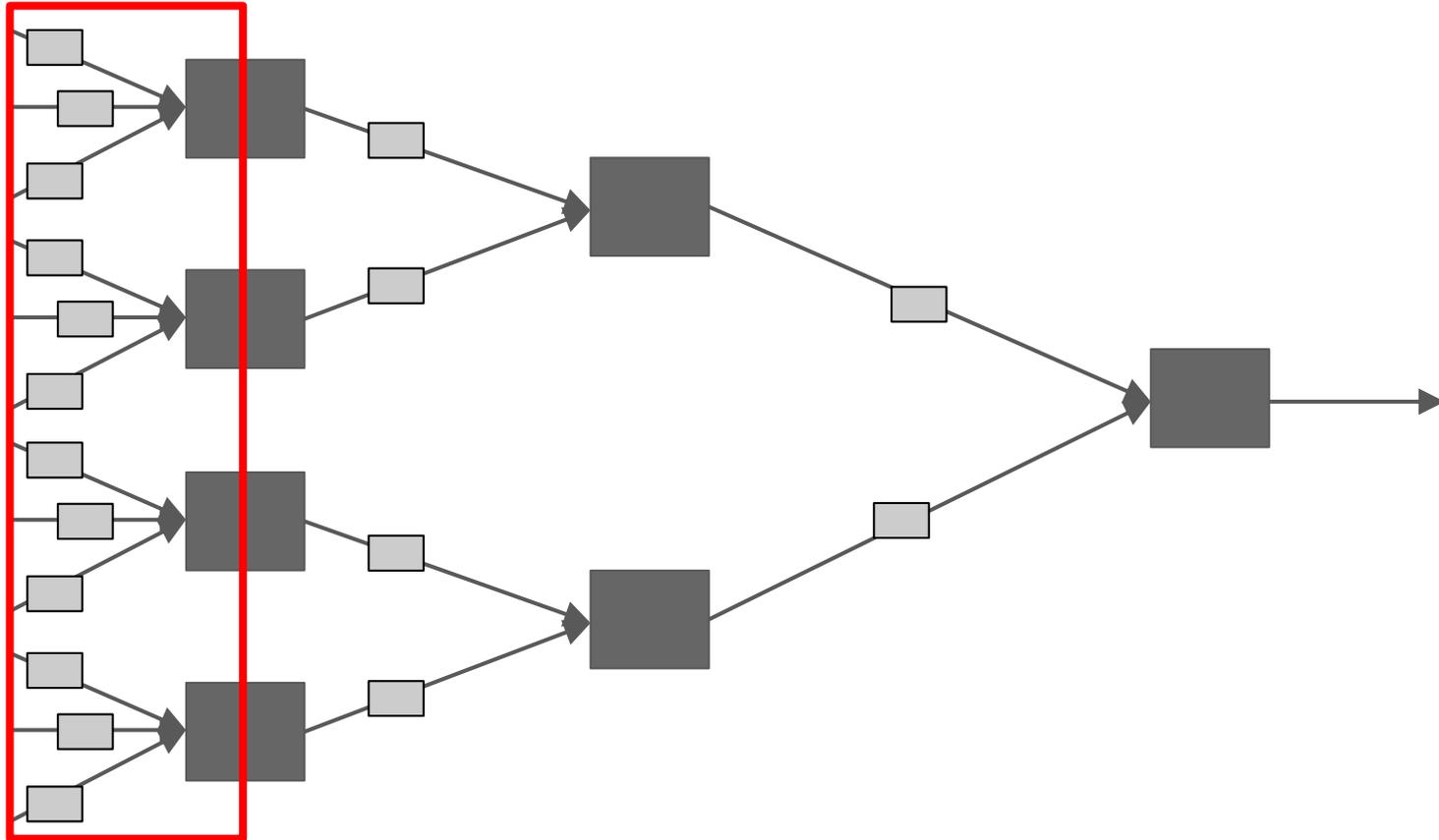


# Convolution Step

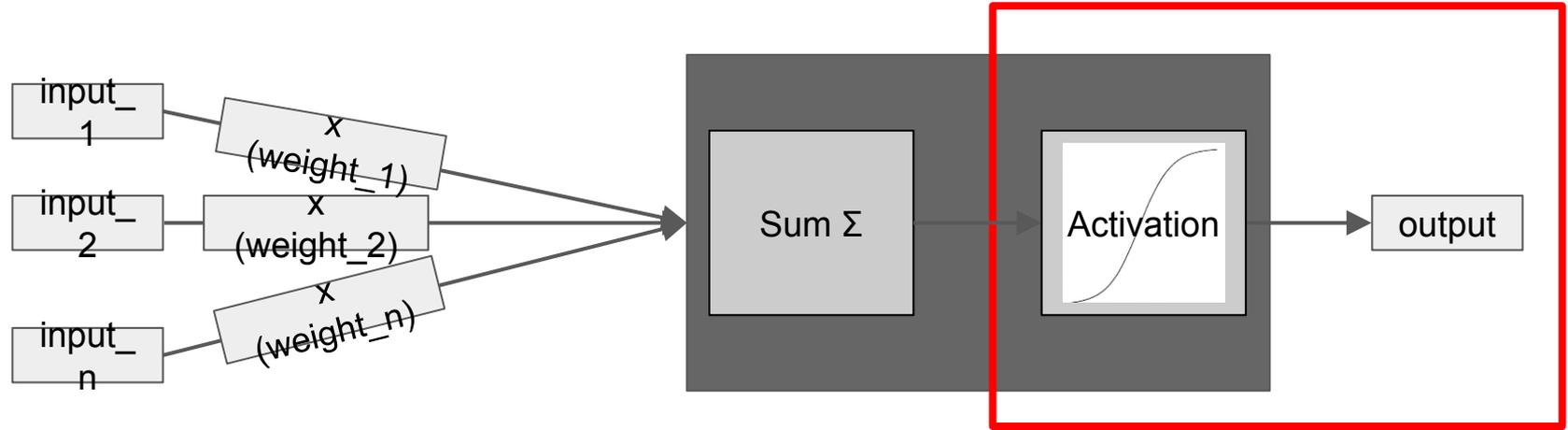


Convolution Step  
(dot product between filter and input)

# Convolutional Layer

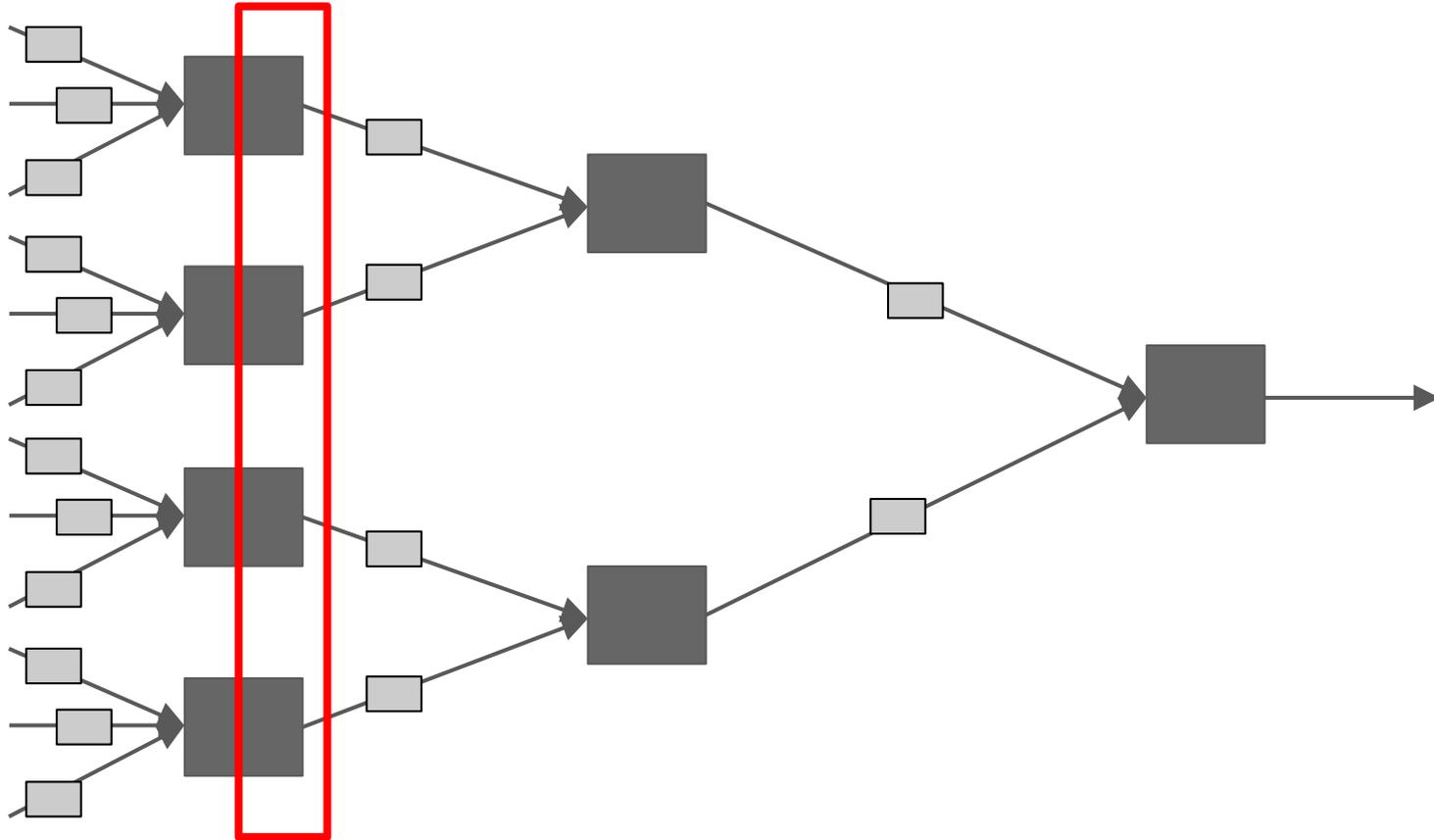


# Activation Step

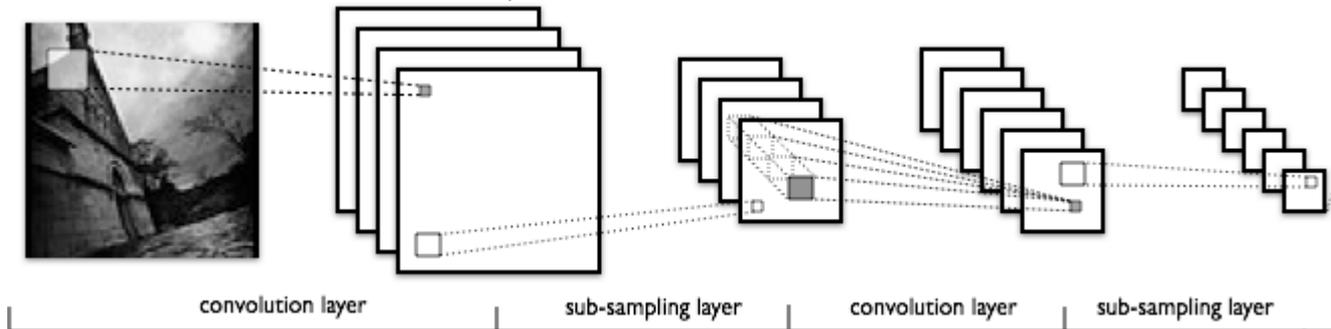
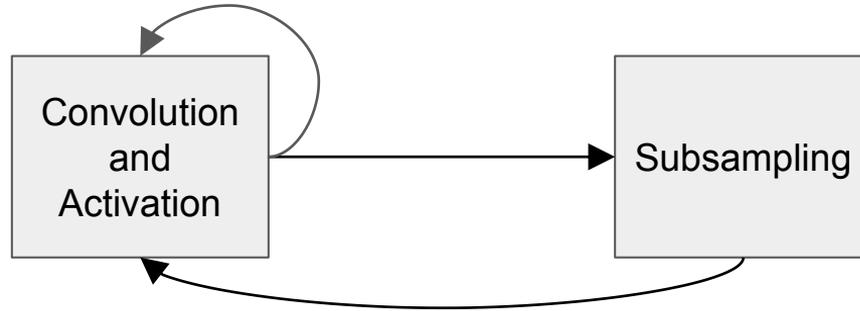


Activation Step

# Activation Layer



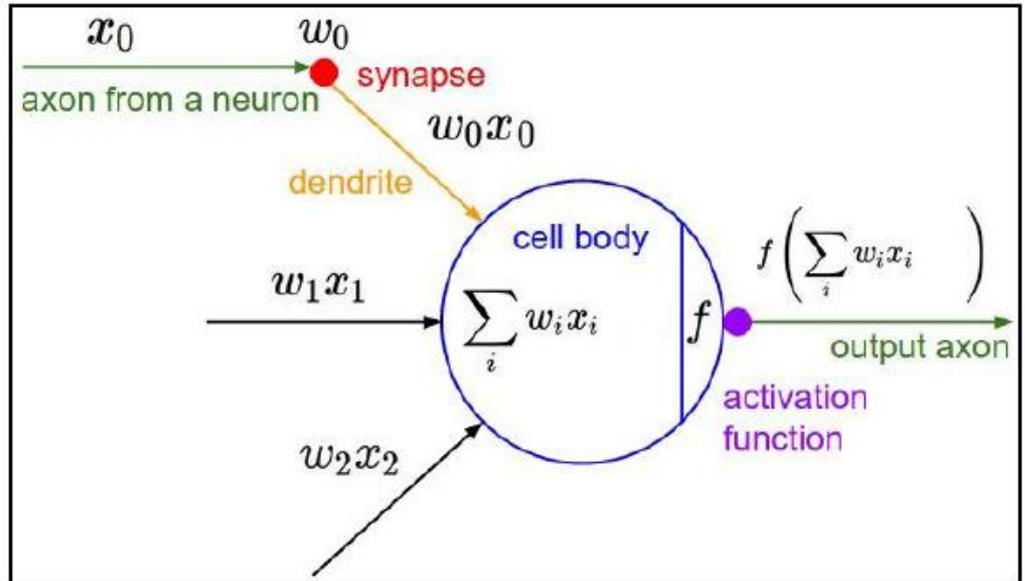
# CNN overview



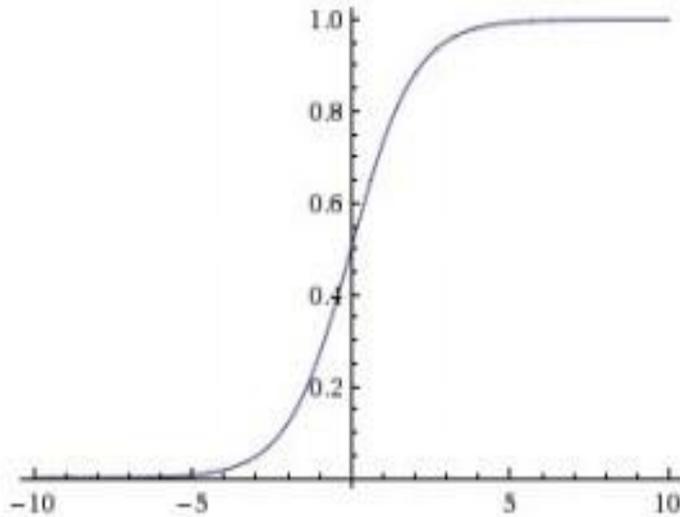
# Activation Step

Each neuron adds up its inputs, and then feeds the sum into a function -- the activation function -- to determine the neuron's output.

Eg : Sigmoid, tanh, ReLu



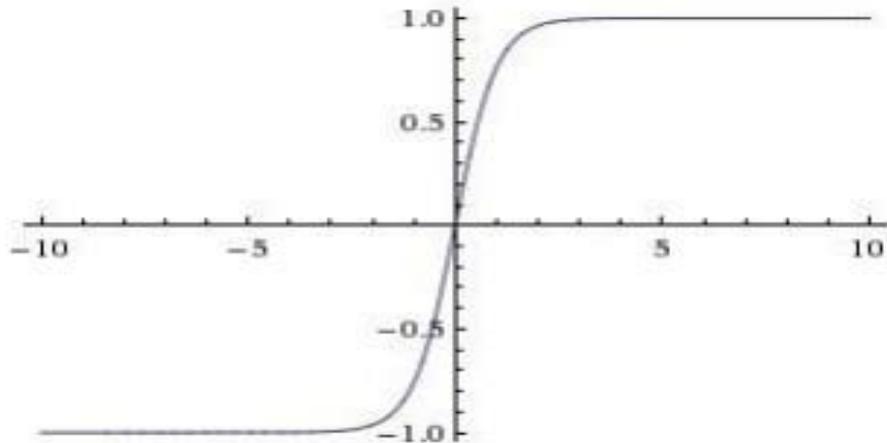
# Activation functions - sigmoid



**sigmoid activation  
function**

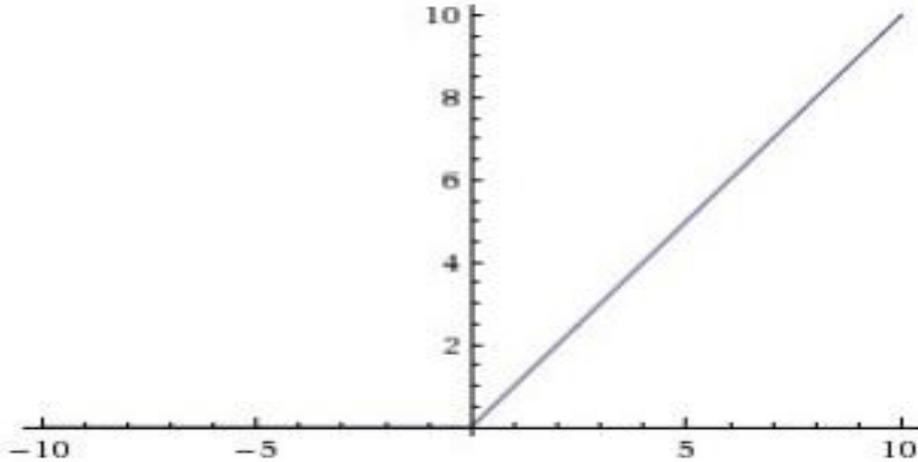
$$\frac{1}{1 + e^{-x}}$$

# Activation function - tanh



**tanh(x)**

# Activation function - ReLU



**ReLU**

$$f(x) = \max(0, x)$$

# Non-linearity Constraint

Activation function is to introduce ***non-linearity into the network***

Without a *nonlinear* activation function in the network, NN, no matter how many layers it has, will behave like a linear system and we will not be able to mimic a 'complicated' function

A neural network may very well contain neurons with linear activation functions, such as in the output layer, but these require the company of neurons with a nonlinear activation function in other parts of the network.

# Convolution Step

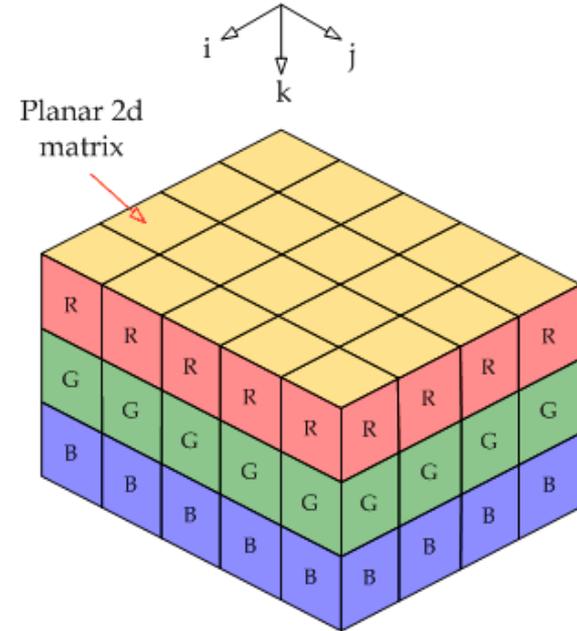
An RGB image is represented by a 3 dimensional matrix

The first channel holds the 'R' value of each pixel

The second channel holds the 'G' value of each pixel

The third channel holds the 'B' value of each pixel

Eg: A 32x32 image is represented by a 32x32x3 matrix



Graphical presentation of  
RGB 3d matrix

Filter 5x5x3

4

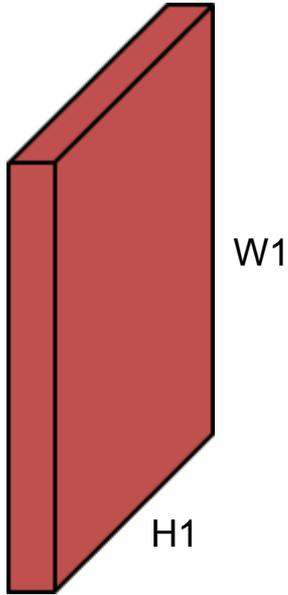


**32x32x3**

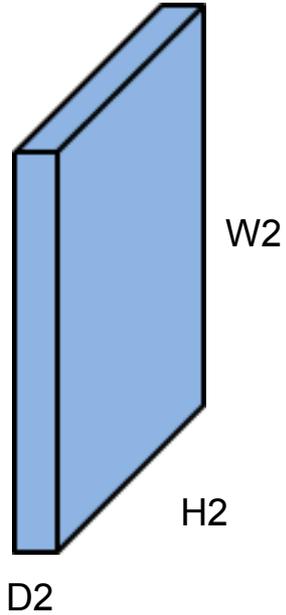


**32x32x3**

# Input Volume vs Output Volume for convolution



Input

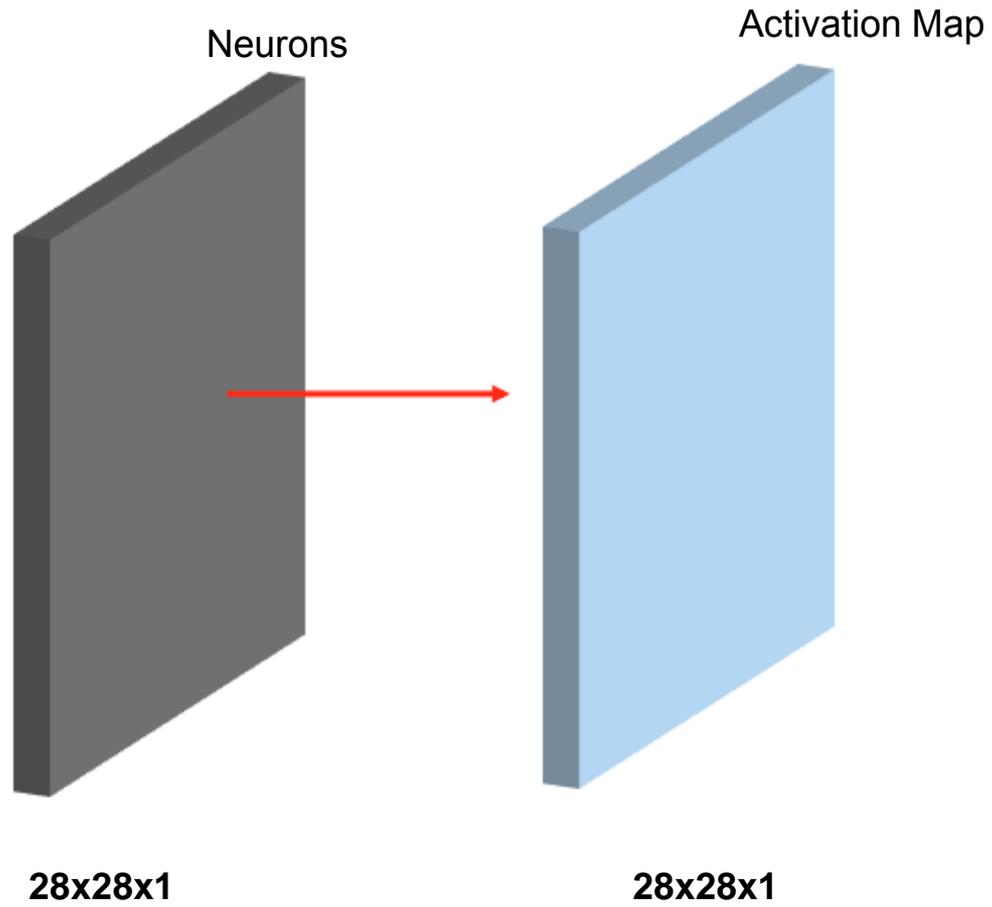


Output

$$W2 = W1 - (\text{filter width}) + 1$$

$$H2 = H1 - (\text{filter height}) + 1$$

$$D2 = 1 \text{ (} D1 = \text{filter depth)}$$





**32x32x3**

**( 28x28x1 ) \* 5**

# Parameters

Input volume:  $32 \times 32 \times 3$

Filter size :  $5 \times 5 \times 3$

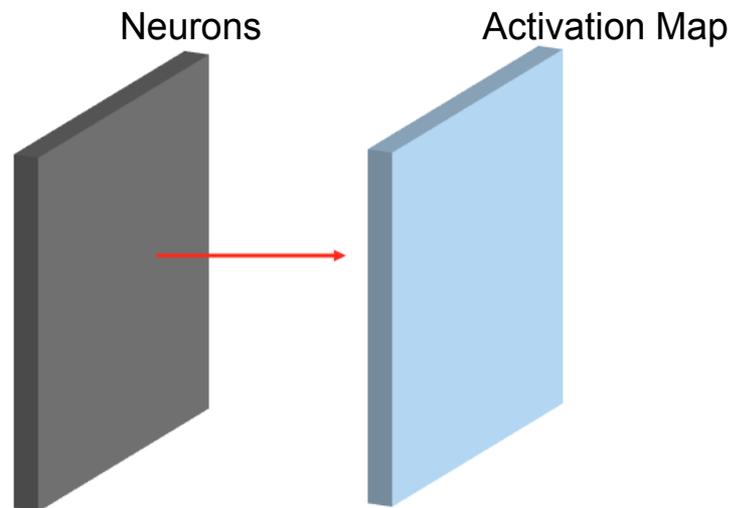
Size of 1 activation map:  $28 \times 28 \times 1$

Depth of first layer: 5

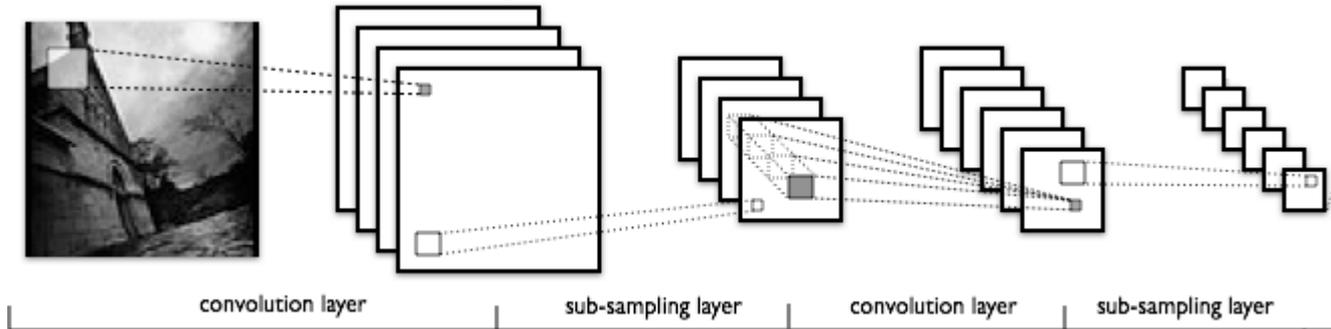
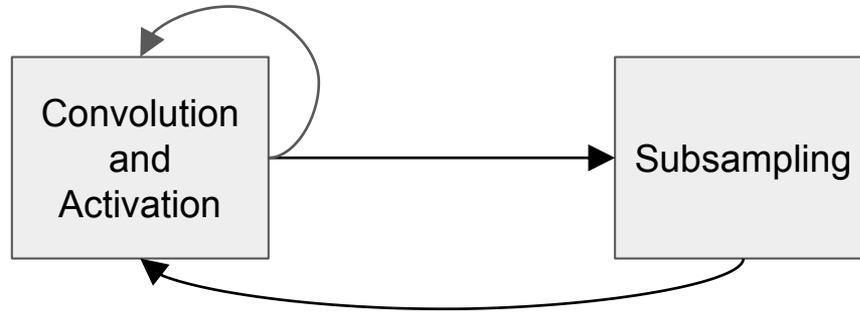
Total Number of neurons:  $28 \times 28 \times 5 = 3920$

Weights per neuron:  $5 \times 5 \times 3 = 75$

Total Number of parameters:  $75 \times 3920 = 294\ 000$



# CNN overview



# Subsampling

## Objectives:

Reduce the size of input/feature space

Keep output of the most responsive neuron of the given interest region.

## Common Methods:

- Max Pooling
- Average Pooling

This involves splitting up the matrix of filter outputs into small non-overlapping grids and taking the maximum/average

### Single depth slice

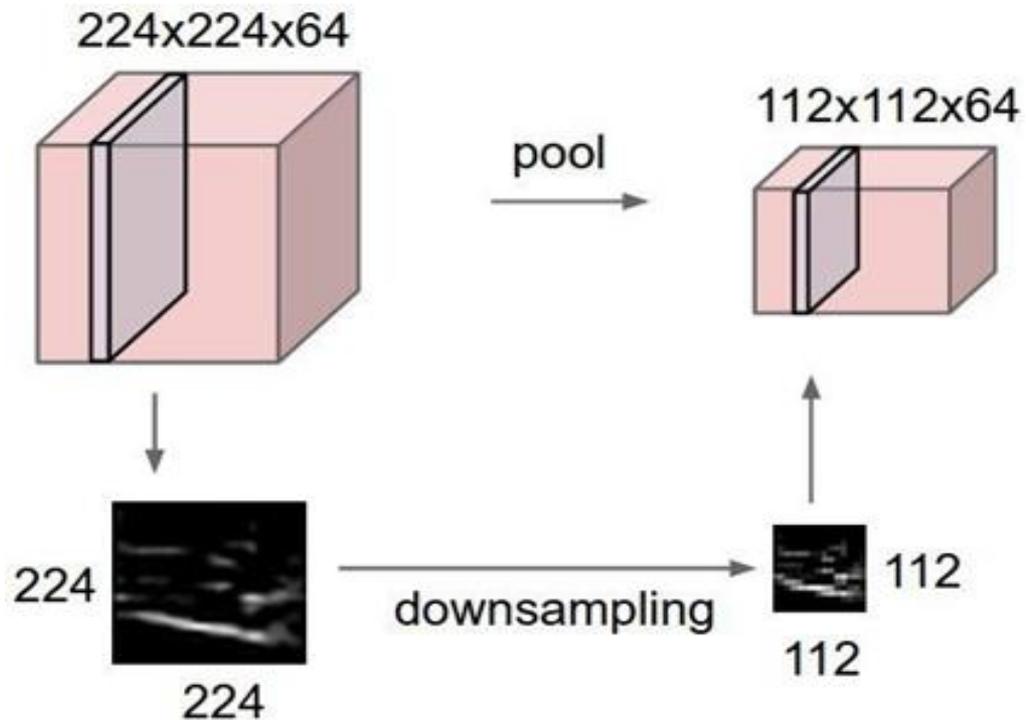
1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

max pool with 2x2 filters  
and stride 2

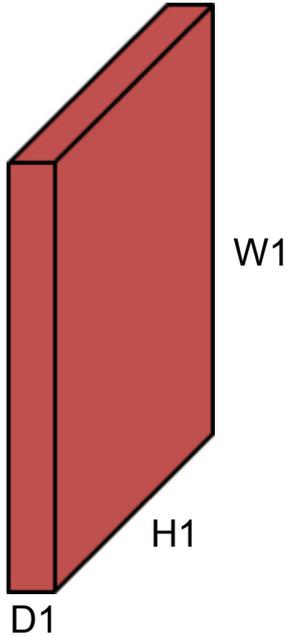


6	8
3	4

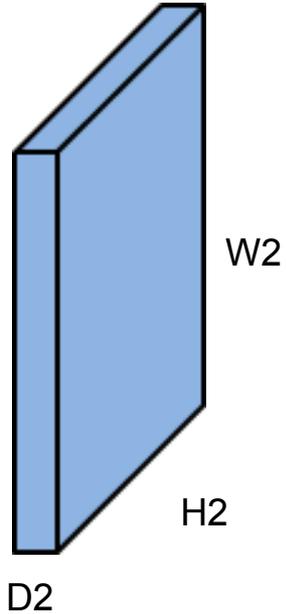
# Max Pooling



# Input Volume vs Output Volume for Max Pooling



Input



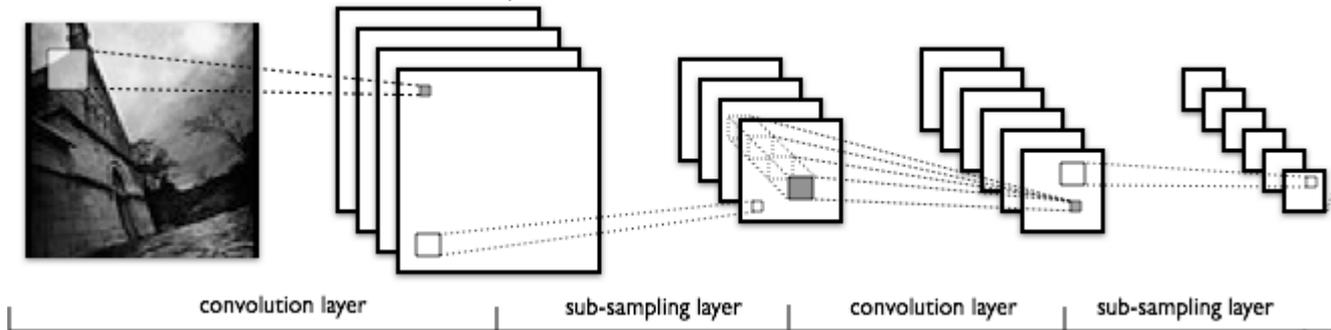
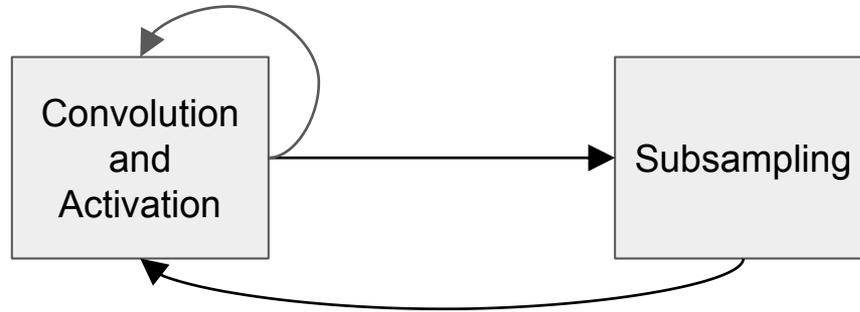
Output

$$W2 = W1 - (\text{pool width}) + 1$$

$$H2 = H1 - (\text{pool height}) + 1$$

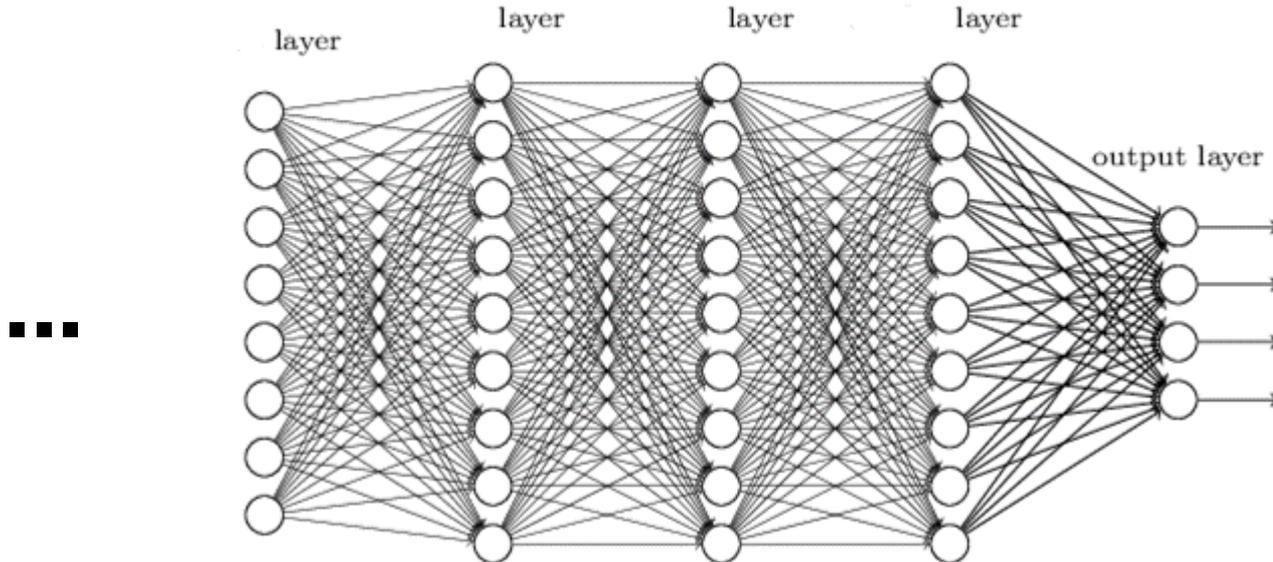
$$D2 = D1$$

# CNN overview



# Fully Connected Layer

Neurons in fully connected layers have full connections to all activations in the previous layer



# Softmax

Typically, output layer has one neuron corresponding to each label/class

The **softmax** function, or **normalized exponential**, "squashes" multi-dimensional vector of arbitrary real values to a multi-dimensional vector of values in the range (0, 1) that add up to 1.

$$P(y = j | \mathbf{x}) = \frac{e^{\mathbf{x}^T \mathbf{w}_j}}{\sum_{k=1}^K e^{\mathbf{x}^T \mathbf{w}_k}}$$

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# Train the Network (setup the problem)

- The training is in fact to find a set of weights (for the filters) that minimize the cost functions,  $C(w,b)$ .
- Normally, gradient descent algorithm is used to find the optimal
- Therefore, we need to find  $\partial C/\partial w_{ljk}$  and  $\partial C/\partial b_{lj}$ , and we update the weights and bias by:

$$w \rightarrow w - \eta \frac{\partial C}{\partial w}$$

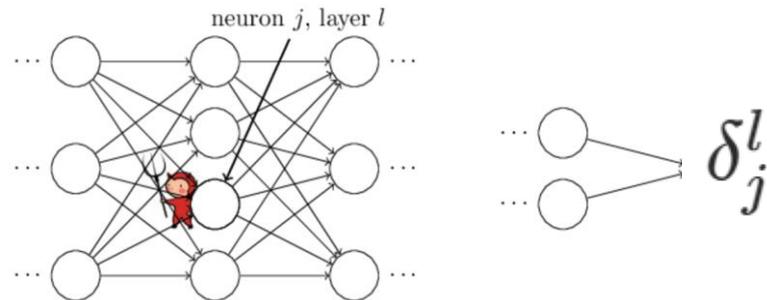
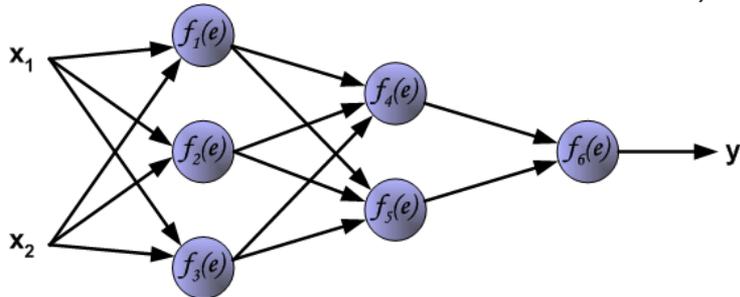
$$b \rightarrow b - \eta \frac{\partial C}{\partial b}$$

# Train the Network (compute the gradient)

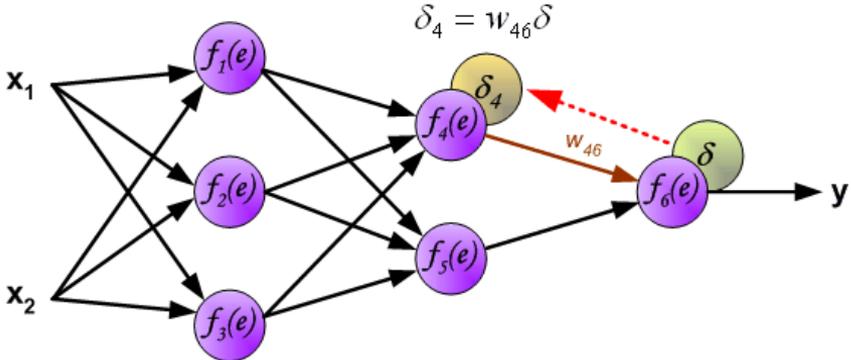
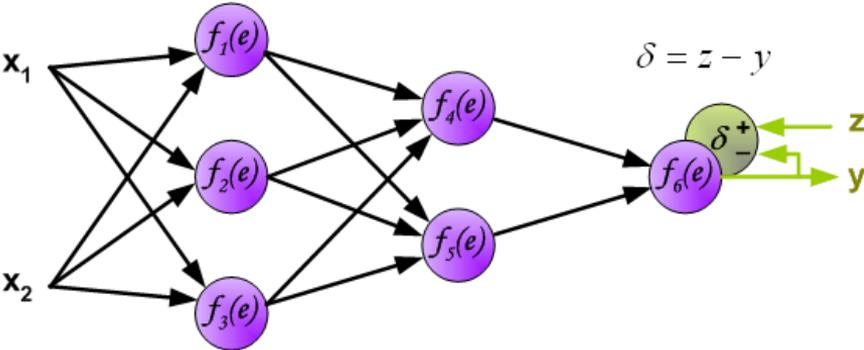
- Traditionally, for **one** training data, If using conventional method (central difference) and we have a **million** weights, the cost function,  $\mathbf{C}(\mathbf{w}, \mathbf{b})$ , will need to be calculated a **million** times !!

$$\frac{\partial C}{\partial w_j} \approx \frac{C(w + \epsilon e_j) - C(w)}{\epsilon}$$

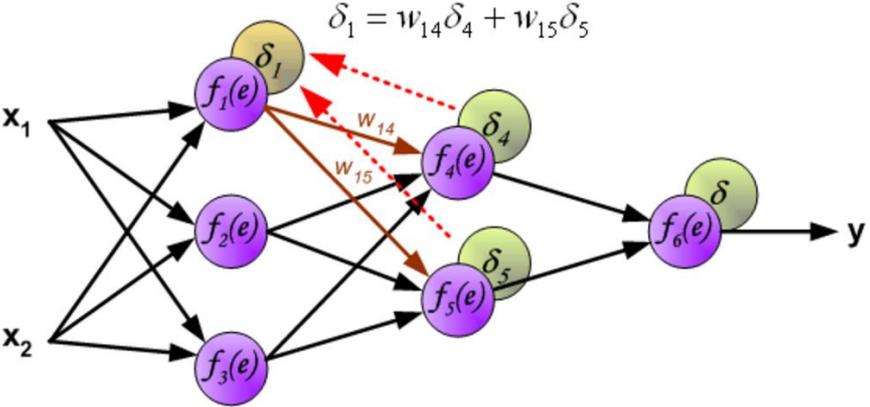
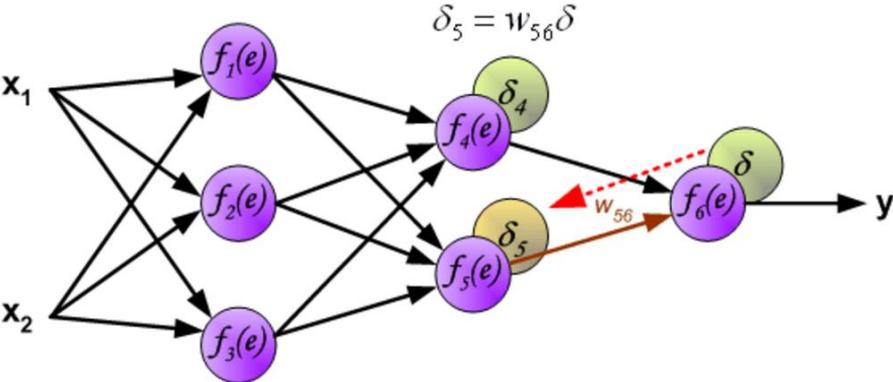
- How can we just calculate  $\mathbf{C}(\mathbf{w}, \mathbf{b})$  once? -- (Backpropagation Algorithm, [Rumelhart](#), [Hinton](#), and [Williams](#), 1986).



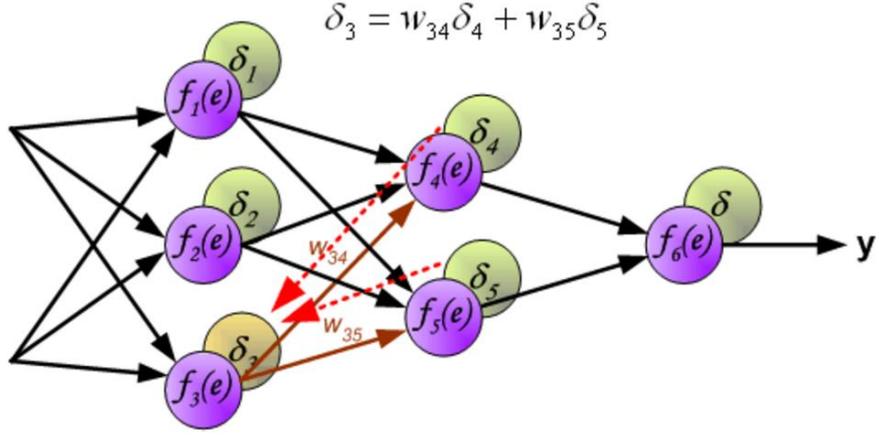
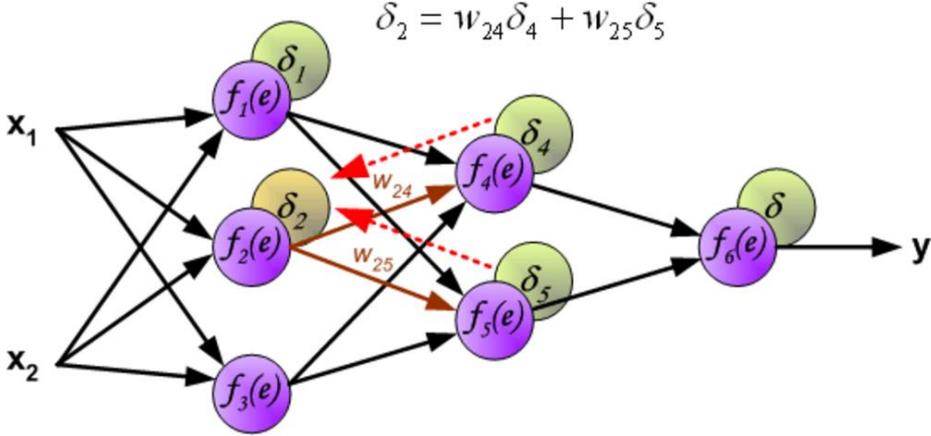
# Backward Propagation of Errors



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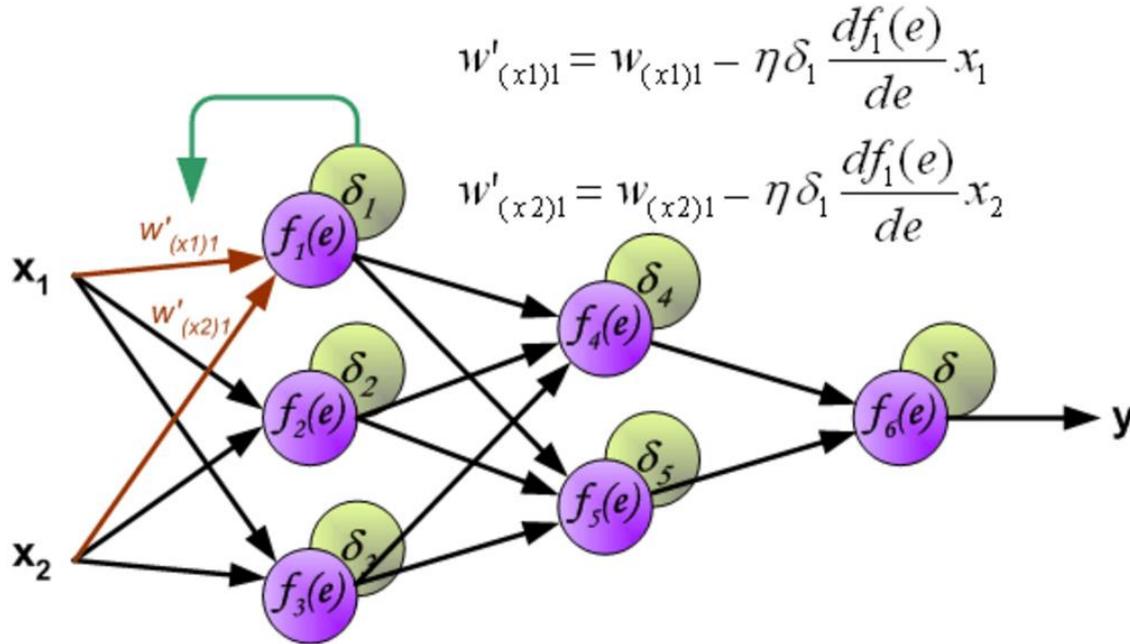
# Backward Propagation of Errors (put it together)

- **Proof:** <http://neuralnetworksanddeeplearning.com/chap2.html>

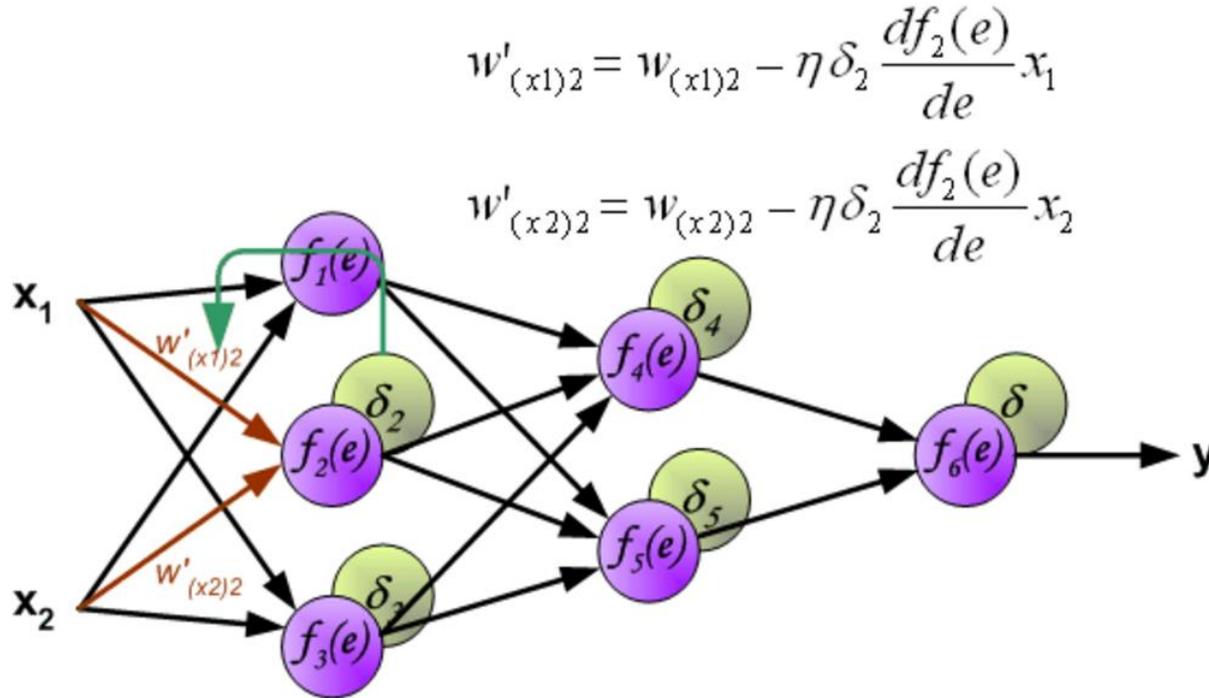
$$\frac{\partial C}{\partial w} = \delta \times \text{derivate of activation function} \\ \times \text{output from the neuron in the previous layer}$$

# Backward Propagation of Errors (put it together)

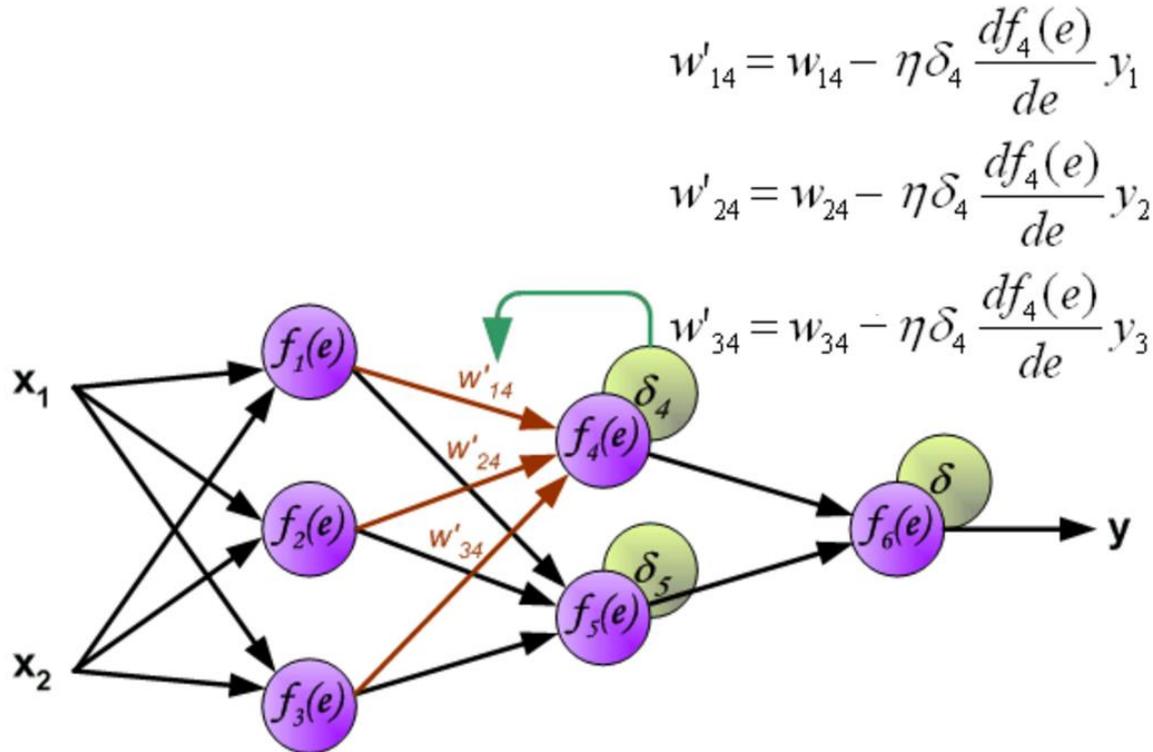
- Tutorial: <http://neuralnetworksanddeeplearning.com/chap2.html>



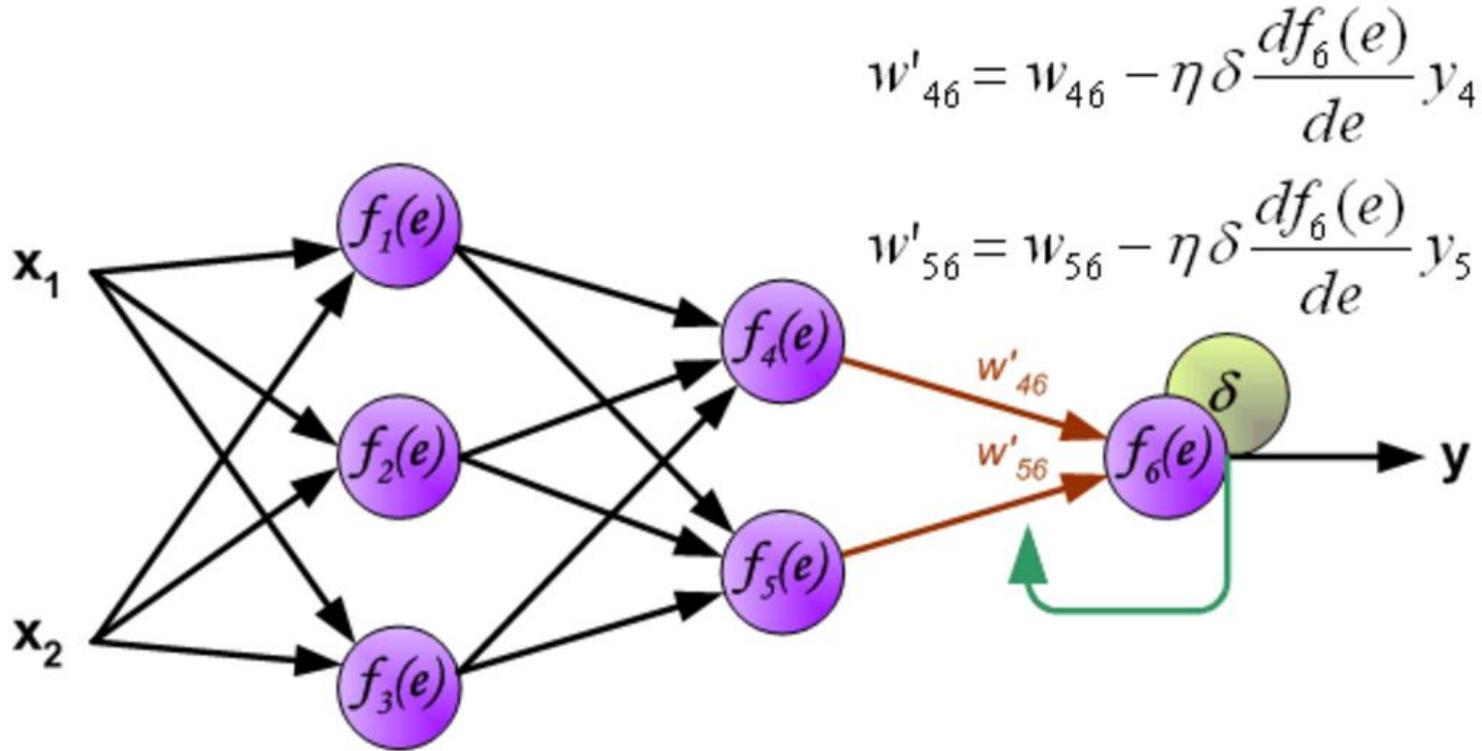
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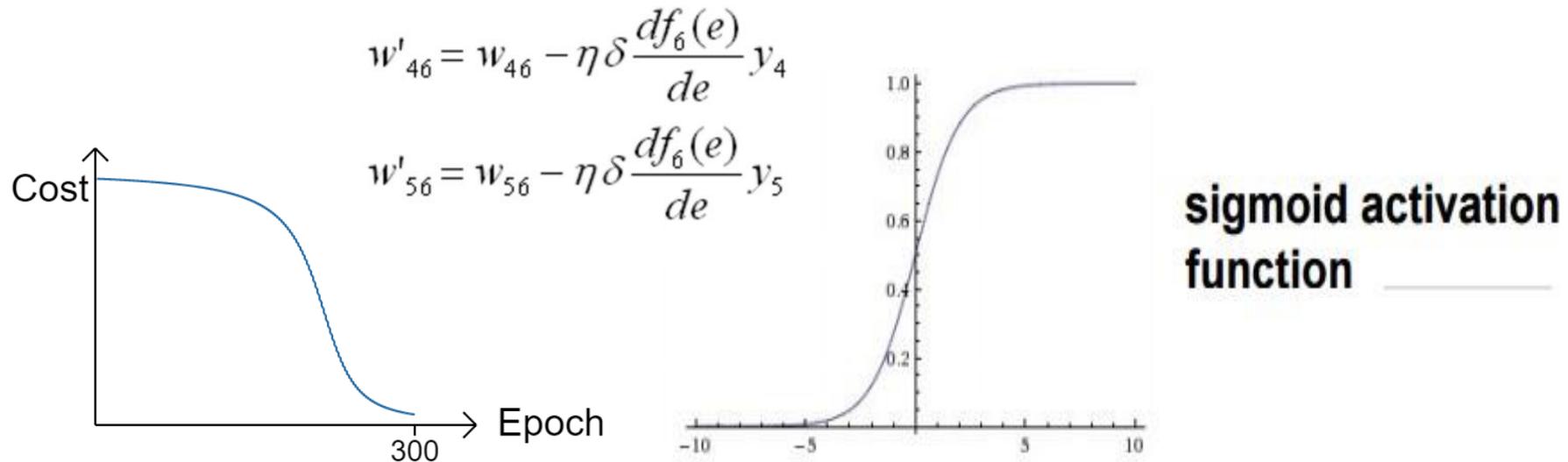


# Backward Propagation of Errors (put it together)



# Train the network (Initializing Weights)

Initialization is need for the gradient descent algorithm and it is critical for the learning performance:

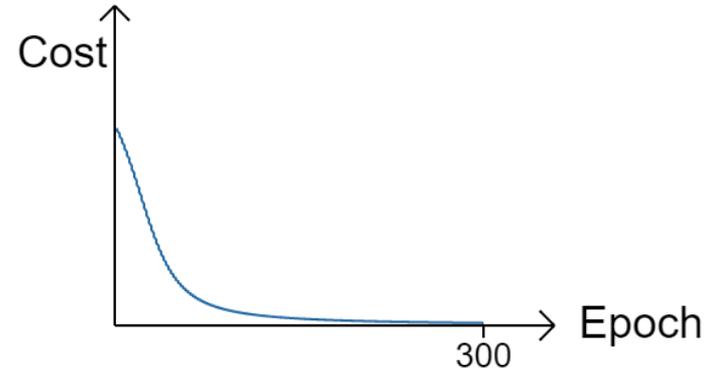
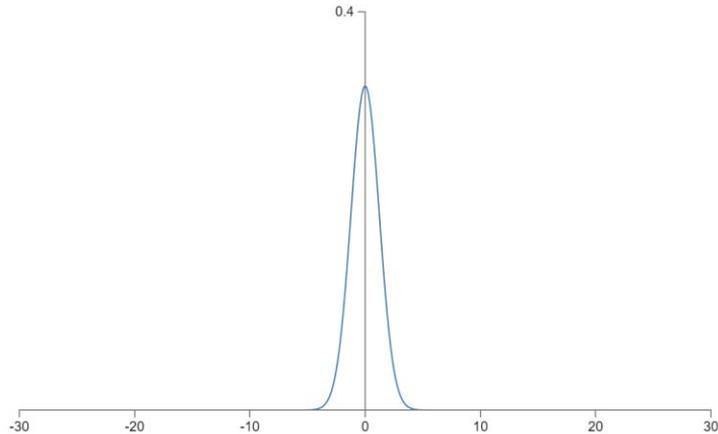


# Initial Weights

We want to stay away from the saturation area.

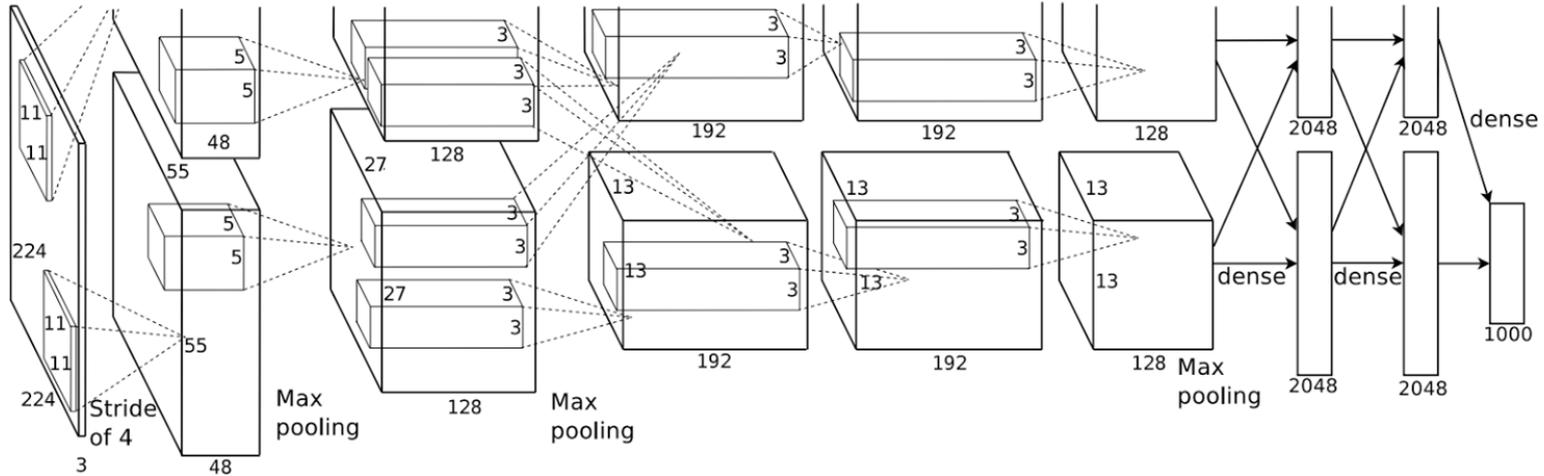
Suppose there is  $n$  weights coming in one Neuron

Best strategy is:  $\text{Normal}(0, 1/\sqrt{n_{\text{in}}})$



# Example architecture

Alex Net, 61 millions weights

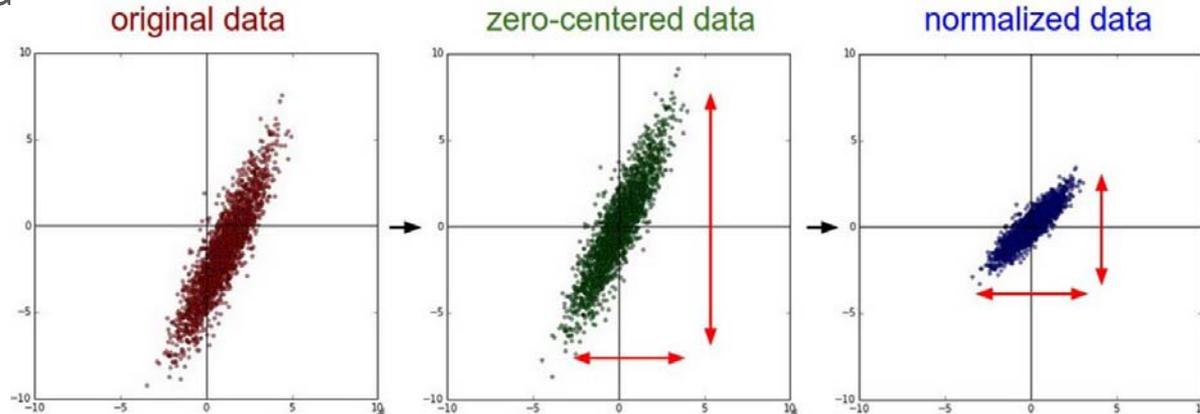


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# Preprocessing Tricks and Tips

Suppose we have dataset  $X = [N \times D]$ , where  $N$  is number of data points, and  $D$  is their dimensionality

1. Mean Image Subtraction: Subtraction of the mean across each individual feature in dataset
2. Normalization for Dimension: Division by standard deviation



# Preprocessing Tricks and Tips

## 3. Principle Component Analysis (PCA) for dimensionality reduction

- Generate covariance matrix across the data
- SVD factorization
- Decorrelation, rotation into Eigenbasis
- Choose a top-k eigenvalues:  $X' = [N \times K]$

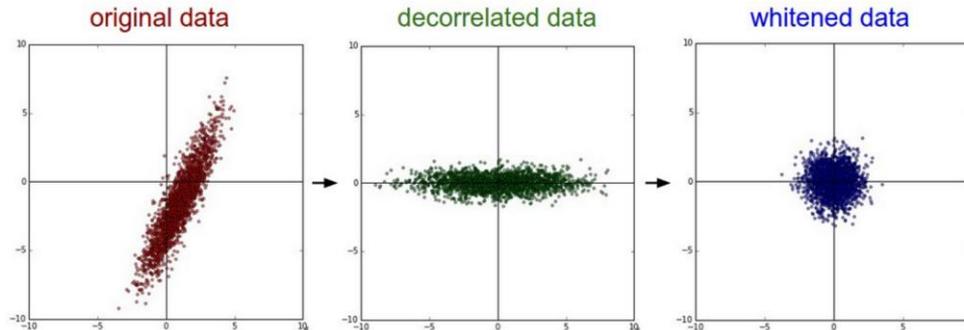
$$\Sigma = \frac{1}{m} \sum_{i=1}^m (x^{(i)})(x^{(i)})^T.$$

$$U = \begin{bmatrix} | & | & \cdots & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & \cdots & | \end{bmatrix}$$

## 4. Whitening

- Divide by eigenvalues (square roots of singular  $v$ )
- Result: Zero mean, Identity Covariance

$$x_{\text{rot}} = U^T x = \begin{bmatrix} u_1^T x \\ u_2^T x \end{bmatrix}$$



# Data Augmentation

1. Rotations
2. Reflections
3. Scaling
4. Cropping
5. Color space remapping



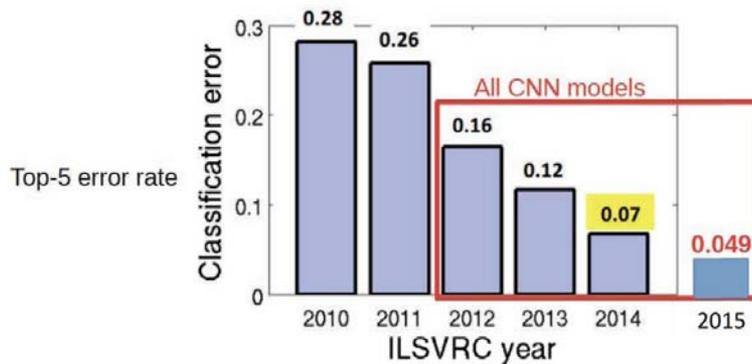
6. *Randomization!*

# Overview

- What is Deep Learning?
- Image Processing
- CNN Architecture
- Training Process
- **Image Classification Results**
- Limitations

# Revisiting the ImageNet Competition (ILSVRC 2010)

Model	Top-1 error rate	Top-5 error rate
<i>Sparse coding</i>	0.47	0.28
<i>SIFT + FVs</i>	0.46	0.26
CNNs	0.37	0.17





**mite      container ship      motor scooter      leopard**

	<b>mite</b> black widow cockroach tick starfish		<b>container ship</b> lifeboat amphibian fireboat drilling platform		<b>motor scooter</b> go-kart moped bumper car golfcart		<b>leopard</b> jaguar cheetah snow leopard Egyptian cat
--	---	--	---	--	--	--	---



**grille      mushroom      cherry      Madagascar cat**

	<b>grille</b> pickup beach wagon fire engine		<b>mushroom</b> agaric mushroom jelly fungus gill fungus dead-man's-fingers		<b>cherry</b> dalmatian grape elderberry ffordshire bullterrier currant		<b>Madagascar cat</b> squirrel monkey spider monkey titi indri howler monkey
--	---	--	--	--	--	--	---

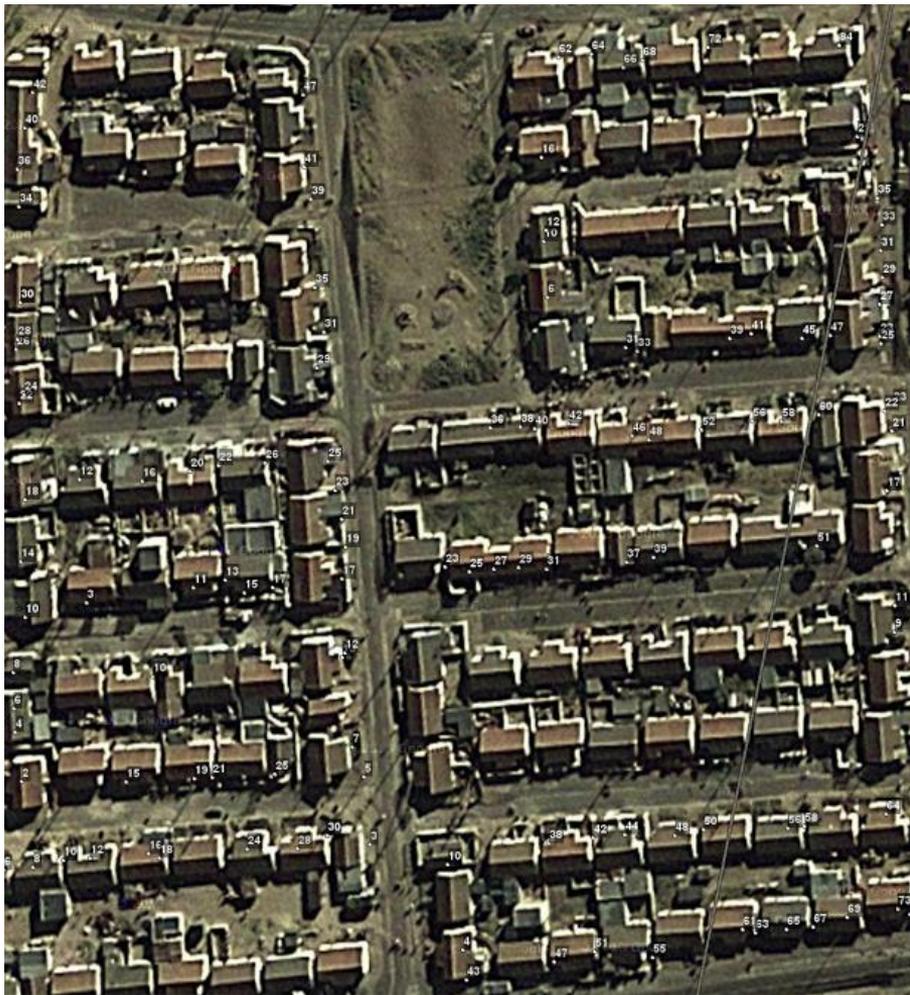
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# Google Street View House Numbers



\*\*"Multi-digit Number Recognition from Street View Imagery using Deep Convolutional Neural Networks" by Ian J. Goodfellow, Yaroslav Bulatov, Julian Ibarz, Sacha Arnoud, Vinay Shet

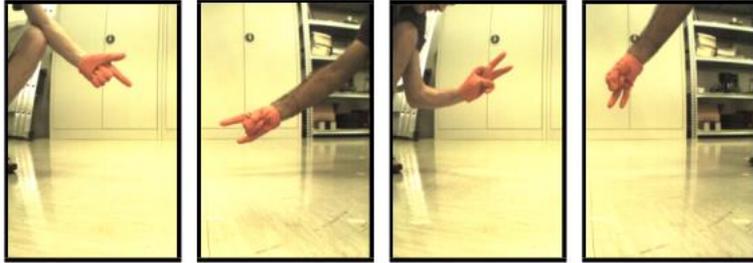
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“Multi-digit Number Recognition from Street View Imagery using Deep Convolutional Neural Networks” by Ian J. Goodfellow, Yaroslav Bulatov, Julian Ibarz, Sacha Arnoud, Vinay Shet

# Recognizing Hand Gestures-HCI application

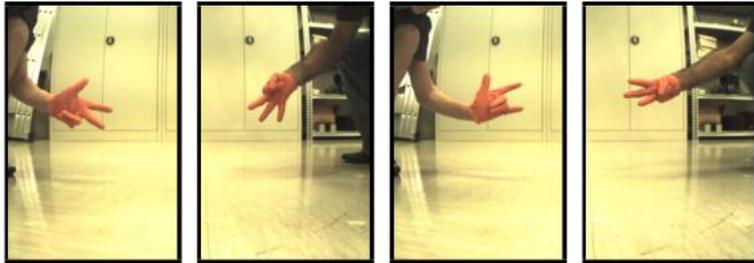


(a)

(b)

(c)

(d)



(e)

(f)

(g)

(h)

N. Jawad, D. Frederick, A. Gianni, C. Dan and M. Ueli, "Max-pooling convolutional neural networks for vision-based hand gesture recognition", IEEE International Conference on Signal and Image Processing Applications, 2011.

# Extended Image Classification: Video Classification

Extend image classification by adding temporal component to classify videos

Note that this adds additional complexity, but the underlying system is the same: Convolutional Neural Nets



track cycling  
cycling  
track cycling  
road bicycle racing  
marathon  
ultramarathon



ultramarathon  
ultramarathon  
half marathon  
running  
marathon  
inline speed skating



heptathlon  
heptathlon  
decathlon  
hurdles  
pentathlon  
sprint (running)



bikejoring  
mushing  
bikejoring  
harness racing  
skijoring  
carting



longboarding  
longboarding  
aggressive inline skating  
freestyle scootering  
freeboard (skateboard)  
sandboarding



ultimate (sport)  
ultimate (sport)  
hurling  
flag football  
association football  
rugby sevens



demolition derby  
demolition derby  
monster truck  
mud bogging  
motocross  
grand prix motorcycle racing



telemark skiing  
snowboarding  
telemark skiing  
nordic skiing  
ski touring  
skijoring



whitewater kayaking  
whitewater kayaking  
rafting  
kayaking  
canoeing  
adventure racing



arena football  
indoor american football  
arena football  
canadian football  
american football  
women's lacrosse



reining  
barrel racing  
rodeo  
reining  
cowboy action shooting  
bull riding



eight-ball  
nine-ball  
blackball (pool)  
trick shot  
eight-ball  
straight pool

New Text

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# Overview

- What is Deep Learning?
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# Even the Best have Issues

Microsoft won the most recent ImageNet competition and currently holds the state-of-the-art implementation

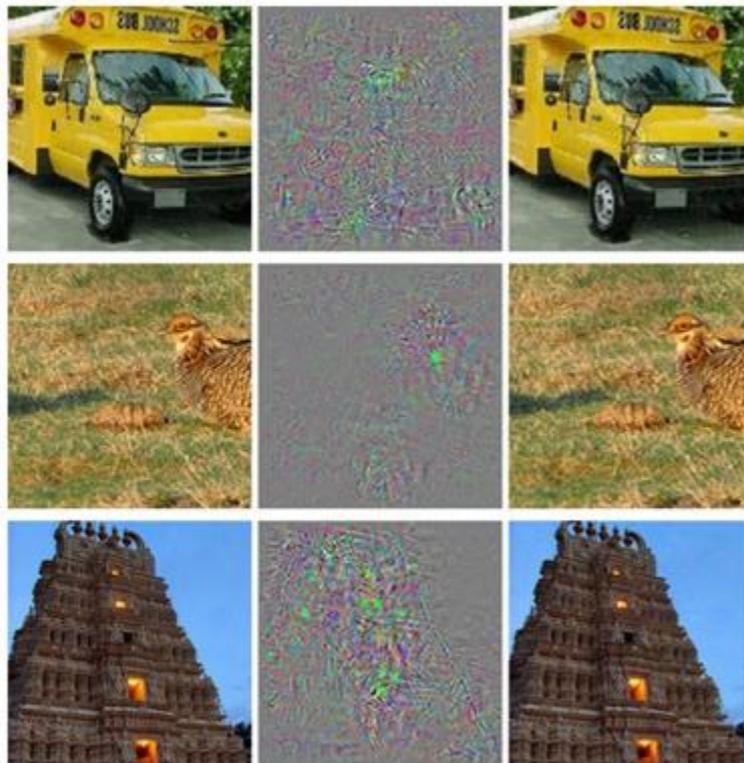
They can recognize 1000 categories of images, extremely reliably.

However:

- 1000 categories does not cover as many objects as you might expect.

- Uses 1.28 million images to train

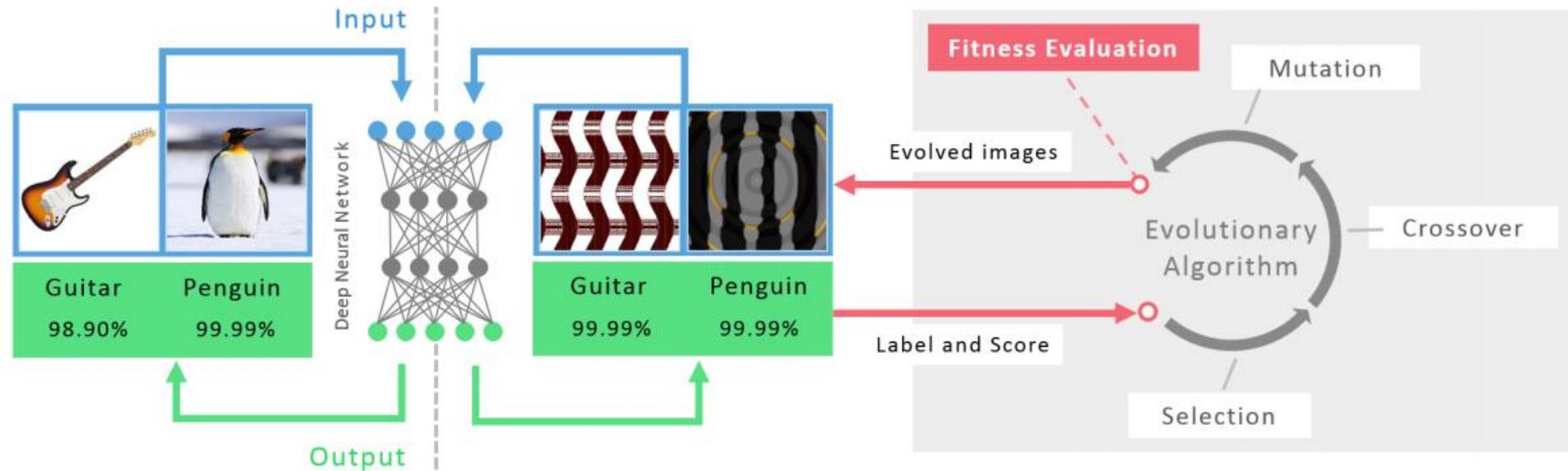
- Takes weeks to train on multiple GPUs, with heavy optimization



correct                      +distort                      ostrich

Courtesy of Szegedy, Christian et al. License: CC-BY.

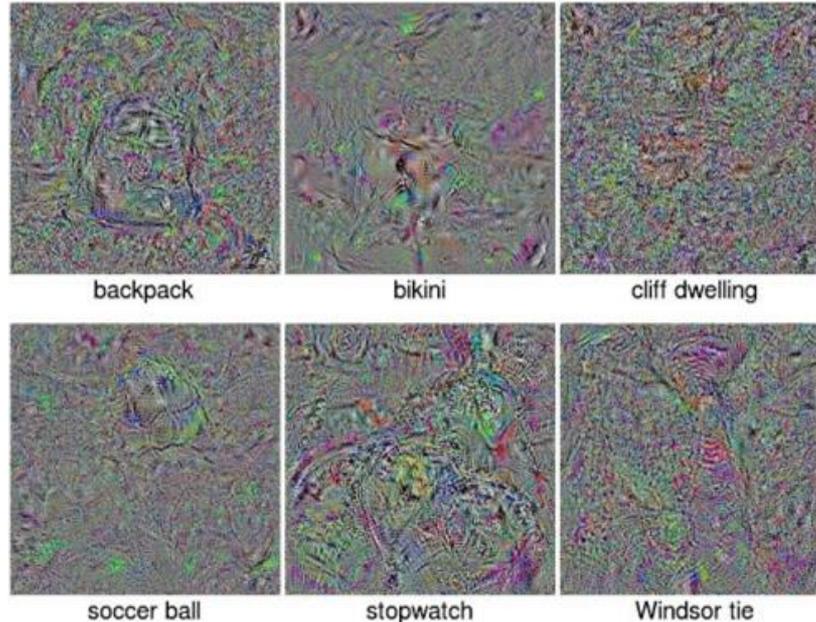
Szegedy et al. Intriguing Properties of Neural Networks. 2014.



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Nguyen A, Yosinski J, Clune J. Deep Neural Networks are Easily Fooled: High Confidence Predictions for Unrecognizable Images. In Computer Vision and Pattern Recognition (CVPR '15), IEEE, 2015.

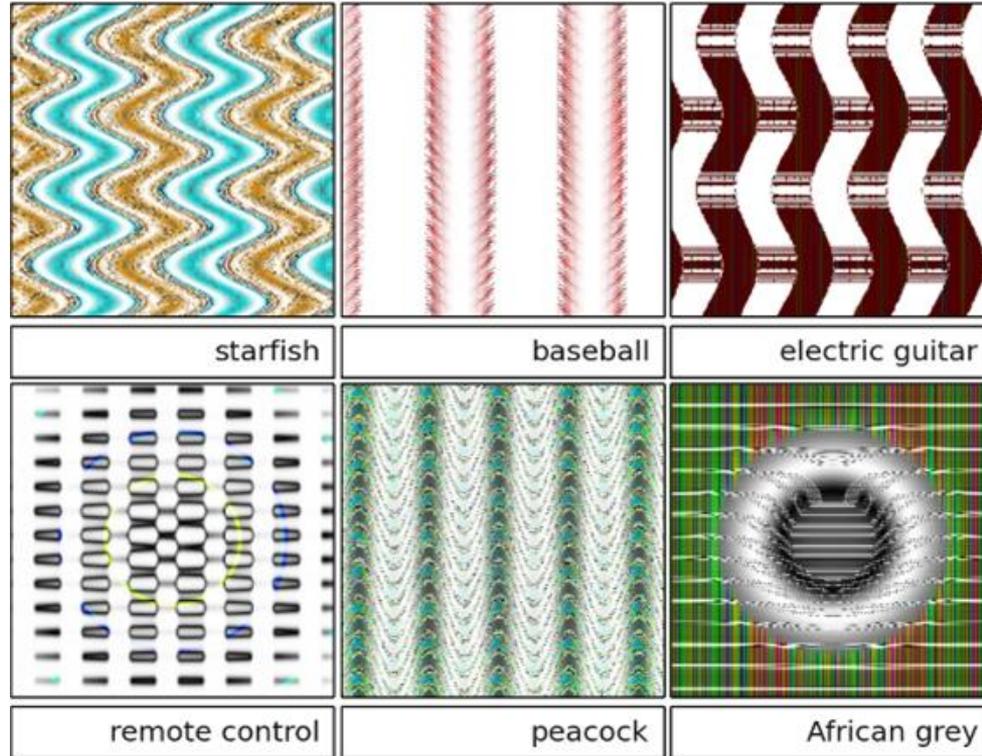
# Gradient Ascent



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Nguyen A, Yosinski J, Clune J. Deep Neural Networks are Easily Fooled: High Confidence Predictions for Unrecognizable Images. In Computer Vision and Pattern Recognition (CVPR '15), IEEE, 2015.

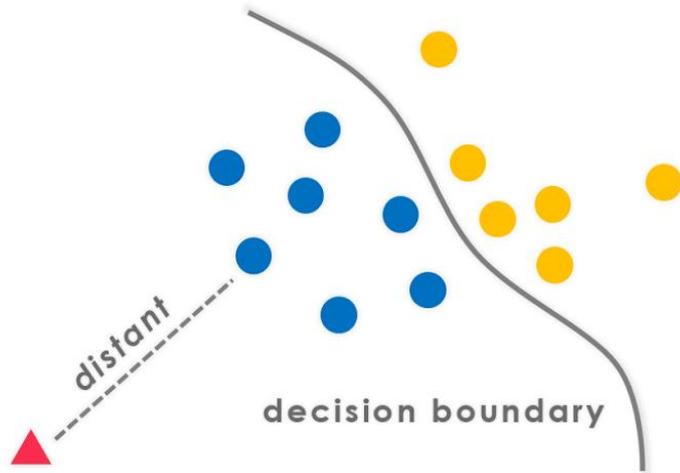
# Indirect Encoding



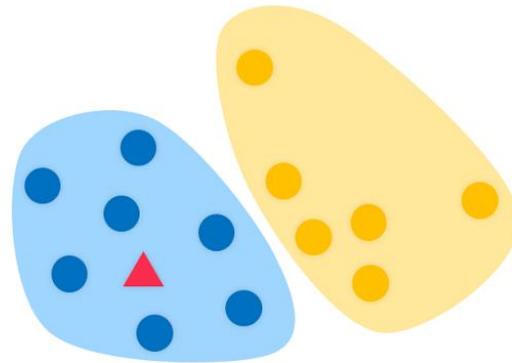
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Nguyen A, Yosinski J, Clune J. Deep Neural Networks are Easily Fooled: High Confidence Predictions for Unrecognizable Images. In Computer Vision and Pattern Recognition (CVPR '15), IEEE, 2015.

**Discriminative**



**Generative**

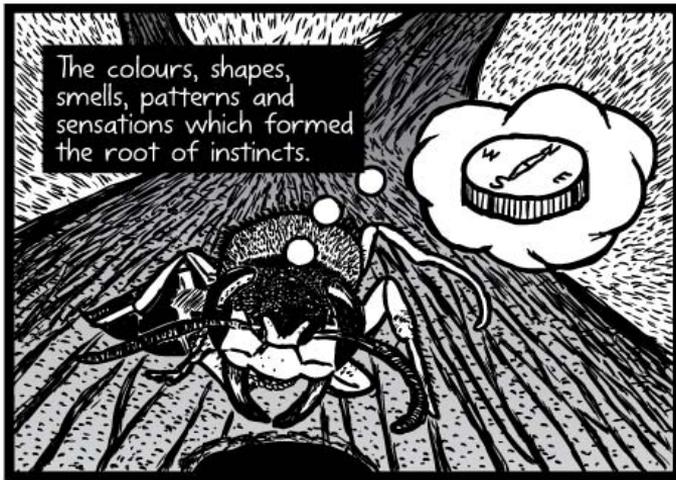




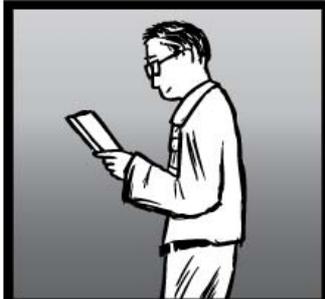








The colours, shapes, smells, patterns and sensations which formed the root of instincts.



Tinbergen succeeded in isolating the traits which triggered certain instincts...



...and then made an interesting discovery.



The instincts had no bounds.

Instead of stopping at a 'sweet spot', the instinctive response would still be produced by unrealistic stimuli.



Once the researchers isolated the instincts' trigger...

...they could create greatly exaggerated dummies which the animals would choose instead of a realistic alternative.



# Takeaways

- Deep Learning is a powerful tool that relies on many iterations of processing
- CNNs outperform all other algorithms for image classification because of the image processing power of convolutional filters
- Backpropagation is used to efficiently train CNNs
- CNNs need tons of data and processing power

# Getting Started With Deep Learning



# References

ImageNet Classification with Deep Convolutional Neural Networks. Alex Krizhevsky, Ilya Sutskever, Geoffrey E. Hinton. <http://www.cs.toronto.edu/~fritz/absps/imagenet.pdf>

Approximation by Superpositions of a Sigmoidal Function. G. Cybenko  
[https://www.dartmouth.edu/~gvc/Cybenko\\_MCSS.pdf](https://www.dartmouth.edu/~gvc/Cybenko_MCSS.pdf)

Backpropagation Tutorial <http://neuralnetworksanddeeplearning.com/chap2.html>

<https://papers.nips.cc/paper/877-the-softmax-nonlinearity-derivation-using-statistical-mechanics-and-useful-properties-as-a-multiterminal-analog-circuit-element.pdf>

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Deep Residual Learning for Image Recognition. Kaiming He, Xiangyu Zhang, Shaoqing Ren, Jian Sun.  
<http://arxiv.org/abs/1512.03385>

# Appendix

# Backward Propagation of Errors

- The gradient of weights and bias can be found by back chaining the auxiliary variable, defined as:

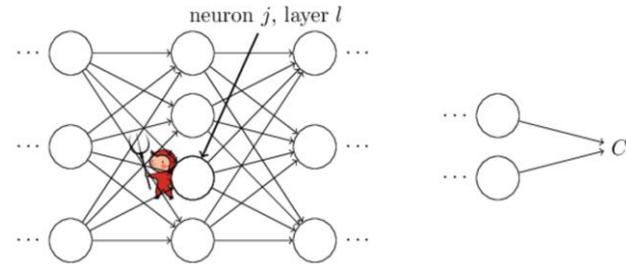
$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l} \quad z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l \quad a^l = \sigma(z^l)$$

- By chain rule:

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \sigma'(z_j^L) \quad \delta^L = \nabla_a C \odot \sigma'(z^L)$$

- The back propagate it (chain rule again):

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$$



# Backward Propagation of Errors (put it together)

**Summary:** the equations of backpropagation

$$\delta^L = \nabla_a C \odot \sigma'(z^L)$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$$

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l$$

# Train the Network (put it together)

1. **Input a set of training examples**

2. **For each training example  $x$ :** Set the corresponding input activation  $a^{x,1}$ , and perform the following steps:

◦ **Feedforward:** For each  $l = 2, 3, \dots, L$  compute

$$z^{x,l} = w^l a^{x,l-1} + b^l \text{ and } a^{x,l} = \sigma(z^{x,l}).$$

◦ **Output error  $\delta^{x,L}$ :** Compute the vector

$$\delta^{x,L} = \nabla_a C_x \odot \sigma'(z^{x,L}).$$

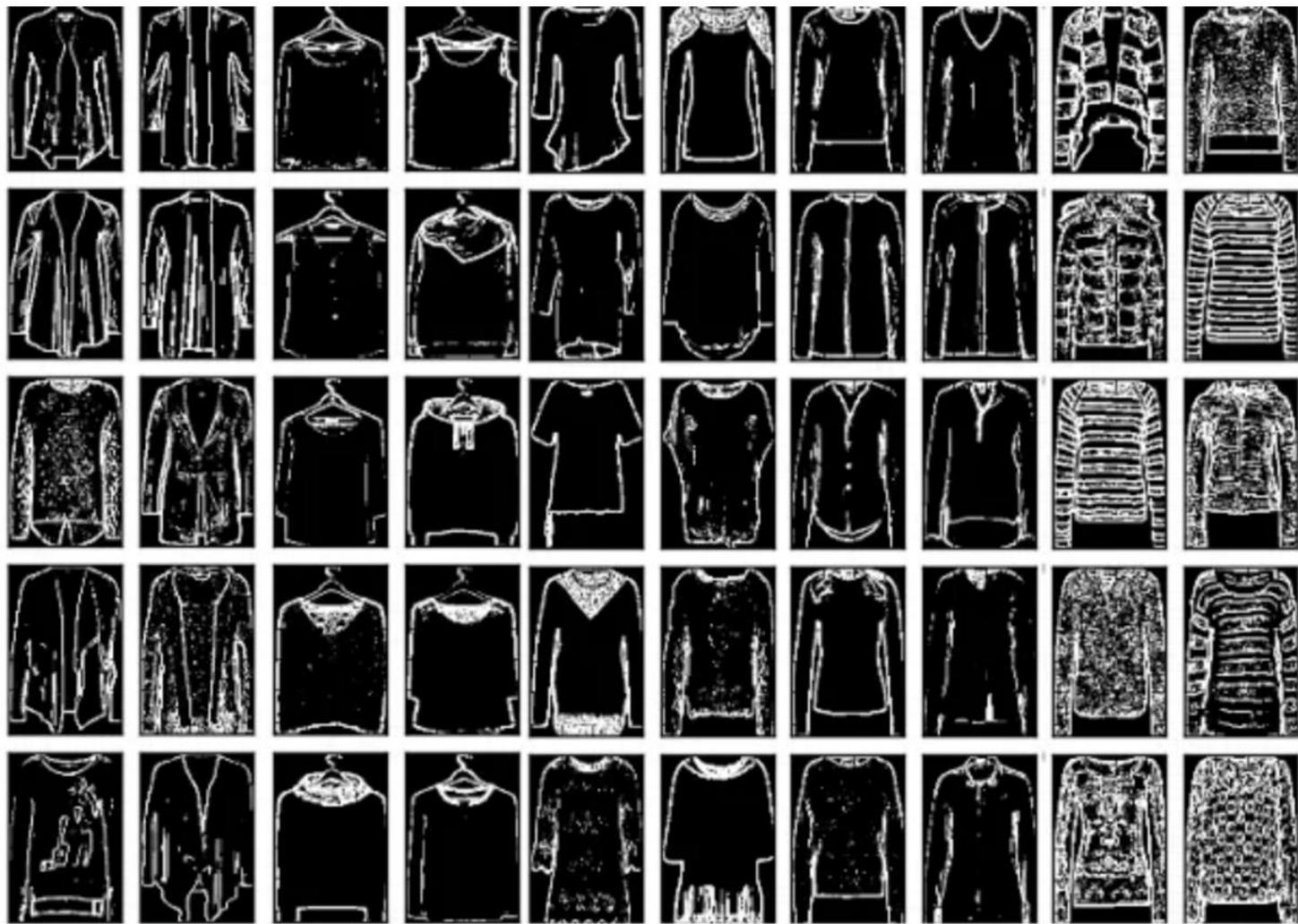
◦ **Backpropagate the error:** For each

$l = L - 1, L - 2, \dots, 2$  compute

$$\delta^{x,l} = ((w^{l+1})^T \delta^{x,l+1}) \odot \sigma'(z^{x,l}).$$

3. **Gradient descent:** For each  $l = L, L - 1, \dots, 2$  update the weights according to the rule  $w^l \rightarrow w^l - \frac{\eta}{m} \sum_x \delta^{x,l} (a^{x,l-1})^T$ , and the biases according to the rule  $b^l \rightarrow b^l - \frac{\eta}{m} \sum_x \delta^{x,l}$ .





Courtesy of Tim Dettmers. Used with permission.

# Convolution: Filters

An output pixel's value is some function of the corresponding input pixel's neighbors

Examples:

Smooth, sharpen, contrast, shift

Enhance edges

Detect particular orientations

Apply Convolution

1	1	1
1	1	1
1	1	1

1/9

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	90	0
0	0	0	90	90	90	90	90	90	0
0	0	0	90	90	90	90	90	90	0
0	0	0	90	90	90	90	90	90	0
0	0	0	90	90	90	90	90	90	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

= 40

# Convolution for 2D matrices

Given two three-by-three matrices, one a kernel, and the other an image piece, convolution is the process of multiplying entries and summing

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = (1*i) + (2*h) + (3*g) + (4*f) + (5*e) + (6*d) + (7*c) + (8*b) + (9*a)$$

The output of this operation constitutes the input to a single neuron in the following layer.

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