

Problem 1

Solid propellant rockets are often equipped with “termination ports” (placed symmetrically), that can be blown open on command to terminate thrust in an emergency. This can be achieved if the internal pressure goes below an “extinction pressure”, of the order of 20 atm in most cases. This problem analyzes the behavior of this system.

(1a) Start with the mass balance equation, written before and after the ports, of total area A_p , are blown open. Before the opening, we assume steady operation, so that

$$a \rho_p A_b P_c^n - \frac{A_t P_c}{c^*} = 0$$

After the opening, the area for flow discharge changes to $A_t + A_p$, and we have

$$\frac{V_c}{RT_c} \frac{dP_c}{dt} = a \rho_p A_b P_c^n - \frac{(A_t + A_p) P_c}{c^*}$$

Since typically the extinction pressure P_{ext} is much less than the initial pressure P_{c0} , the linearization made in class is not accurate enough, and a full non-linear solution is needed. To facilitate the algebra, define the non-dimensional variables

$$\tau = \frac{t}{t_{ch}}, \quad \text{with} \quad t_{ch} = \frac{V_c c^*}{RT_c A_t}$$

$$\alpha = \frac{A_p}{A_t}; \quad y = \frac{P_c}{P_{c0}}$$

to obtain the Bernoulli-type nonlinear differential equation

$$\frac{dy}{d\tau} = y^n - (1 + \alpha)y$$

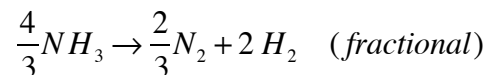
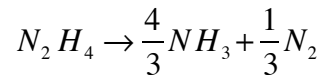
This becomes linear with the change $u = y^{1-n}$. Show that the solution satisfying $y(0)=1$ is

$$y = \left[\frac{1 + \alpha e^{-(1+\alpha)(1-n)\tau}}{1 + \alpha} \right]^{\frac{1}{1-n}}$$

(1b) Assume $P_{c0} = 70$ atm., $c^* = 1,800$ m/s, $\rho_p = 1,700$ kg/m³, $V_c / A_t = 10$ m and $T_c = 3400$ K. Calculate the minimum port area ratio, α_{min} , that will achieve extinction (in infinite time), and then calculate the extinction times for a few values of $\alpha > \alpha_{min}$.

Problem 2

As was mentioned in class, monopropellant Hydrazine rockets often operate with catalytic decomposition chambers that are purposely sized too small for full decomposition into N_2 and H_2 , in which case the gas temperature ends up being higher than with full decomposition, due to the presence of endothermic Ammonia (NH_3) among the products. The net reaction can be split into a fast hydrazine “disproportionation” reaction into Ammonia and nitrogen, followed by a fractional decomposition of the Ammonia in a slower reaction:



(b1) Assume only 40% of the Ammonia decomposes in the second reaction, and formulate the net reaction. (Hint: add the first reaction plus 0.4 times the second).

(b2) Since the temperature may be too high for a constant specific heat assumption (linear enthalpy vs. T), use the following quadratic fits for the relevant gas temperatures (in J/mole, with $\theta = T / 1000K$):

$$h_{N_2} = -11.84 + 32.42\theta + 0.76\theta^2$$

$$h_{H_2} = -8.23 + 27.61\theta + 1.54\theta^2$$

$$h_{NH_3} = -70.40 + 51.66\theta + 4.11\theta^2$$

The heat of formation of Hydrazine, in the liquid state, is $\Delta H_{N_2H_4,l}^f = 50,630 J / mol$, and the liquid is assumed to be injected at the standard temperature of 298.2K. Calculate the adiabatic temperature of the final gas, its mean molecular mass, and its mean specific heat, in J/kg/K.

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