Homework 9: Mean-Line Compressor Design

a) At the compressor inlet (station 2), the flow area is:

$$A_2 = \pi(r_{T2}^2 - r_{H2}^2) \qquad (1)$$

The mean radius (same everywhere) is:

$$\bar{r} = \frac{r_{T2}+r_{H2}}{2} \qquad (2)$$

Hence:

$$A_2 = \pi\bar{r}^2 \frac{r_{T2}^2-r_{H2}^2}{\left(\frac{r_{T2}+r_{H2}}{2}\right)^2} = 4\pi\bar{r}^2 \frac{1-\left(\frac{r_H^2}{r_T^2}\right)_2}{\left(1+\left(\frac{r_H}{r_T}\right)_2\right)^2} \qquad (3)$$

Using $A_2 = 0.1385\ m^2$ and $\left(\frac{r_H}{r_T}\right)_2 = 0.4$:

$$0.1385 = 4\pi\bar{r}^2 \frac{1-0.4^2}{(1+0.4)^2}$$

$$\bar{r} = 0.1604\ m$$

$$0.1604 = r_{T2}\frac{1+\left(\frac{r_H}{r_T}\right)_2}{2} = 0.7r_{T2}$$

$$r_{T2} = 0.2291\ m$$

$$r_{H2} = 0.4r_{T2} = 0.09164\ m$$

Blade height:

$$h_2 = r_{T2} - r_{H2} = 0.1375\ m$$

b) We can calculate the axial velocity at station 2:

$$w = M_2\sqrt{\gamma R T_2} = M_2\sqrt{\frac{\gamma R T_{t2}}{1+\frac{\gamma-1}{2}M_2^2}} = 0.4\sqrt{\frac{1.4\times 287\times 279.8}{1+0.2\times 0.4^2}} = 192.02\ m/s \qquad (4)$$

Keeping $w_2 = w_3 = w$:

$$\frac{A_3}{A_2} = \frac{\rho_2}{\rho_3} = \frac{P_{t2}}{P_{t3}}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{1}{\gamma-1}} \qquad (5)$$

$$\frac{P_{t2}}{P_{t3}} = \frac{P_{t3}}{P_{t2}}\frac{T_{t3}}{T_{t2}} \qquad (6)$$

We also have:

$$M_3\sqrt{T_3} = M_2\sqrt{T_2} \qquad (7)$$

$\frac{M_3}{M_2} = \sqrt{\frac{T_2}{T_3}} = \sqrt{\frac{T_{t2}}{T_{t3}}\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}}$ (8)

$\frac{M_3}{1+\frac{\gamma-1}{2}M_3^2} = \frac{T_{t2}}{T_{t3}}\frac{M_2^2}{1+\frac{\gamma-1}{2}M_2^2}$ (9)

$\frac{M_3}{1+0.2M_3^2} = \frac{279.8}{559.7}\frac{0.4^2}{1+0.2\times0.4^2} = 0.077506$

$M_3^2 = 0.077506 + 0.015501M_3^2$

$M_3 = 0.2806$

Then:

$\frac{A_3}{A_2} = \left(\frac{279.8}{559.7}\right)^{3.5-1}\left(\frac{1+0.2\times0.2806^2}{1+0.2\times0.4^2}\right)^{2.5} = 0.1698$

$A_3 = 0.02352\ m^2$

As in part a), $\frac{A_3}{4\pi\bar{r}^2} = \frac{1-\left(\frac{r_H}{r_T}\right)_3^2}{\left(1+\left(\frac{r_H}{r_T}\right)_3\right)^2} = 0.072748$

To solve for $\left(\frac{r_H}{r_T}\right)_3 = x$:

$1 - x^2 = 0.072748(1 + 2x + x^2)$

$x = 0.86437 = \frac{r_{H3}}{r_{T3}}$

Then:

$0.1604 = r_{T3}\frac{1+0.86437}{2}$

$r_{T3} = 0{,}1721\ m$

$r_{H3} = 0.1487\ m$

Blade height:

$h_3 = r_{T3} - r_{H3} = 0.02337\ m$

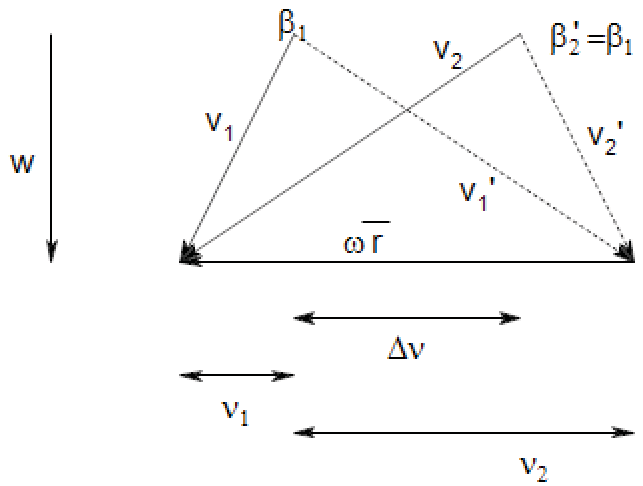
c) Euler's equation for one stage gives:

$c_p(\Delta T_t)_3 = \omega\bar{r}(v_2 - v_1) = (\omega\bar{r})^2\left(\frac{v_2-v_1}{\omega\bar{r}}\right)$ (10)

Since we will use repeating stage, the factor $\frac{v_2-v_1}{\omega\bar{r}}$, which depends on angles only, will be the same for all stages. In addition, $\omega\bar{r}$ is also the same, so $(\Delta T_t)_3$ will be the same for all stages. Using N stages,

$(\Delta T_t)_3 = \frac{T_{t3}-T_{t2}}{N} = \frac{559.7-279.8}{N} = \frac{279.9}{N}$ (11)

The velocity triangle can be sketched from the condition that the rotor and the stator provide the same flow turning and, in their frame, the same deceleration:



$v_2 - v_1 = \Delta v$ (12)

$v_1 = w \tan \beta_1$ (13)

$v_2 = \omega \bar{r} - w \tan \beta_1$ (14)

$\Delta v = \omega \bar{r} - 2w \tan \beta_1$ (15)

$\tan \beta_1 = \frac{\omega \bar{r} - \Delta v}{2w}$ (16)

We can now try values of N and follow these steps:

$\Delta T_{t3} = \frac{279.9}{N}$

$\Delta v = \frac{c_p (\Delta T_t)_3}{\omega \bar{r}} = \frac{1004.5 (\Delta T_t)_3}{300}$

$\tan \beta_1 = \frac{300 - \Delta v}{2 \times 132.0}$

$v_2' = v_1 = \frac{132}{\cos \beta_1}$

$v_2 = 300 - 132 \tan \beta_1$

$v_1' = v_2 = \sqrt{w^2 + v_2^2}$

The diffusion factor (the same for stator and rotor) is:

$D = 1 - \frac{v_2'}{v_1'} + \frac{\Delta v}{2 \sigma v_1'}$

$(\sigma = 1.5)$

Some results follow:

N	$(\Delta T_t)_3\ [K]$	$\Delta v\ [m/s]$	$\beta_1\ [°]$	$v_2'\ [m/s]$	$v_2\ [m/s]$	$v_1'\ [m/s]$	D
8	34.99	117.2	34.71	160.6	208.6	246.8	0.5077
9	31.10	104.1	36.57	164.6	202.1	241.4	0.4628
10	27.99	93.7	38.00	167.5	196.9	237.0	0.4250

These are all probably acceptable designs. We select N = 9.

d)

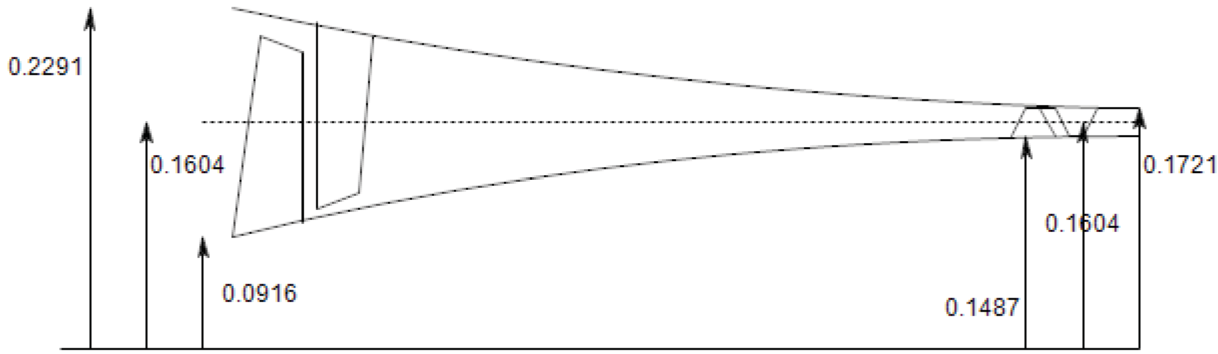


Figure 1: Nondimensional cross-section of compressor (dimensions in *m*)

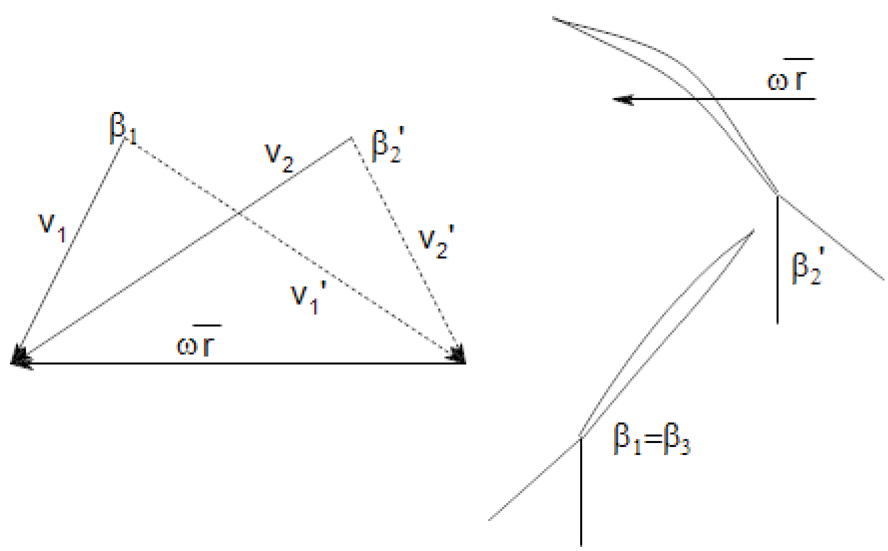


Figure 2: Velocity triangles (any stage) and blade profiles

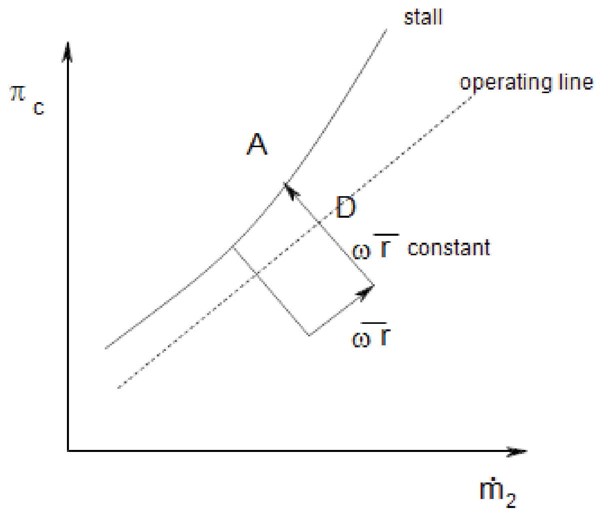
Concept Questions

1) The temperature *increments* are uniform. The stage temperature ratios are then:

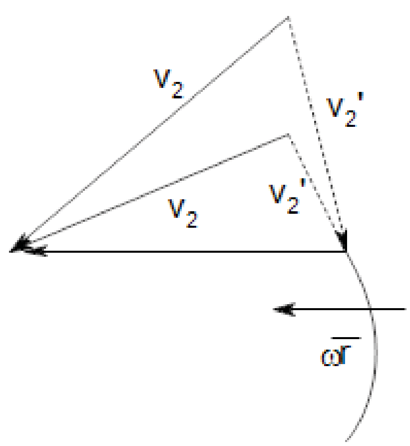
$\tau_s = 1 + \frac{(\Delta T_t)_s}{T_{ts,in}}$

$\pi_s = \tau_s^{\frac{\gamma}{\gamma-1}}$

As $T_{ts,in}$ increases, τ_s and π_s *decrease*. So the first stage has the highest pressure ratio across it, and the last stage has the lowest.



2) Actually, the statement of the question is not correct, unless it refers to points along the compressor operating line. The problem with this interpretation is that as $\dot{m}$ is reduced along the operating line, so is $\omega\bar{r}$, and *no problem arises.* Problems do arise if one reduces $\dot{m}$ *along a constant speed line*, like along DA in the sketch. To see this, take the design velocity triangle, leave the base $\omega\bar{r}$ unchanged, but reduce the height w: The incidence angle to a rotor blade is seen to *increase*, and the same happens to the stator. At some point, the blades *stall*.



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