

## 16.55 Ionized Gases

### Problem Set #5

#### Problem 1: A probe in a non-Maxwellian plasma

The theory of cold probes, as explained in class, applies to plasmas with Maxwellian electron and ion distributions. However, probes can also be useful in more general situations, especially if the distribution can be still assumed to be isotropic. This problem is concerned with one such situation.

The attached figure, taken from a book on electric lamps, shows voltage-current curves (electrons only) obtained with a small ( $0.16 \text{ mm}^2$  surface) planar probe at different distances from the cathode of a fluorescent lamp. Some of the curves imply a Maxwellian distribution (curve D), while others, especially those taken very near the cathode, seem to indicate an extra number of high energy electrons. Since there is a 15 V potential drop through the cathode sheath, electrons emitted by the hot filament are expected to have mostly energies of about 15 eV after having been accelerated in the sheath and scattered directionally into an isotropic distribution, and before additional elastic and inelastic collisions bring them into equilibrium with the general electron background. A reasonable model for the electron distribution function near the cathode is therefore a Maxwellian distribution of density  $n_{e2}$  and temperature  $T_e$ , superimposed on an isotropic and mono-energetic population  $n_{e1}$  (the “primaries”) characterized by a single velocity  $w_0$ . Show that the measurements in case A are consistent with this model, and devise a way to extract from the data the approximate values of  $n_{e1}$ ,  $n_{e2}$ ,  $T_e$  and  $w_0$ . It is suggested that a linear plot of  $I$  vs.  $V$  will help separate the primary and secondary populations. You can verify this by using Medicus’ method for a mono-energetic plus Maxwellian population.

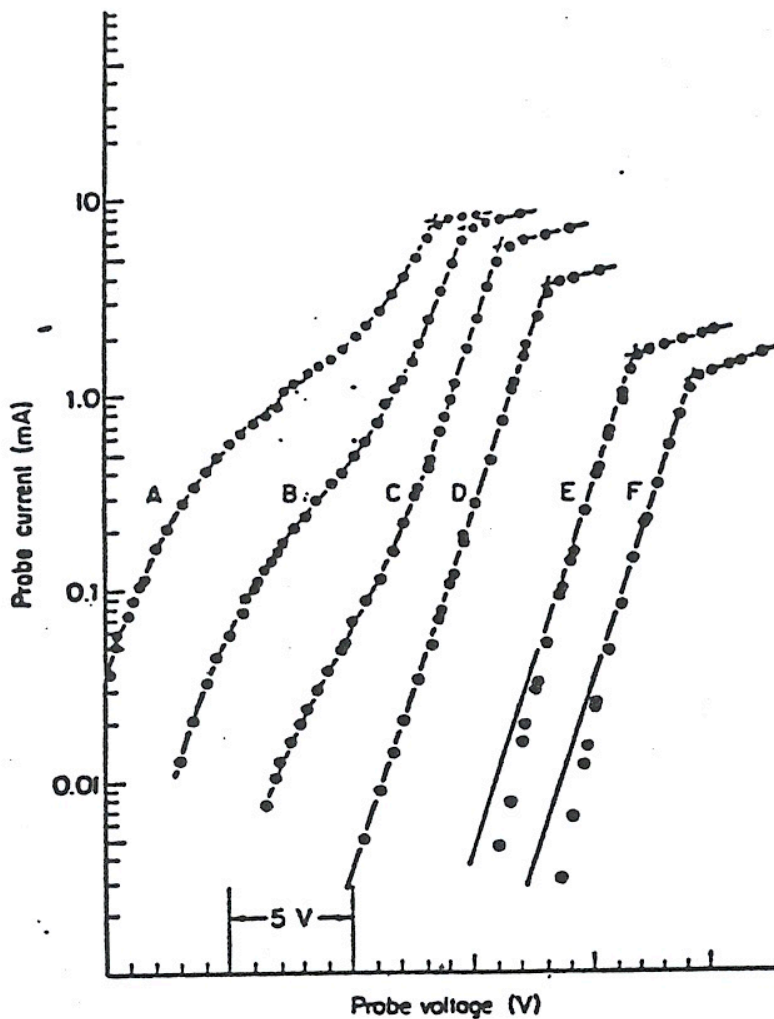


Figure 4.8 Logarithmic plots of electron current to a probe versus probe potential. A straight-line plot such as Curve D corresponds to a Maxwellian distribution of electron energies. Plots such as Curve A correspond to a Maxwellian group with a superposed high-energy group of electrons (the "primary" electrons), while plots such as Curves E and F (for which the probe was in the Faraday dark space) correspond to a Maxwellian distribution with a deficiency of high-energy electrons. These plots are arbitrarily displaced along the voltage axis to avoid confusion; they were taken from a discharge in 1.0 Torr of neon plus saturated mercury vapor at 40°C. Curve A, 0.25 cm from cathode; Curve B, 0.75 cm; Curve C, 1.5 cm; Curve D, 2.0 cm; Curve E, 3.0 cm; Curve F, 4.25 cm.

Waymouth, J. *Electrical Discharge Lamps*. © 1971  
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**Problem 2: Charging of a small drop in a plasma**

A small spherical droplet of radius  $R$  is surrounded by a neutral plasma of temperature  $T$  and electron density  $n_e$ . The drop is much smaller than the Debye length, which in turn is smaller than the electron and ion mean free paths. Starting from neutrality, the drop collides more frequently with electrons than with ions, and a negative charge accumulates on it, eventually leading to a steady state negative charge. Calculate the droplet charge as a function of time, including  $t \rightarrow \infty$ . Assume any electron arriving at the drop is absorbed, and any ion arriving is neutralized. Ignore the effects of image forces and the motion of the drop.

### Problem 3: He<sup>4</sup> as a Bose-Einstein condensate

Take a system of  $N$  atoms of He<sup>4</sup> contained in a “box” of volume  $V$ . These atoms have a resultant spin of zero and therefore do not obey Pauli’s exclusion principle. The intermolecular forces in He<sup>4</sup> are weak enough, so that its boiling point ( $T_B = 4.21$  K) is the second to lowest in nature (only the isotope He<sup>3</sup> is lower) and it remains as a liquid even at  $T = 0$  K, at least for moderate pressures. Liquid He<sup>4</sup> however, suffers an extraordinary transition at some temperature  $T_\lambda < T_B$  at which exhibits practically no flow resistance at all, i.e., zero viscosity. It is said that the liquid becomes a “superfluid”. He<sup>4</sup> atoms start to condense (Bose-Einstein condensation) in the lowest energy state once  $T_\lambda$  is reached.

Assume that He<sup>4</sup> atoms could be described as a dilute, non-interacting system (not exactly true for a liquid) with translational degrees of freedom only, such that,

$$\varepsilon_{n_x n_y n_z} = \frac{\pi^2 \hbar^2}{2mL^2} n^2 \quad \text{where} \quad n^2 = n_x^2 + n_y^2 + n_z^2 \quad \text{and} \quad [n_x, n_y, n_z] = 1, 2, 3, \dots$$

Define the lowest level (where particles condense) with zero energy. The system energy is then  $\varepsilon = \varepsilon_{n_x n_y n_z} - \varepsilon_{111}$ . At a given temperature,  $N$  will be distributed among the ground state  $N_0$  and excited levels  $N_{exc}$  such that  $N = N_0 + N_{exc}$ .

Assume that, as the temperature decreases, no particles will move into the ground state until  $T_\lambda$  is reached and the maximum possible number of particles in excited states  $N_{exc}^{max}$  (note that for high temperatures,  $N_{exc}^{max} \gg N$ ) is precisely equal to  $N$ . Once this maximum number of particles reaches a value less than  $N$ , particles will have to move into the ground state, thus triggering the condensation process.

1. Find  $T_\lambda$ , taking the liquid density as  $\rho_{LHe} = 124.91$  kg/m<sup>3</sup>.
2. Comment on the Bose-Einstein condensation in the gas phase. Assume the density to be equal to  $\rho_{GHe} = 0.178$  kg/m<sup>3</sup>.

Hints:

1.

$$\int_0^\infty \frac{\sqrt{u} du}{e^u - 1} = \Gamma(3/2) \zeta(3/2) = \left( \frac{\sqrt{\pi}}{2} \right) \quad (2.612)$$

2. Note from Bose-Einstein statistics that the number of particles in the ground state will depend only on the chemical potential, and that this number is maximum when  $\mu \rightarrow 0$ . Equivalently, the maximum number of particles in excited levels  $N_{exc}^{max}$  will be reached when the chemical potential vanishes.

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