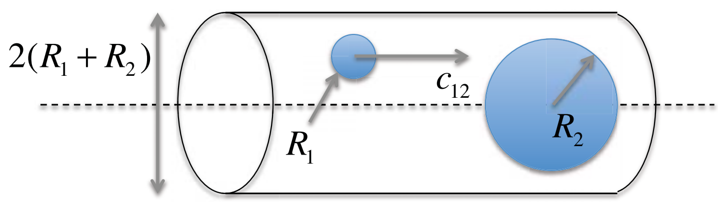
Fundamentals of Plasma Physics

Definition of Plasma: A gas with an ionized fraction ($n \rightarrow i^+ + e^-$). Depending on density, $\vec{E}$ and $\vec{B}$ fields, there can be many regimes.

Collisions and the Mean Free Path (mfp)
For a simple view of the **mfp**, consider two hard spheres (R_1, R_2) and look from sphere 2. Sphere 1 approaches at a relative speed c_{12}. If the relative line of motion intercepts a sphere of radius $R_1 + R_2$, there will be a collision, leading to scattering:



The volume swept by 1 per unit time is $\pi(R_1+R_2)^2 c_{12}$. Taking the product with the number of spheres 2 per unit volume n_2 results in the Collision Frequency of 1 with all 2's:

$$\nu_{12} = \pi(R_1 + R_2)^2 c_{12} n_2$$

where n_2 is the number density of particles 2. Call $Q_{12} = \pi(R_1+R_2)^2$ the Collision Cross-Section for 1-2 collisions (symmetric to interchange of 1 and 2); then we have in general,

$$\nu_{12} = Q_{12} c_{12} n_2$$

Notice that the collision frequency of one 2 with all 1's is $\nu_{21} = Q_{12} c_{12} n_1$, which is different only in exchanging the densities.

The mean free path of a particle 1 with all particles 2 if they were the only colliding partners is,

$$\lambda_{12} = c_{12} \frac{1}{\nu_{12}} = \frac{1}{Q_{12} n_2}$$

and for a particle of type 2, we would have $\lambda_{21} = 1/(Q_{12} n_1)$.

All of this has assumed "hard collisions", but the results are valid for "soft collisions" as well, with proper definitions of the cross-section, which we will study later. A different kind of generalization is that the collisions may be inelastic, like ionization or excitation; collision frequency and mean free path can be defined for these processes as well.

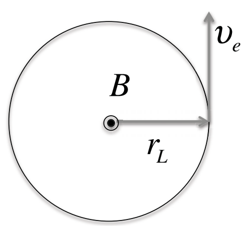
Depending on the importance of collisions, we can have the following regimes:

1a. Highly collisional plasmas
Electron mean free path (mfp) << linear size of plasma. Examples are:
(i) Soldering arcs; $L \approx 1cm$, $n_n \approx 2 \times 10^{24} m^{-3}$, $Q_{en} \approx 10^{-19} m^2 \rightarrow \lambda_{en} \approx 5\mu m$
(ii) Arcjet thrusters; $D \approx 1mm$, $n_n \approx \times 10^{24} m^{-3}$, $Q_{en} \approx 10^{-19} m^2 \rightarrow \lambda_{en} \approx 10\mu m$

1b. Weakly ionized plasmas
(i) Hall thrusters (Xe); $n_n \approx 10^{19} m^{-3}$, $Q_{en} \approx 10^{-19} m^2 \rightarrow \lambda_{en} \approx 1m$
Here, $L \approx 1 - 10cm$, but due to magnetic guiding, $L_{eff} \approx 1m$. So, even though $Q_{ioniz} \approx 10^{-20} m^2$ only, the mfp for ionization is also about $1m$, and ionization does occur.
(ii) Ionospheric plasma; $n_n \approx 10^{12} m^{-3}$, $Q_{en} \approx 10^{-19} m^2 \rightarrow \lambda_{en} \approx 10^7 m = 10,000km$
and ionization is not an issue either.

Degree of magnetization and the Larmor radius

In the absence of an electric field, the motion of an electron in a magnetic field is circular around a magnetic line in the anti-clockwise, or threading sense (plus arbitrary velocity parallel to the line). The radial force balance then gives,



$$ev_\perp B = \frac{m_e v_\perp^2}{r_{Le}}$$

$$r_{Le} = \frac{m_e v_\perp}{eB}$$

which is the electron Larmor, or gyro radius. The frequency of this gyration is,

$$\omega_{ce} = \frac{eB}{m_e}$$

which turns out to be independent of velocity (faster electrons describe bigger circles in equal time).
For ions, the picture is the same, but the gyration is in the opposite sense (clockwise, or un-threading). The ion gyro radius and gyro frequency are,

$$r_{Li} = \frac{m_i v_{\perp i}}{eB} \ ; \ \omega_{ce} = \frac{eB}{m_i}$$

and clearly, the ion gyro frequency is lower by m_e/m_i than the electron gyro frequency. The ion gyro radius is bigger than the electron gyro radius by $m_i v_i / m_e v_e$, which is less than the mass ratio because ions move generally slower (by $\sqrt{\frac{m_e T_i}{m_i T_e}}$, if the motion is thermal).

Depending on conditions, plasmas can be:

2a. Strongly (electron) magnetized if $r_{Le} << (L, \lambda_e)$, with a similar condition to be strongly ion magnetized. Examples are:
(i) Fusion plasmas (Hydrogen)
$v_e \approx 3 \times 10^6 m/s$, $v_i \approx 3 \times 10^5 m/s$, $B \approx 10T \rightarrow r_{Le} \approx 2\mu m$, $r_{Li} \approx 200\mu m$
Also $n_i \approx 10^{20} m^{-3}$, $Q_{ei} \approx 10^{-21} m^2 \rightarrow \lambda_{ei} \approx 10m$, while $L \approx 1 - 10m$. So, in a fusion plasma both electrons and ions are strongly magnetized.
(ii) Hall thrusters (Xe)
$v_e \approx 10^6 m/s$, $v_i \approx 10^4 m/s$, $B \approx 0.02T \rightarrow r_{Le} \approx 0.3\mu m$, $r_{Li} \approx 7cm$
Also we have $\lambda_e \approx 1m$ and $\lambda_i \approx 1m$, plus $L \approx 1 - 10cm$. Here, electrons are strongly

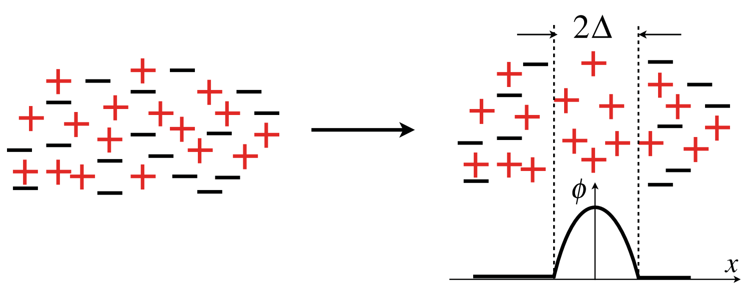
magnetized, but ions are not.

2b. Un-magnetized, or weakly magnetized plasma, when r_{Le} is not the shortest characteristic distance (other than the usually much smaller Debye length, see below). An example is the plume of a small plasma thruster in low earth orbit (LEO): $v_e \approx 10^6 m/s$, $B \approx 3 \times 10^{-5} T$ $\rightarrow r_{Le} \approx 20cm$, which is typically more than the plume diameter (or of the same order).

Quasi-neutrality, shielding and the Debye length

Due to the strong attraction between ions and electrons, plasmas are almost always quasi neutral, meaning $|n_i - n_e| / n_e << 1$. Some exceptions are:
(i) Within a sheath, very near a wall, where $n_e << n_i$
(ii) Near an emitting cathode (dark space), where $n_e >> n_i$
(iii) During very fast (GHz) oscillations (Langmuir oscillations).

In steady state, a plasma can be non-neutral only over very thin layers. Suppose some agent keeps electrons and ions apart so that only ions occur over a thickness 2Δ,



We can now solve Gauss' law ($\nabla \cdot \vec{E} = \rho_{ch}/\epsilon_0$ with $\epsilon_0 = 8.854 \times 10^{-12} F/m$) in the layer. Putting $\vec{E} = -\nabla\phi$ and $\rho_{ch} = en_i = en_{e,\infty}$,

$$\frac{d^2\phi}{dx^2} = -\frac{en_{e,\infty}}{\epsilon_0} \; ; \; \phi(\pm\Delta) = 0$$

with solution (for $-\Delta < x < \Delta$),

$$\phi = \frac{en_{e\infty}}{2\epsilon_0}(\Delta^2 - x^2) \; ; \; \phi(0) = \frac{en_{e\infty}\Delta^2}{2\epsilon_0}$$

Now, suppose only the electron thermal energy ($kT_e/2$ per direction, with $k = 1.38 \times 10^{-23} J/K$, Boltzmann's constant and T in degrees Kelvin). This energy is then equal to $e\phi(0)$, and solving for Δ,

$$\Delta = \sqrt{\frac{\epsilon_0 k T_e}{e^2 n_{e\infty}}}$$

This distance is called the Debye length, $\lambda_D = \Delta$. In MKS units, with T_e in K, $\lambda_D \approx 69\sqrt{T_e/n_{e\infty}}$, a very small distance in general. Thermal energy alone cannot separate charges over long distances. Even when an external potential $V > kT_e/e$ is applied, we obtain

$\lambda_D = \sqrt{2\epsilon_0 V / e n_{e\infty}}$, which cannot be large unless V is very large or n_e is very small.

In a dynamics sense, when plasma is placed in contact with a wall that can neutralize arriving ions and absorb arriving electrons, the initial arrival rate of electrons is much faster than that of ions (faster electrons), and the wall accumulates negative surface charge until its potential is of the order of kT_e/e (or $T_e(eV)$), at which point there is enough electron repelling force to equalize the fluxes. The potential well extends only a distance of the order of λ_D into the plasma (the sheath); the plasma shields itself from the wall disturbance, except inside the thin sheath. The same conclusion follows even if the wall is forced to some (not extremely high) potential: the plasma sees this applied potential only within distances of the order of λ_D (somewhat more, due to the applied potential).

Examples:
(i) The lower ionosphere. Here the density is very small (say, $n_e = 10^{12} m^{-3}$), and so is the electron temperature ($T_e \approx 1000K = 0.09eV$). Even with this low density, $\lambda_D \approx 0.8cm$, smaller than most spacecraft features.
(ii) Ion or Hall thrusters. In an Ion thruster, $T_e \approx 5eV \approx 60,000K$, and $n_e \approx 10^{18} m^{-3}$, giving $\lambda_D \approx 17\mu m$ only. In a Hall thruster, the temperature is about 4 times higher, yielding $\lambda_D \approx 34\mu m$. In both cases, this is microscopic, and most of the plasma is quasi-neutral.

Why do we say **quasi**neutral, not simply neutral? This is because there remains enough (although very small) non-neutrality to allow relatively weak electric fields to exist. If n_e were literally equal to n_i, Poisson's equation $\nabla^2\phi = e(n_i - n_e)/\epsilon_0$ would have the simple solution $\phi = constant$ (or at most $\vec{E} = constant$). The key is how small the constant ϵ_0 is. This becomes clearer if we read Gauss' law backwards: $\rho_{ch} = \epsilon_0 \nabla \cdot \vec{E}$, which says that even with reasonably large fields, the net charge density must be very small.

Langmuir oscillations, plasma frequency

We just found that when ions predominate in a thin region (order 2Δ), the electric potential is $\phi = \frac{e n_{e\infty}}{2\epsilon_0}(\Delta^2 - x^2)$, and the field is $E_x = -d\phi/dx = e n_{e\infty} x/\epsilon_0$. An electron placed in this field would move according to,

$$m_e \frac{d^2x}{dt^2} = -eE_x = -\frac{e^2 n_{e\infty}}{\epsilon_0} x$$

which is a harmonic oscillator with a natural frequency (called the Plasma Frequency), of,

$$\omega_{pe} = \sqrt{\frac{e^2 n_{e\infty}}{\epsilon_0 m_e}} \approx 56.5\sqrt{n_{e\infty}}$$

We note that ω_{pe} is independent of the size of Δ, and so it is a fundamental property of the plasma itself. We will see later that it plays an important role in the propagation of waves. Very roughly, a passing wave can "shake" the electrons if its frequency is below this plasma natural frequency. but not if it is above. Thus, waves with frequency above ω_{pe} will propagate freely through the plasma (although with some phase shift), but slower waves will be reflected, because the plasma will have time to shield itself during each period, and the wave fields cannot penetrate. In Hz, the plasma frequency is $f_{pe} = \omega_{pe}/2\pi \approx 8.99\sqrt{n_e}$, so the ionosphere will reflect waves below some $9MHz$.

In considering "slow" effects, these very fast oscillations are usually ignored, or "filtered out".