16.61 Handout #7
Prof. J. P. How April 3, 2003
Prof. J. Deyst **Due:** April 10, 2003

16.61 Homework Assignment #7

1. In the March 2001 edition of the National Geographic magazine, one of the questions to the *Ask us* column is

   Is it true that when you flush a toilet, water circulates clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere?

   What do you think led the reader to ask this question, and what would your answer be, and why?

2. Consider a body with

$$I = \begin{bmatrix} 30 & -I_{xy} & -I_{xz} \\ -10 & 20 & -I_{yz} \\ 0 & -I_{zy} & 30 \end{bmatrix}$$

   and angular velocity

$$\vec{\omega} = 10\vec{i} + 10\vec{j} + 10\vec{k}$$

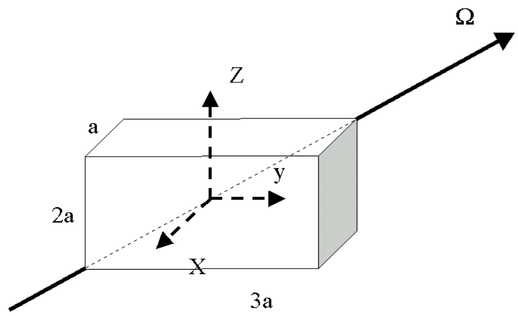
   If

$$\vec{H} = 200\vec{i} + 200\vec{j} + 400\vec{k}$$

   Find:

   (a) Values for I_{zy}, I_{xy}, I_{xz}, I_{yz}
   (b) The moment of inertia about $\vec{\omega}$
   (c) Rotational kinetic energy
   (d) Principal moments of inertia

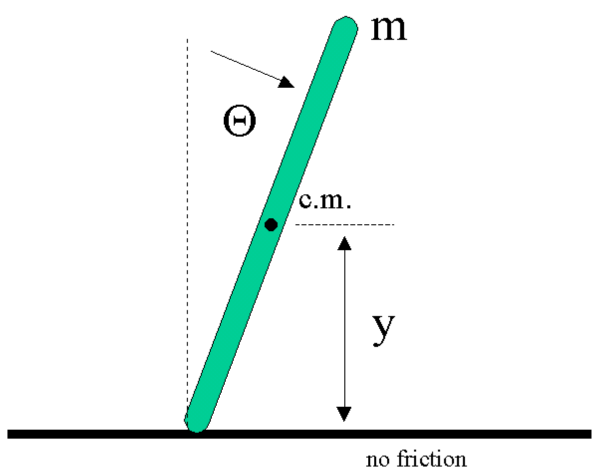
3. Consider a solid, uniform cube (mass M) with sides a, $2a$, and $3a$ as shown in the figure. The body is rotating in such a way that the angular velocity vector Ω passes through the diagonal as shown.



(a) Find the inertia matrix for this system.

(b) Find the angular momentum vector for this system.

4. A thin rod of mass m and length l is balancing vertically on a smooth horizontal floor. It is disturbed slightly and falls.



(a) Given that the rod is a rigid body (6 degrees of freedom) explain why there is only one degree of freedom in this problem. (i.e. List the 5 constraints.)

(b) Derive an expression for the motion of the rod by:

- Using θ as the generalized coordinate,
- Finding T and V,
- Finding L, the Lagrangian Function, and
- Applying Lagrange's Equation to solve for the equation of motion in terms of θ and it's time derivatives.