# 16.61 Midterm Exam #1

February 27, 2003

Jonathan How and John Deyst

1. There are 2 questions and you have ~~1~~ 1.25 hour. The questions are both worth the same.
2. Pay strict attention to the FARM methodology when frames are asked for in the question. It will play an important part (roughly 3/8) of the marking for those questions.
3. This is a closed-book exam, but you are allowed 1 sheet of notes (double sided).

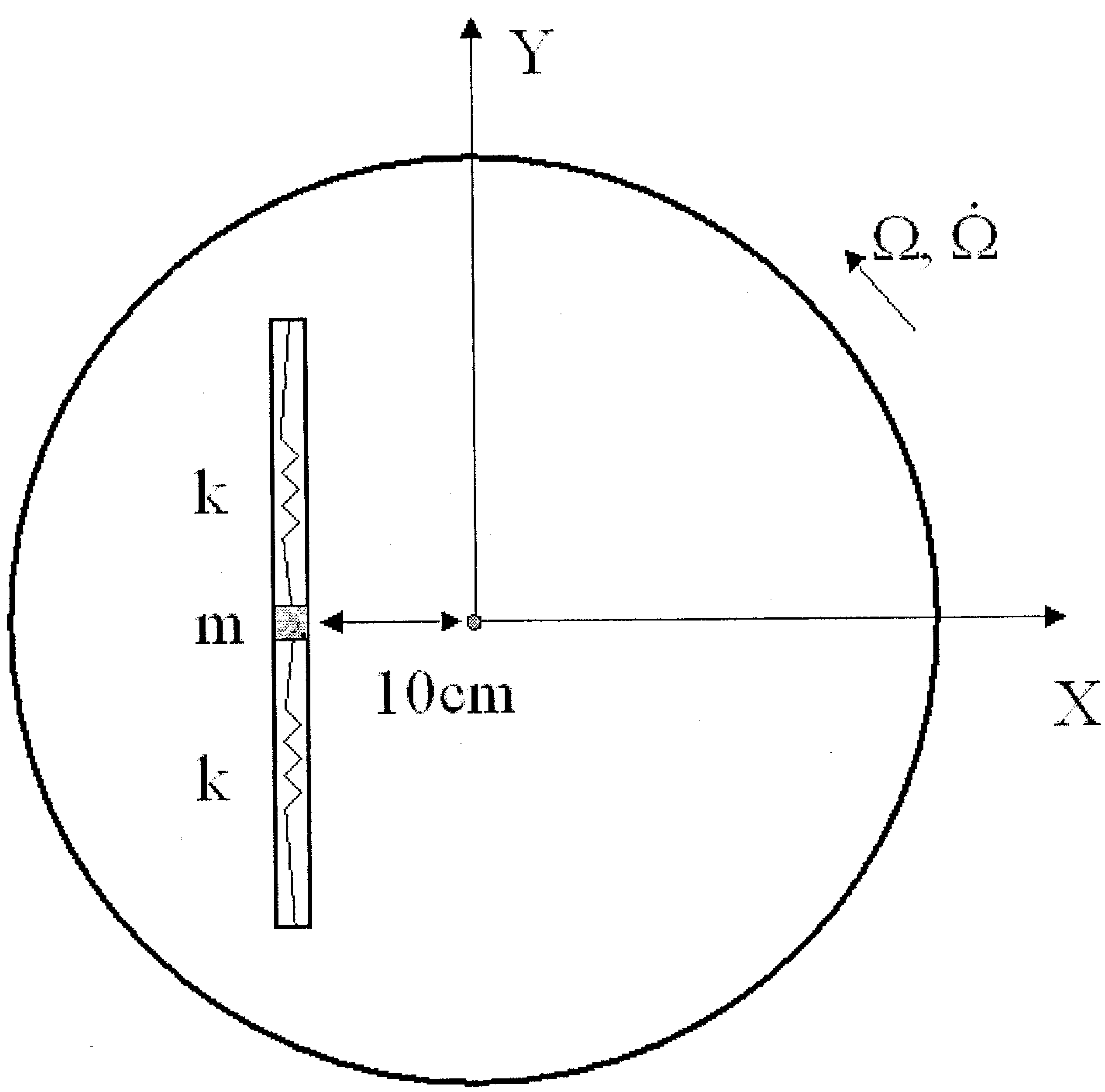
   **Please hand in these notes at the end of the test – you will get them back**

Something you might need:

$$
\begin{aligned}
{}^{PI}\ddot{\vec{r}}^{\,I} = \; & {}^1\ddot{\vec{r}}^{\,I} + {}^2\ddot{\vec{r}}^{\,1} + {}^3\ddot{\vec{r}}^{\,2} + 2([{}^1\vec{\omega} + {}^2\vec{\omega}] \times {}^3\dot{\vec{r}}^{\,2}) + 2({}^1\vec{\omega} \times {}^2\dot{\vec{r}}^{\,1}) +. \\
& {}^1\dot{\vec{\omega}}^{\,I} \times {}^2\vec{r} + [{}^1\dot{\vec{\omega}}^{\,I} + {}^2\dot{\vec{\omega}}^{\,1}] \times {}^3\vec{r} + \\
& {}^2\vec{\omega} \times ({}^2\vec{\omega} \times {}^3\vec{r}) + {}^1\vec{\omega} \times ({}^1\vec{\omega} \times [{}^2\vec{r} + {}^3\vec{r}]) + 2\;{}^1\vec{\omega} \times ({}^2\vec{\omega} \times {}^3\vec{r})
\end{aligned}
$$

1. Consider a variation of a merry-go-round (MGR) problem studied in class. In this case, the mass is free to move back and forth along a track oriented as shown in the figure (the $X$ and $Y$ axes are attached to the disk). The disk of the MGR is rotating about the $Z$-axis with rate $\Omega = 4\text{rad/sec}$ and $\dot{\Omega} = -1\text{rad/sec}^2$. There are two springs acting on the mass ($m = 2$), as shown. Derive the equations of motion for this scenario.

   What is the side force required to keep the mass in the track?




Remember - define the appropriate frames and focus on the FARM approach (frame selection, angles, angular rates, and matrix formulation).

2. The thin circular disk of mass $m$ and radius $r$ is rotating about its $z$-axis with a constant angular velocity $p$ (relative to the yoke) and the yoke in which it is mounted rotates about the $X$-axis through OB with a constant angular velocity ${}^{1}\vec{\omega}$. At the same time, the entire assembly is rotating around the fixed $Y$-axis through O with a constant angular velocity ${}^{2}\vec{\omega}$.

- Determine the absolute acceleration of the point $A$ on the rim of the disk as it passes the position shown. You should solve this problem using the equations given for two rotating frames, but be careful in establishing the motion of point $A$ in the second frame.