# Solution to Quiz #2
# Matrix Experiments

1) Does the following matrix experiment have the balancing property with respect to control factors A and B?
**Yes. All levels of B are equally represented at each level of A.**

2) Does the following matrix experiment have the balancing property with respect to control factors B and C?
**Yes. This one is slightly harder to see right away, but all levels of B are equally represented at each level of C.**

3) Does the following matrix experiment have the balancing property with respect to control factors A and D?
**NO. All levels of D are not equally represented at each level of A. For example, rows 4-6 are all the rows with level A2. In these rows we have D2 twice, D1 once and no instance of D3.**

4) Is the matrix an orthogonal array?
**NO. If any pair of columns in a matrix experiment are not orthogonal, then the matrix is not orthogonal. Columns A and D have neither the balancing property nor proportional balancing. Hence they are not orthogonal. Also, we can check the inner product of the contrast of the two columns directly. Since the inner product is non-zero, the matrix is not orthogonal**

| | Control Factors | | | |
|---|---|---|---|---|
| Expt. No. | A | B | C | D |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 |
| 4 | 2 | 1 | 2 | 2 |
| 5 | 2 | 2 | 3 | 1 |
| 6 | 2 | 3 | 1 | 2 |
| 7 | 3 | 1 | 3 | 3 |
| 8 | 3 | 2 | 1 | 3 |
| 9 | 3 | 3 | 2 | 1 |

$$\begin{Bmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{Bmatrix} \cdot \begin{Bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 1 \\ -1 \end{Bmatrix} = 1 + 0 - 1 + 0 + 0 + 0 + 1 + 1 - 1 = -1$$

5) After completing a matrix experiment, if an additive (factors separable) model holds, and one wishes to maximize the S/N ratio, one should
   a) Select the control factor settings in the row of the array corresponding to the experiment with the highest response.
   b) Select the noise factor levels that maximize the inner product of the array.
   c) Select the level of each control factor level independently based on its average effect on the S/N ratio.
   d) Select the noise factor levels that minimize the variance of the response.

**C is the correct answer.** If an additive model holds, then the factor effects on the S/N ratio obey the superposition principle. Therefore, to maximize the signal to noise ratio, simply pick the level of each control factor independently based on its mean effect on the S/N ratio. If working from a factor effects plot, just pick the level with the highest value of S/N.

**A is incorrect.** In most cases, the row in the array with the highest S/N ratio is better than the baseline S/N ratio. However, by picking the factor levels based on mean effects as described in C, you can usually do better than the best experimental value achieved in the rows of the array. If the additive model holds, then you can certainly do no worse than the best row of the array.

**B is incorrect.** This answer is basically nonsense. The inner product operator applies to vectors (e.g., columns of a matrix) not to whole matrices. Further, the control factor settings can never affect the orthogonality of the columns of a matrix experiment.

**D is incorrect.** If you were to plot the control factor effects on the variance, you could certainly select those settings that minimize the variance. This may even be a good strategy in some contexts. However, this approach is not guaranteed to maximize the S/N ratio since the means may change as you change the control factor settings. These mean shifts may offset your gains in S/N due to reduced variance.

6) In many cases, matrix experiments using orthogonal arrays may be preferred to one factor at a time experiments because
   a) The balancing property provides a better trade-off among desired system quality characteristics.
   b) One factor at a time experiments do not provide the flexibility to progressively adapt the experimental design to the data.
   c) Orthogonal arrays give more reliable estimates of factor effects in the same number of experiments or fewer as compared to one factor at a time experiments.
   d) One factor at a time experiments cannot determine factor effects when an additive model is used to approximate the system response.

**C is the correct answer.** In a matrix experiment, the error variance in the factor effects will be less than that in a comparable one-factor-at-a-time experiment by a factor of 1/(replication number). The replication number is the number of times any individual control factor level is tested in the matrix experiment (e.g. in an L9, the replication number is three for all four control factors).

**A is incorrect.** The balancing property is the property than ensures that main effects will not be confounded with other main effects in the experiment. If there are multiple system quality characteristics that must be traded off, one must take separate measures to define an appropriate trade strategy. The balancing property has nothing to do with these issues.

**B is incorrect.** One factor at a time experiments are reasonably flexible. Their weakness lies primarily in efficiency in resolving factor effects.

**D is incorrect.** One factor at a time experiments can resolve the main effects of a system and can allow one to make predictions based on an additive model.

7) In order to conduct a *full factorial* experiment on a system with 4 control factors each having 3 levels (with no replicates), how many experiments would one would need to conduct?

For a full factorial experiment, you would need to test every combination of all the factor levels. In this case **3^4=81 experiments**.

8) In order to resolve only the main effects in a system with 4 control factors each having 3 levels (with no replicates), how many experiments would one would need to conduct?

**1+(3-1)*4=9 experiments**.  This case exactly fits with an $L_9$.

9) The verification experiment primarily serves to
   a) Confirm that the assumptions in the experiment actually hold and that therefore the predicted improvement in system performance can be realized
   b) Establish that the experiment was indeed orthogonal
   c) Provide additional tuning of the noise factor levels
   d) Provide data needed for ANOVA

**A is the correct answer.**  The confirmation experiment shows that the predicted results can actually be realized. It also provides some evidence that the additive model holds for the system, although it is not a perfect guarantee.

**B is incorrect.** The matrix on which the experiment is based can be shown to be orthogonal before ever running an experiment.

**C is incorrect.**  Noise factors don't strictly speaking have "levels" at all since we cannot control them.  We can define some limits on the variation, but the width of these tolerance bands are set during the tolerance design phase. In robust design, this phase always follows the parameter design phase.

**D is incorrect.**  All the data required for ANOVA comes from the matrix experiment. The confirmation experiment is not required to construct an ANOVA table.

10) A response $y$ is known to be governed by the relationship

$$y=ax+b$$

where $x$ is a random variable with a mean of 2 and standard deviation of 3. The losses $L$ due to deviation of $y$ from its target $m$ are given by

$$L(y)=k(y-m)^2$$

Which settings of $a$ and $b$ among those listed will give the lowest expected value of quality loss?

a) $a=3, \ b=m$
b) $a=3, \ b=m-3$
c) $a=2, \ b=m$
d) $a=2, \ b=m-2$


**D is the right answer.** The principle here is the fundamental principle of two-step optimization. First, reduce sensitivity of the system to noise, then put the system on target. The sensitivity of the response $y$ to the noise factor $x$ is just the derivative $dx/dy=a$. So that narrows it down to answer c or d.

Now work to get on target. The mean value of y is

$$E(y) = E(ax + b) = aE(x) + b = 2a + b = 4 + b$$

Now, the target value of $E(y)=m$. Unfortunately, neither value of $b$ from the two will get you exactly on target, but $b=m-2$ will get you closer than $b=m$.