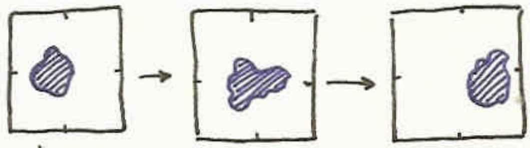
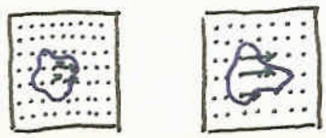


t=0 t=1 t=2

DESCRIPTION:
- movement of C of M
- deformation described as movement expansion

ALTERNATIVE DESCRIPTION:
- vector field flow



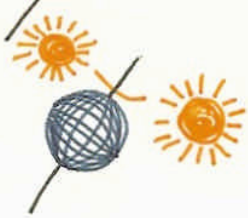
OPTICAL FLOW: measuring "motion" in visual field through changes in image across time
– usually, but not always, equivalent to motion of objects

2 PATHOLOGICAL CASES (exceptions, rather than rule)

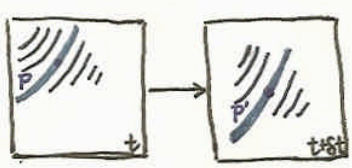
1. Uniform Gray Sphere

- optical flow zero
- but object is moving

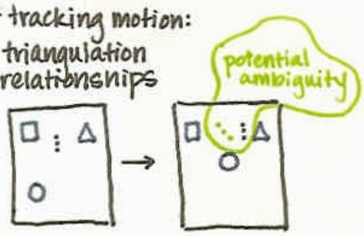
2.

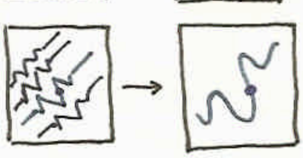


- moving light $\Rightarrow$ optical flow
- but object is still



2 ways of tracking motion:
1. Global triangulation
2. Local relationships





Imagine a series of 2D images with t as 3rd dime

Intensity of each pixel: $E(x,y,t)$

Our flow field: $\begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$ at time t

Most naive assumption

At t+δt, the location where some index pixel moves to has the same intensity as its index location at time t

$$E(x,y,t) = E(x+\delta x, y+\delta y, t+\delta t) = E(x+u\delta t, y+v\delta t, t+\delta t)$$

$\delta x = u\delta$
$\delta y = v\delta$

Note:
1. At best, only true for small δt
2. Later will deal with issue that intensity may be different in new location
3. Need at least one more constraint to solve for (u,v)

Re-express 1st constraint:

$$E(x,y,t) = E(x,y,t) + \frac{\partial E}{\partial x}\delta x + \frac{\partial E}{\partial y}\delta y + \frac{\partial E}{\partial t}\delta t + \ldots$$

$$\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

$$\Rightarrow E_x u + E_y v + E_t = 0$$

Express a "smoothness" constraint

Error function for smoothness

$$e_s = \iint \left[(u_x^2 + u_y^2) + (v_x^2 + v_y^2) \right] dx\,dy$$

$u_x = \frac{\partial}{\partial}$
⋮

$$e_c = \iint (E_x u + E_y v + E_t)^2 \, dx\,dy$$

We will minimize: $e_s + \lambda e_c$

Minimize $\iint F\,dx\,dy$

where $F = \left[(u_x^2 + u_y^2) + (v_y^2 + v_x^2)\right] + \lambda (E_x u + E_y v + E_t)$

if $E_x = E_y = 0$

LAPLACE'S EQN FORM: $\nabla^2 u + \nabla^2 v = 0$

For mathematics & details:
Berthold Horn Robot Vision, Ch. 12