9.07 Introduction to Probability and Statistics for Brain and Cognitive Sciences
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Homework Assignment 3
September 22, 2016
Due September 28, 2016 by 5:00pm

1. Suppose that the probability density function of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{(1-x)}{2} & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   A. Sketch $f(x)$.

   B. Find $F(X)$.

   C. Compute $E(X)$ and $Var(X)$.

   D. Compute $\Pr(X > -\frac{1}{2})$.

2. Suppose that the lifetime of an electron component follows an exponential distribution with parameter $\lambda = 0.1$.

   A. Find the probability that the lifetime is less than 10.

   B. Find the probability that the lifetime is between 5 and 15.

   C. Find t such that the probability that the lifetime is greater than t is 0.01.

3. If X is a Gaussian random variable with mean μ and variance σ^2 draw a diagram and compute

   A. $\Pr(|X-\mu| \leq 0.675\sigma)$
   B. $\Pr(X > \mu + \sigma)$
   C. $\Pr(X = \sigma)$ (Explain).

4. Suppose X is a binomial random variable with parameters $n = 40$ and $p = 0.4$.

A. Assume that you can generate random numbers on the interval $(0,1)$. Write an algorithm to simulate X.

B. Carry out a simulation of 200 samples of X in MATLAB® using the algorithm in A.

C. How many successes did you observe in your sample? How does this compare with the expected number of successes?

5. Suppose that in a certain population, individuals' heights are approximately normally distributed with $\mu = 70$ inches and $\sigma = 3$ inches.

   A. What proportion of the population is over 6 feet tall?

   B. What proportion of the population is between 5'5" and 5'10''?

   C. Show that the mode of this distribution is 70 inches.

6. To verify the calculations of the variance of a Gaussian random variable on **page 10** of **Lecture 3**, we use properties of the gamma function. We want to show that

$$\int_{\infty}^{\infty} y^2 e^{-y^2/2} dy = \sqrt{2\pi}. \qquad (1)$$

We proceed in 5 steps.

A. Draw a graph to explain why

$$\int_{\infty}^{\infty} y^2 e^{-y^2/2} dy = 2\int_0^{\infty} y^2 e^{-y^2/2} dy.$$

B. Show that

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx = 2\int_0^{\infty} u^{2\alpha-1} e^{-u^2} du$$

by making the change of variable $x = u^2$.

C. Show that

$$\int_0^{\infty} y^2 e^{-y^2/2} dy = 2\sqrt{2}\int_0^{\infty} v^2 e^{-v^2} dv$$

by making the change of variable $\sqrt{2}v = y$.

D. Use B to show that

$$2\int_0^{\infty} v^2 e^{-v^2} dv = \Gamma(\tfrac{3}{2})$$

E. Use D and the fact that

$$\Gamma(\tfrac{3}{2}) = \frac{\sqrt{\pi}}{2}$$

to establish (1).

7. Take the first 400 observations from the MEG data set on the class website in the file MEG.data.

A. Compute the five-number summary.

B. Compute a boxplot.

C. Compute the sample mean $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ and the sample standard deviation $\hat{\sigma} = \left[n^{-1}\sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{½}$.

D. !Assuming a Gaussian distribution with mean $\bar{x}$ and standard deviation $\hat{\sigma}$ compute a Q-Q plot for these data. Does this sample agree with a Gaussian distribution?

Hint: It might be useful in **Problem 8** to use functions boxplot and icdf, To see the utilization of this function, type "help (name of function)" in the MATLAB command prompt.