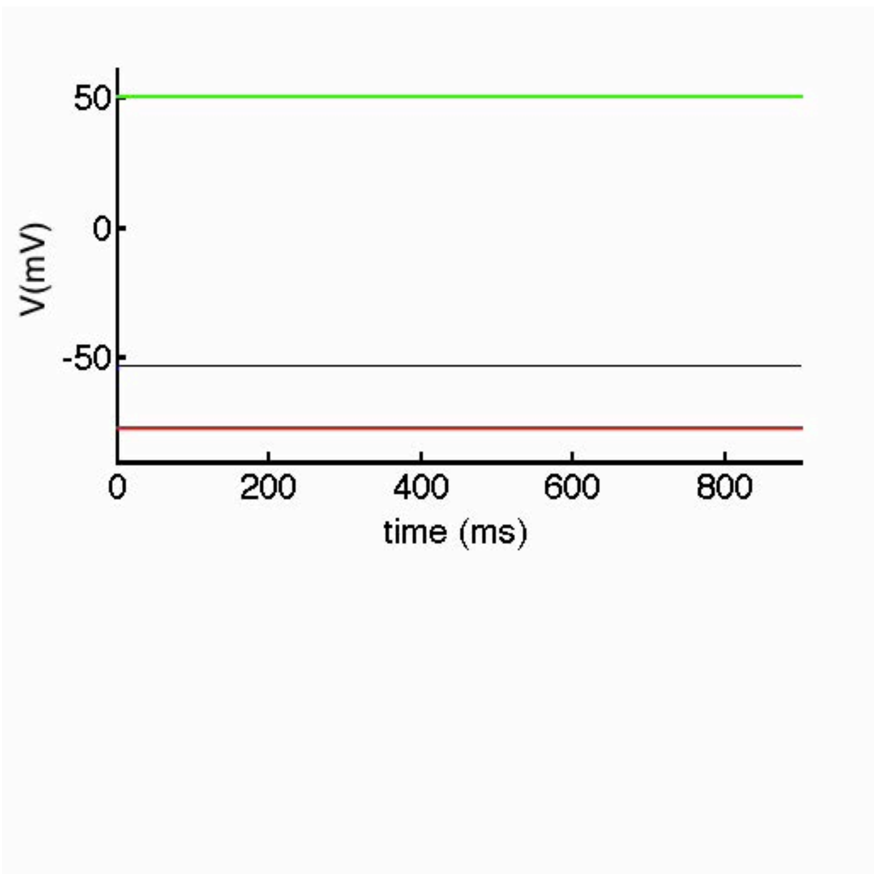
Introduction to Neural Computation

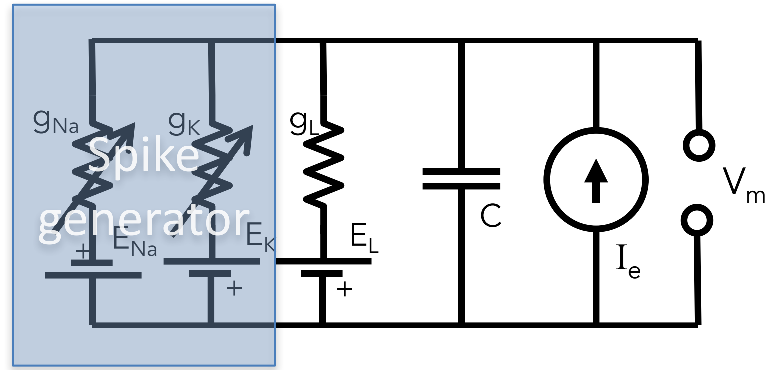
Prof. Michale Fee
MIT BCS 9.40 — 2018

Lecture 5

Hodgkin-Huxley model of action potential generation

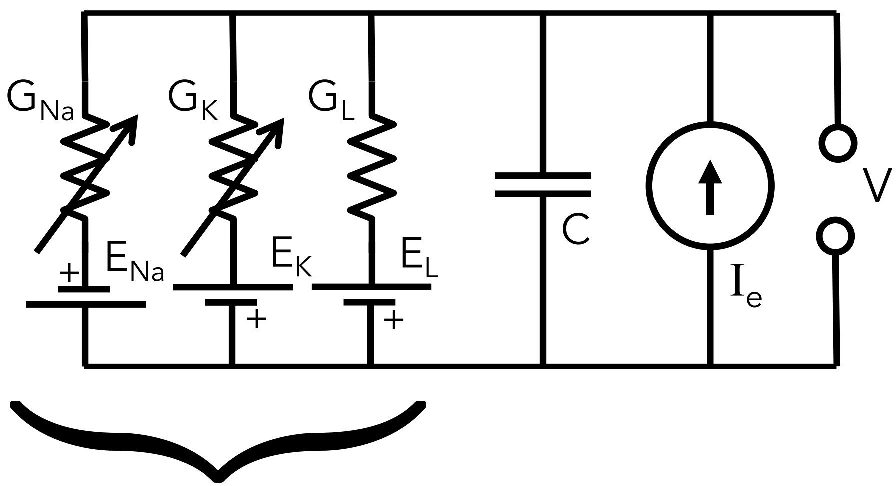
Voltage and time-dependent ion channels are the 'knobs' that control membrane potential.





Removed due to copyright restrictions: Figure 1a: The first intracellular recording of an action potential, from squid axon. Häusser, M. "The Hodgkin-Huxley theory of the action potential." *Nature Neuroscience* 3 (2000).

Hodgkin-Huxley model of action potential generation



$$I_m = I_{Na} + I_K + I_L$$

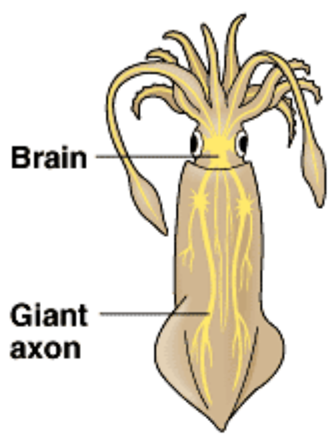
This is the total membrane ionic current, and it includes the contribution from —sodium channels, potassium channels and a 'leak' conductance.

The equation for our HH model neuron is

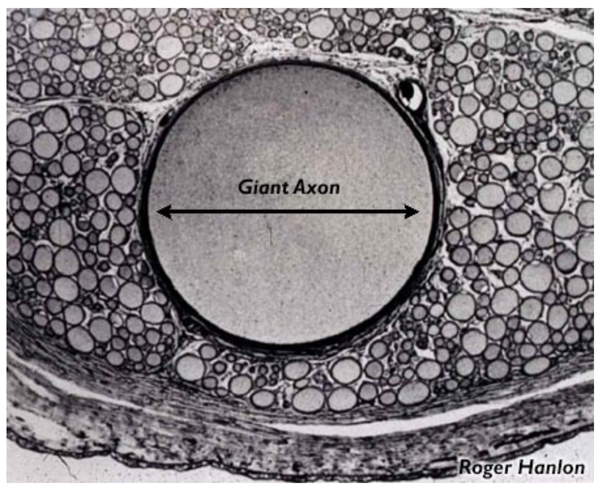
$$I_m(t) + C\frac{dV(t)}{dt} = I_e(t)$$

Voltage and Time dependence

- Voltage and time-dependent ion channels are the 'knobs' that control membrane potential.
- H&H studied the properties of K and Na channels in the squid giant axon. In particular they wanted to study the voltage and time dependence of the K and Na channels.



(g) Squid (mollusk)

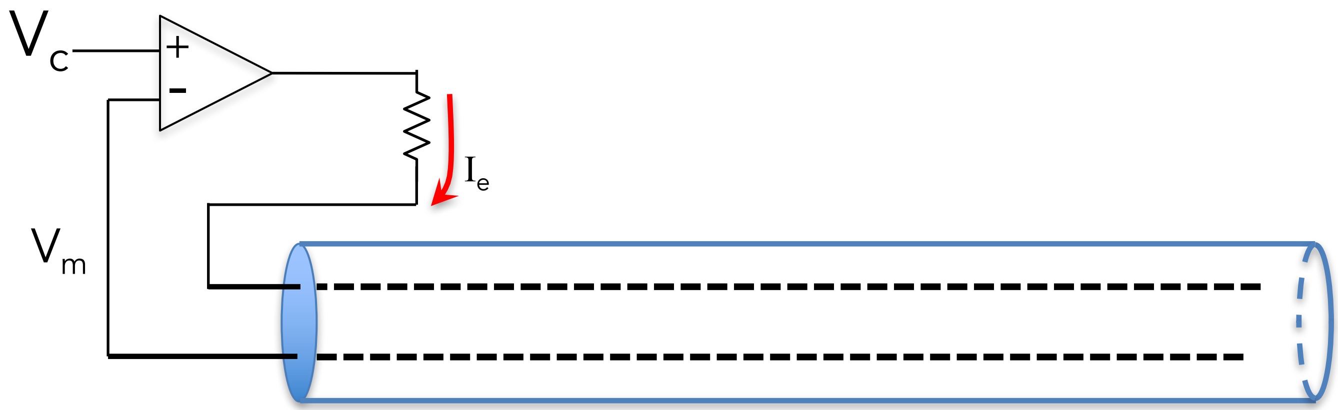


1mm diameter!

Removed due to copyright restrictions: Figure 1a: The first intracellular recording of an action potential, from squid axon. Häusser, M. "The Hodgkin-Huxley theory of the action potential." *Nature Neuroscience* 3 (2000).

Hodgkin and Huxley, 1938

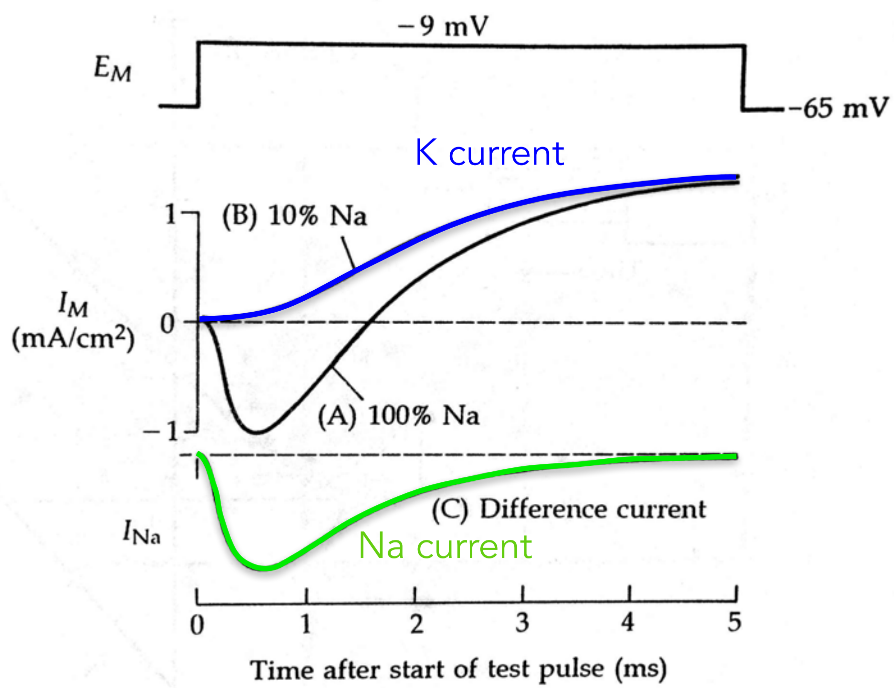
Ionic currents



Ionic currents

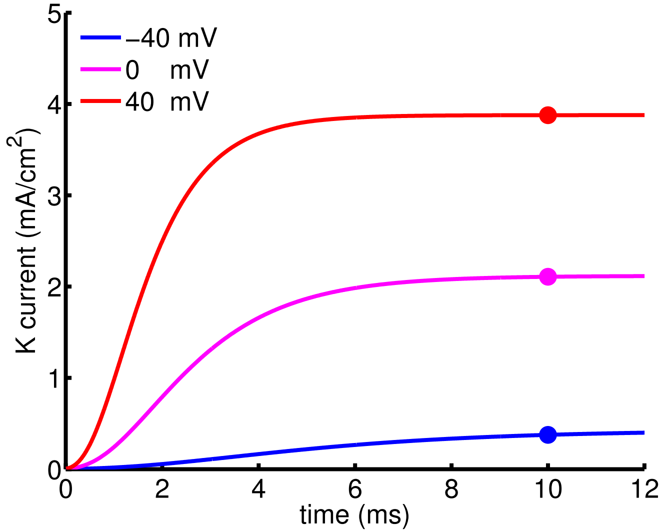
How do we figure out the contribution of Na and the contribution of K?

Ionic substitution (e.g. replace NaCl with choline chloride)

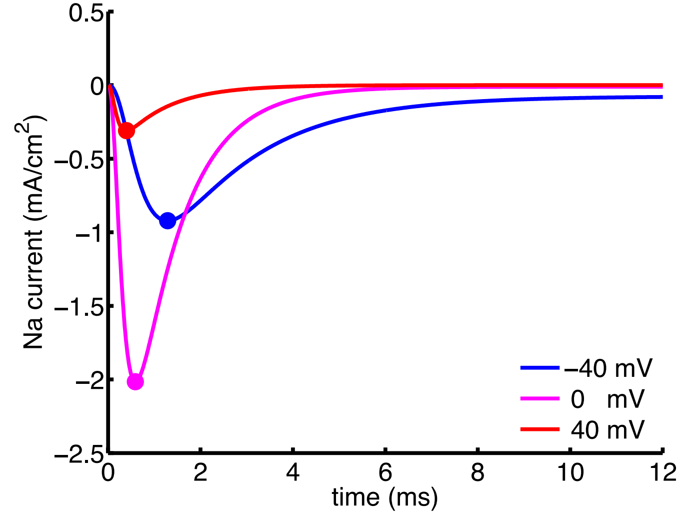


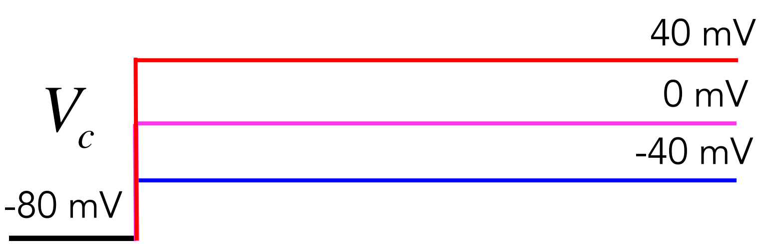
Ionic currents

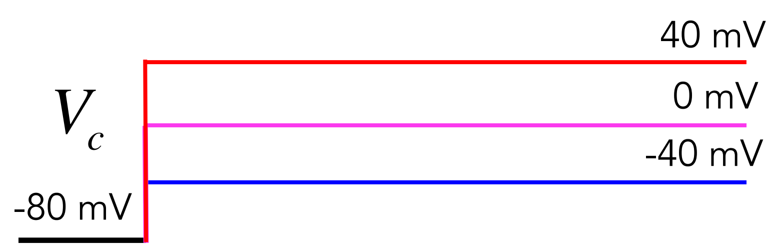
K current



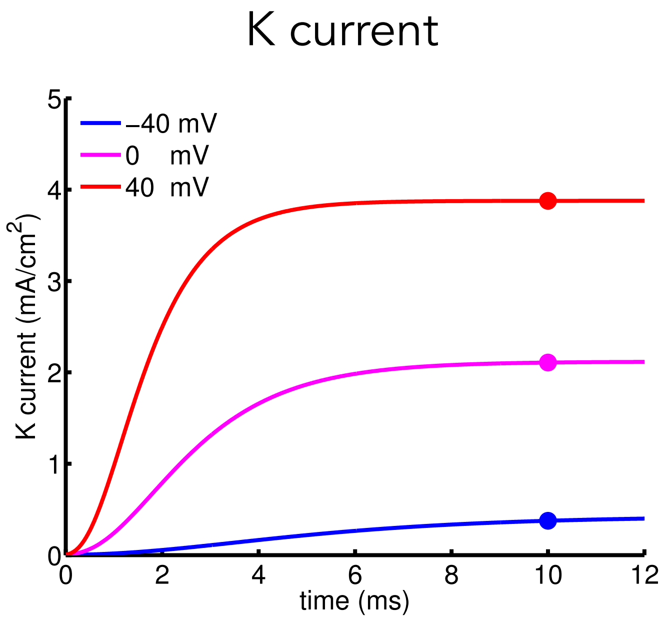
Na current

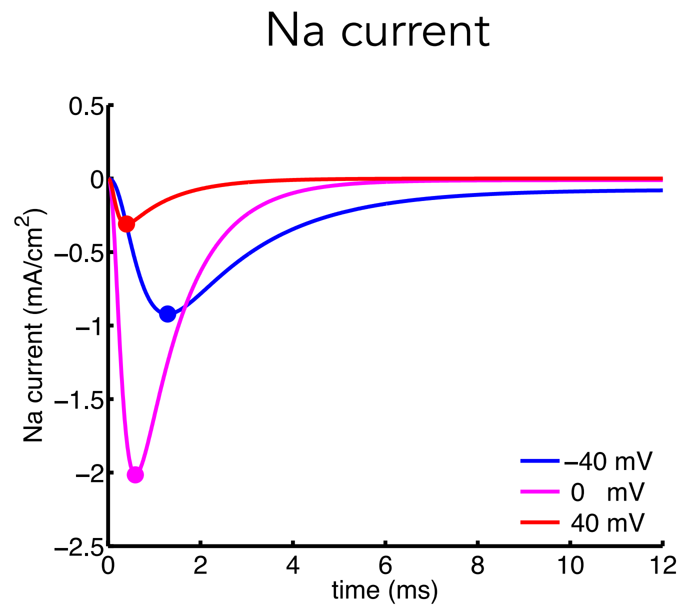


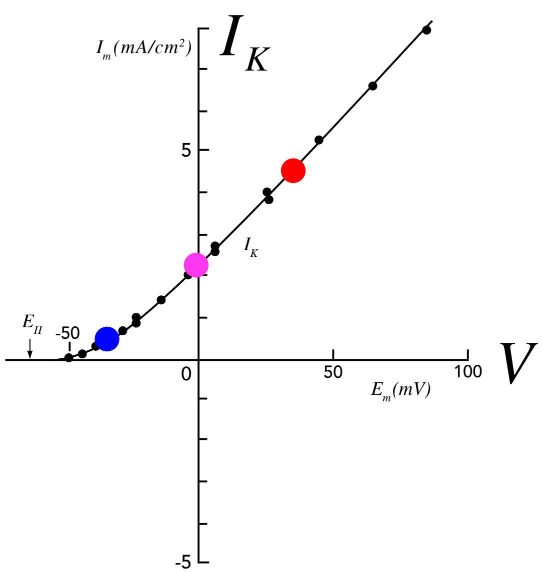


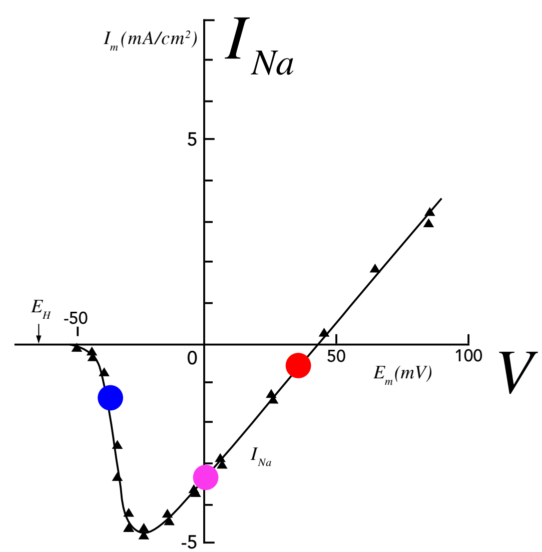


Ionic currents

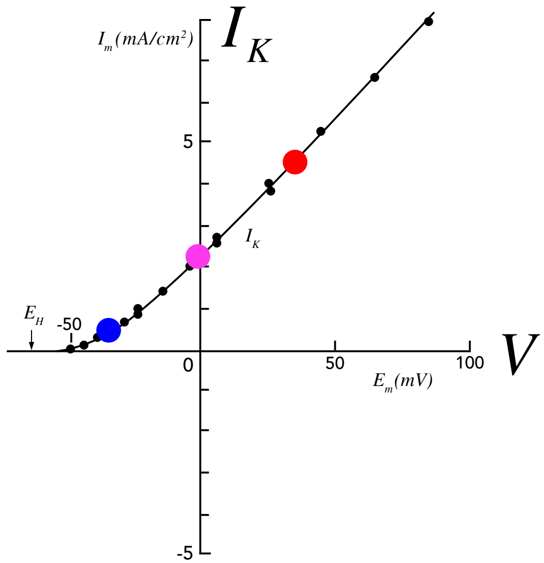


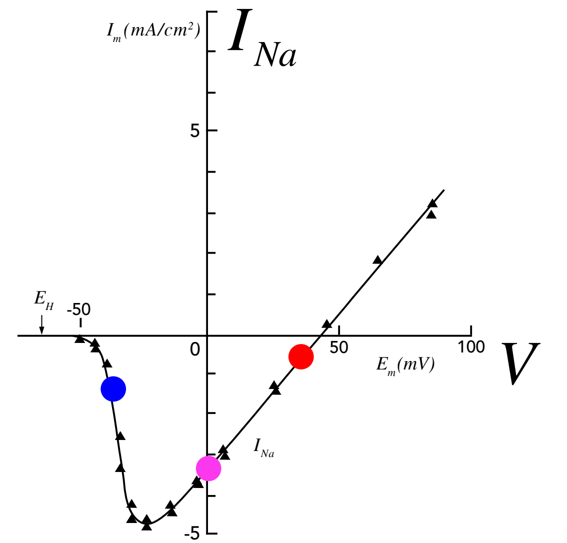






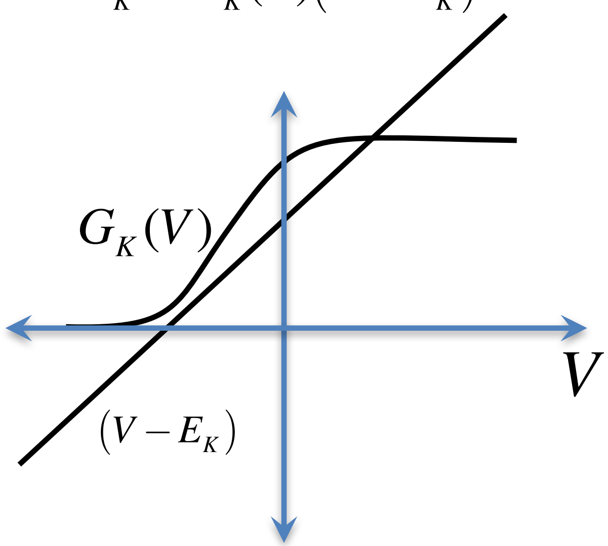
Ionic currents (Voltage dependence)

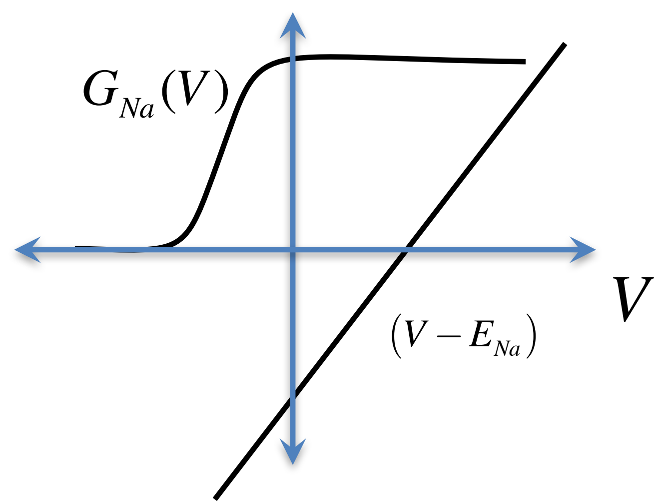




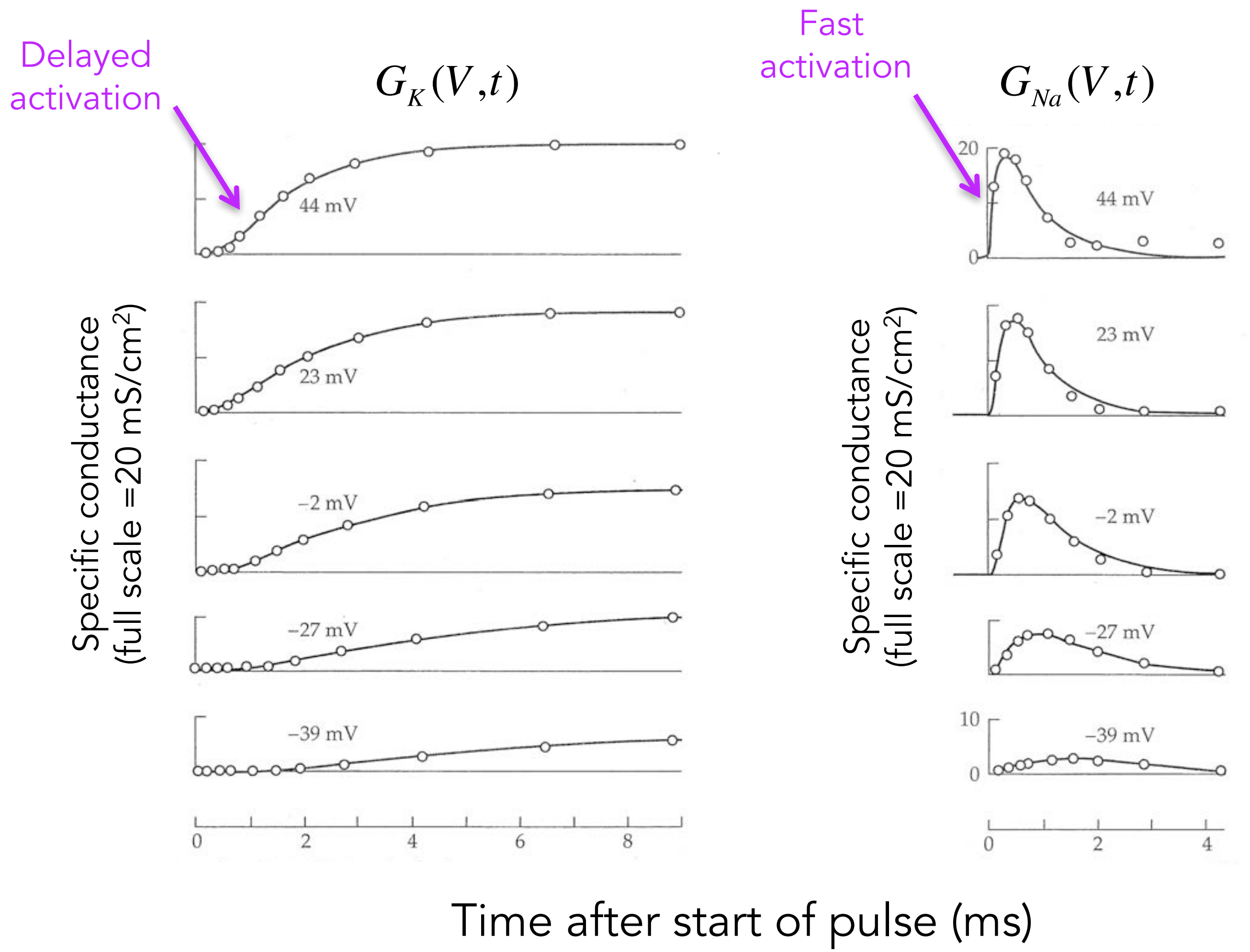
$$I_K = G_K(V)\left(V - E_K\right)$$

$$I_{Na} = G_{Na}(V)\left(V - E_{Na}\right)$$

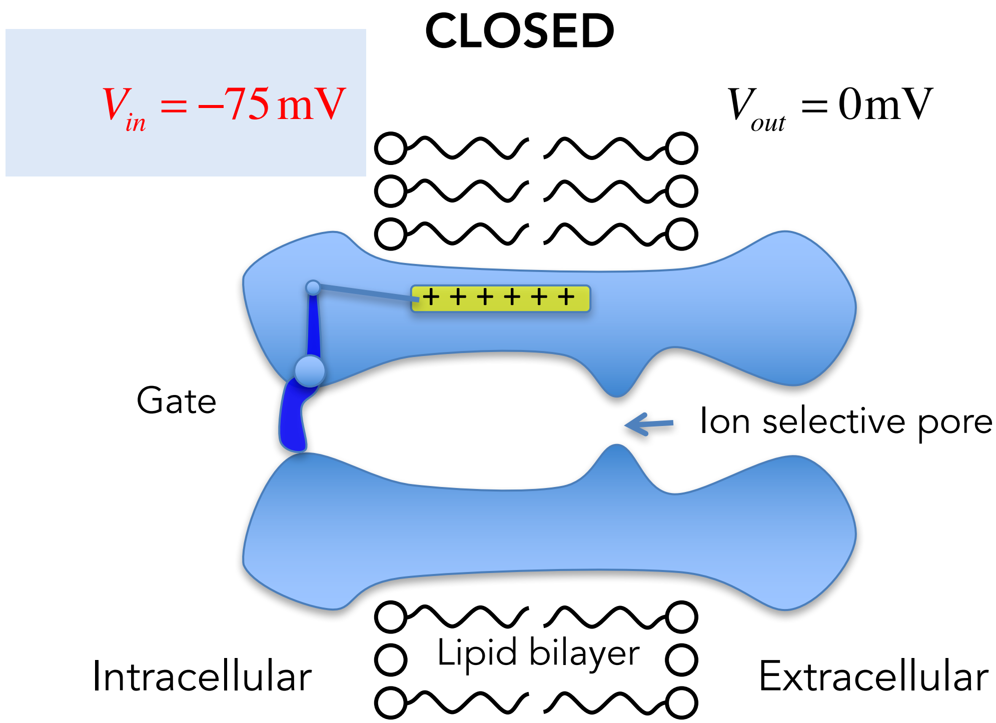




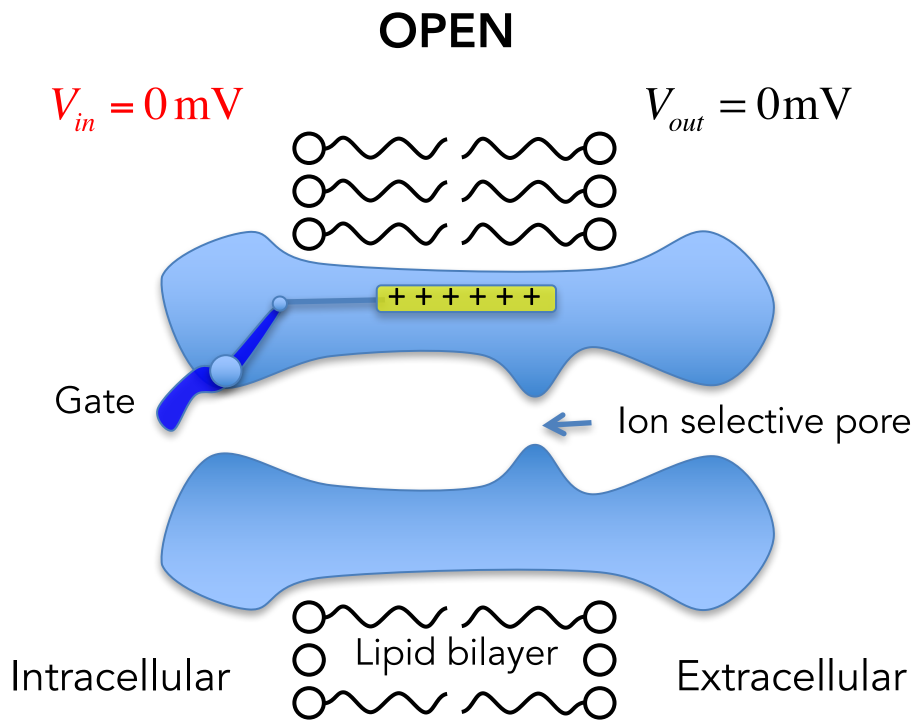
Ionic currents (time and voltage dependence)



Voltage-dependent conductance use voltage sensors



Voltage-dependent conductance use voltage sensors



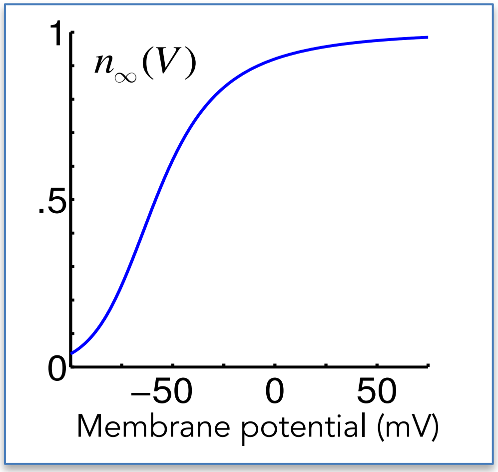
K and Na conductances

We modeled changes in conductance as transitions between 'closed' and 'open' states of ion channels.

K-conductance

$$\underset{1-n}{\text{'closed'}} \underset{\beta_n}{\overset{\alpha_n}{\rightleftharpoons}} \underset{n}{\text{'open'}}$$

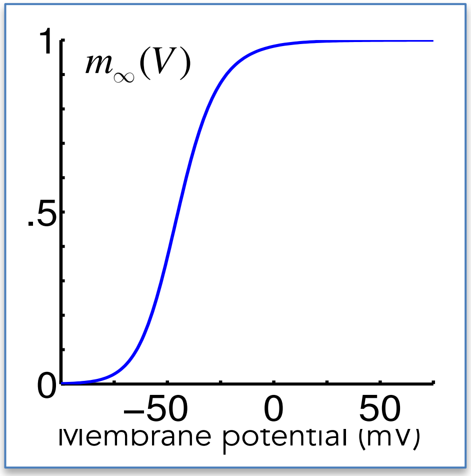
$$\tau_n \frac{dn}{dt} = n_\infty - n \qquad n_\infty = \frac{\alpha_n}{(\alpha_n + \beta_n)}$$



Na-conductance

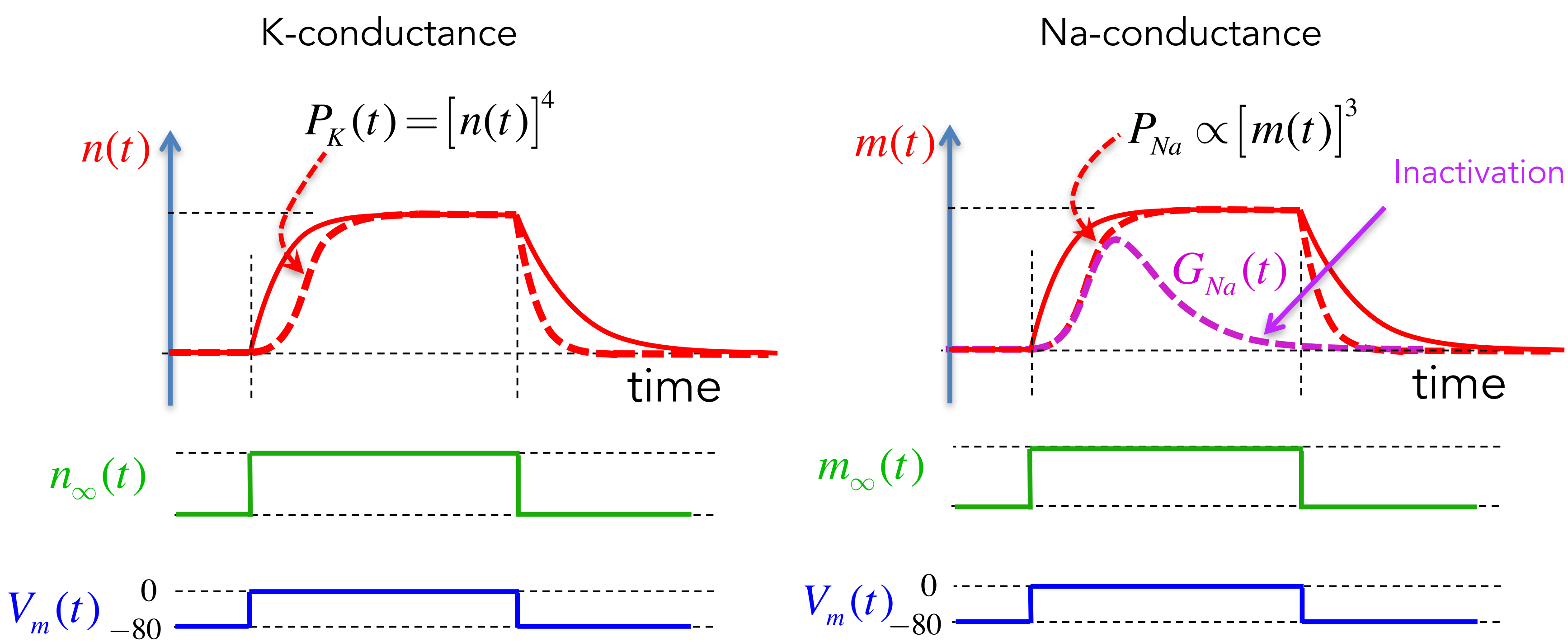
$$\underset{1-m}{\text{'closed'}} \underset{\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} \underset{m}{\text{'open'}}$$

$$\tau_m \frac{dm}{dt} = m_\infty - m \qquad m_\infty = \frac{\alpha_m}{(\alpha_m + \beta_m)}$$



Gating variables

The activation of both Na and K conductances is represented by 'gating variables' m and n



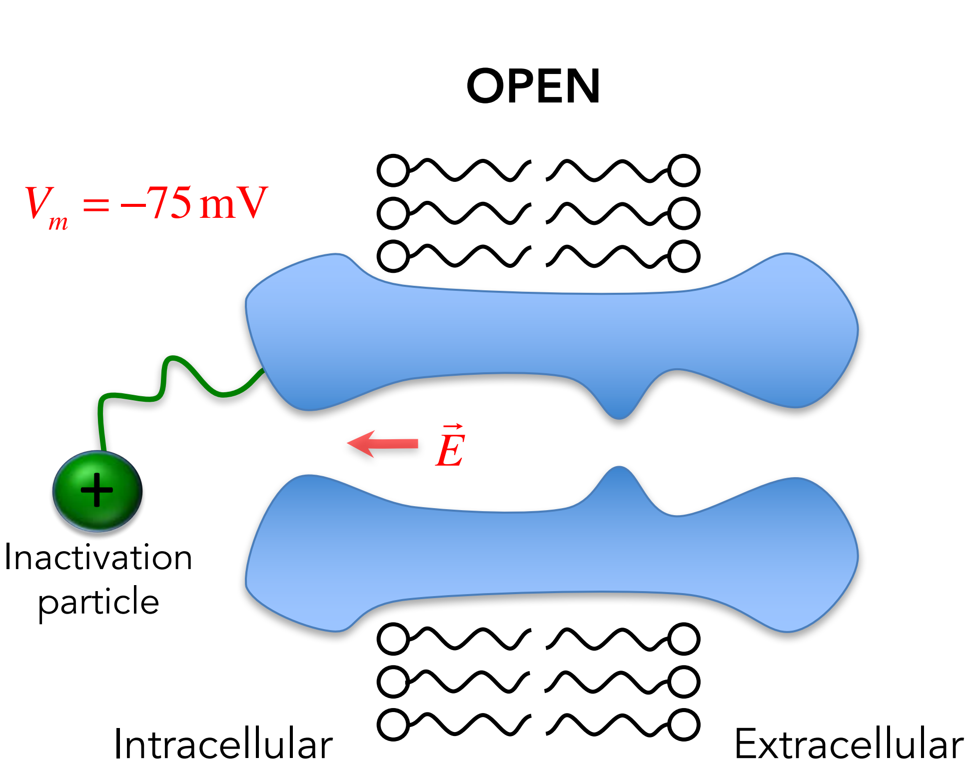
Activation gating variable

Inactivation gating variable

$$P_{Na} = m^3 h$$

Sodium channel inactivation

HH postulated an additional voltage-dependent inactivation gate.

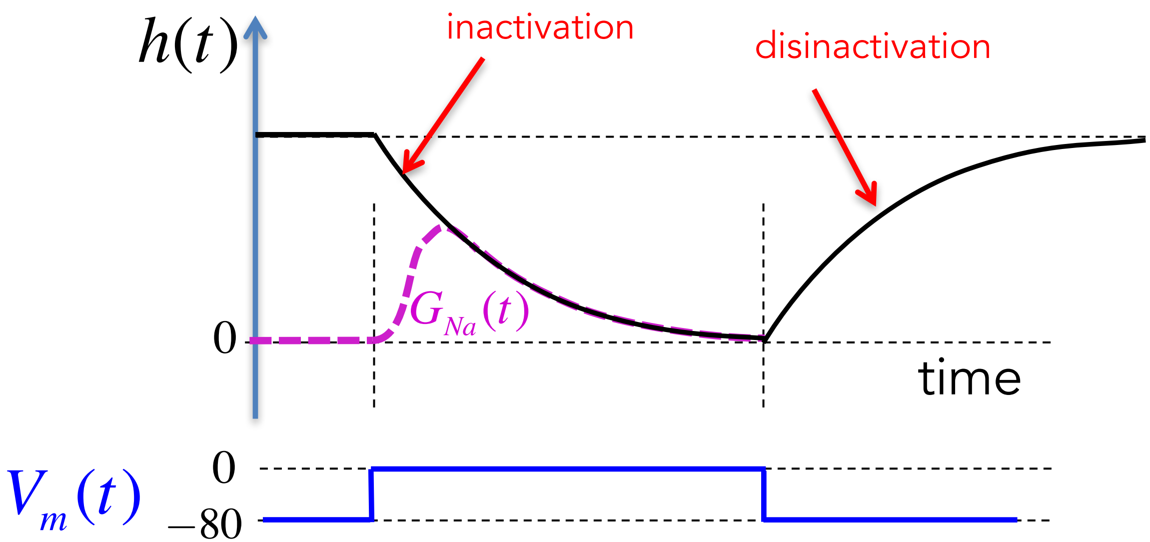


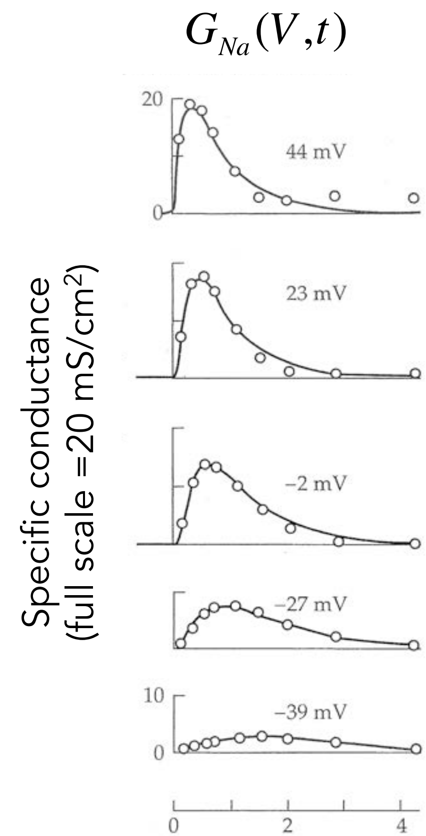
$$\underset{1-h}{\text{'closed'}} \underset{\beta_h}{\overset{\alpha_h}{\rightleftharpoons}} \underset{h}{\text{'open'}} \qquad \tau_h \frac{dh}{dt} = h_\infty - h$$

Sodium channel inactivation

Dynamics of inactivation are captured by a new gating variable ‘h’.

$$\tau_h \frac{dh}{dt} = h_\infty - h$$





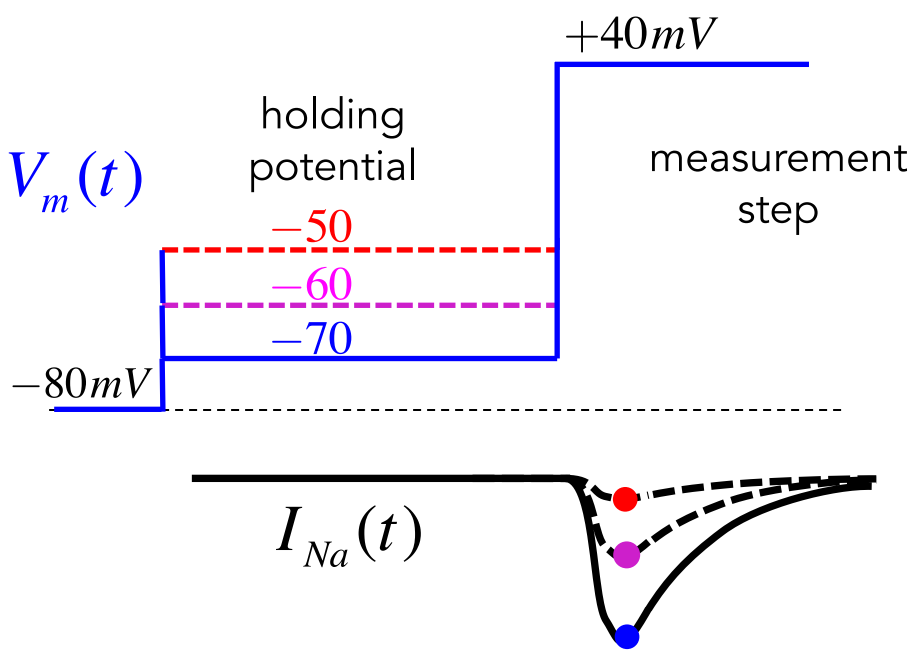
$$P_{Na} = m^3 h$$

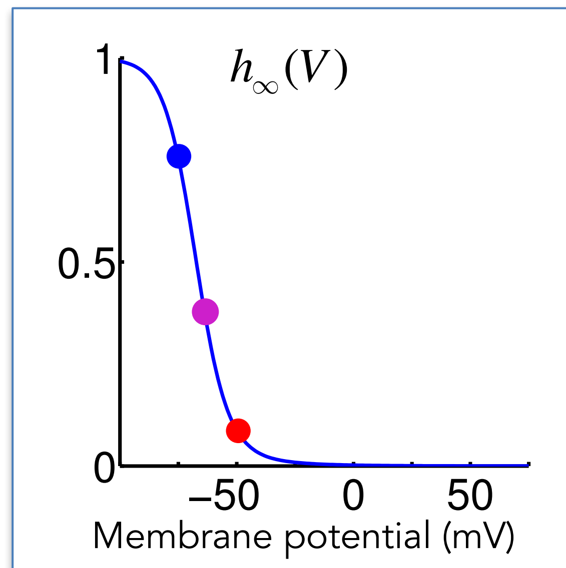
Measuring the parameters

How do we measure inactivation and recovery from inactivation?

1. Hold V_m at different values
2. Let the Na channels inactivate
3. Then measure the Na current!

$$\tau_h \frac{dh}{dt} = h_\infty - h$$





The sodium conductance

Putting our two Na-channel gating variables together, we get:

The probability of having a Na channel open is:

$$P_{Na} = m^3 h$$

Note independence

NOT !

But it's not so bad

The sodium conductance is:

$$G_{Na} = \bar{G}_{Na} m^3 h$$

And the sodium current is:

$$I_{Na} = \bar{G}_{Na} m^3 h (V - E_{Na})$$

Putting it all together!

Start with initial contition $V_m = V_0$ at time step t_0

Compute:

$n_\infty(V)$ and $\tau_n(V)$ $\quad$ $m_\infty(V)$ and $\tau_m(V)$ $\quad$ $h_\infty(V)$ and $\tau_h(V)$

$n(t) = n(t-1) + \frac{dn}{dt}\Delta t$ $\quad$ $m(t) = m(t-1) + \frac{dm}{dt}\Delta t$ $\quad$ $h(t) = h(t-1) + \frac{dh}{dt}\Delta t$

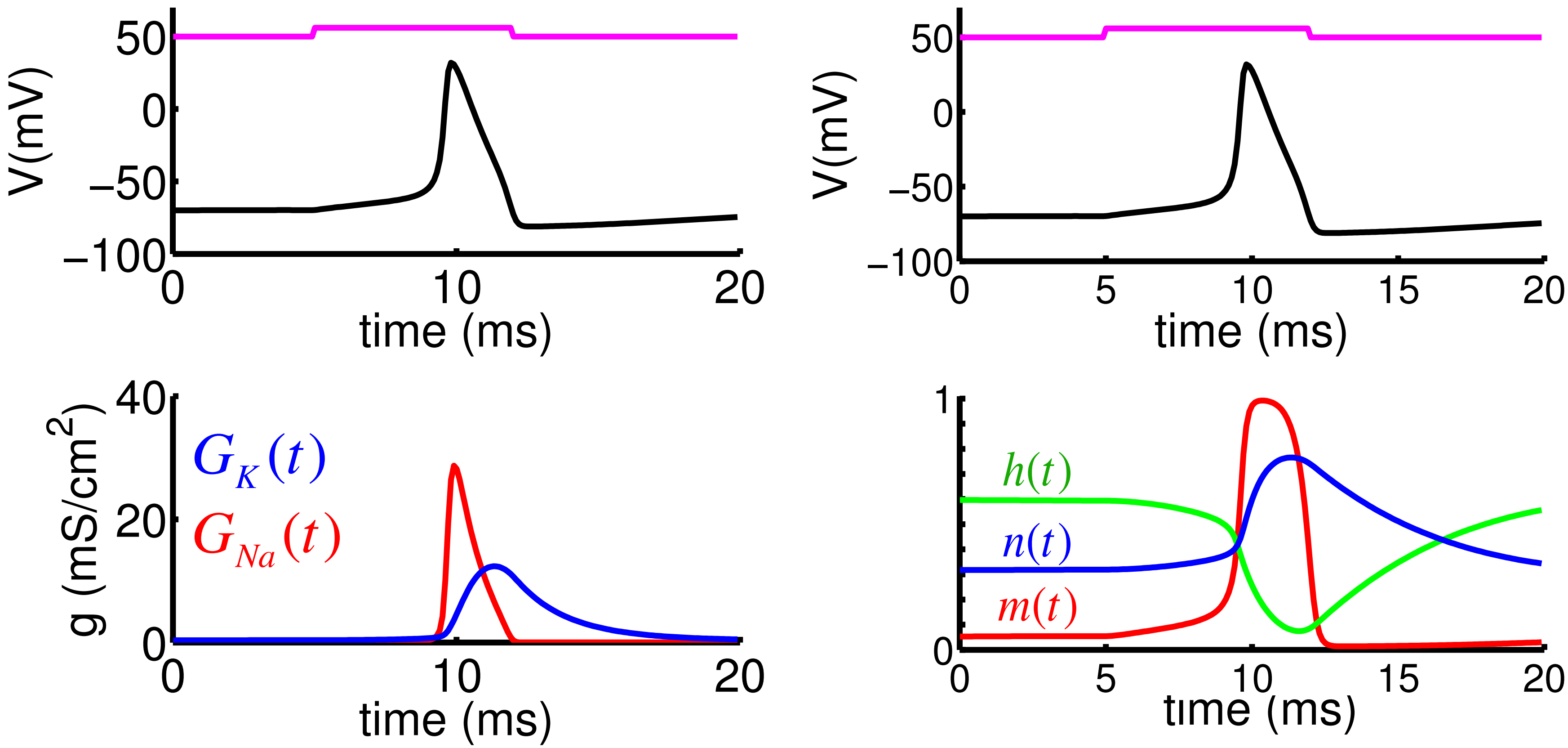
$I_K = \bar{G}_K n^4 (V - E_K)$ $\quad$ $I_{Na} = \bar{G}_{Na} m^3 h (V - E_{Na})$ $\quad$ $I_L = \bar{G}_L (V - E_L)$

Total membrane current $I_m = I_K + I_{Na} + I_L$

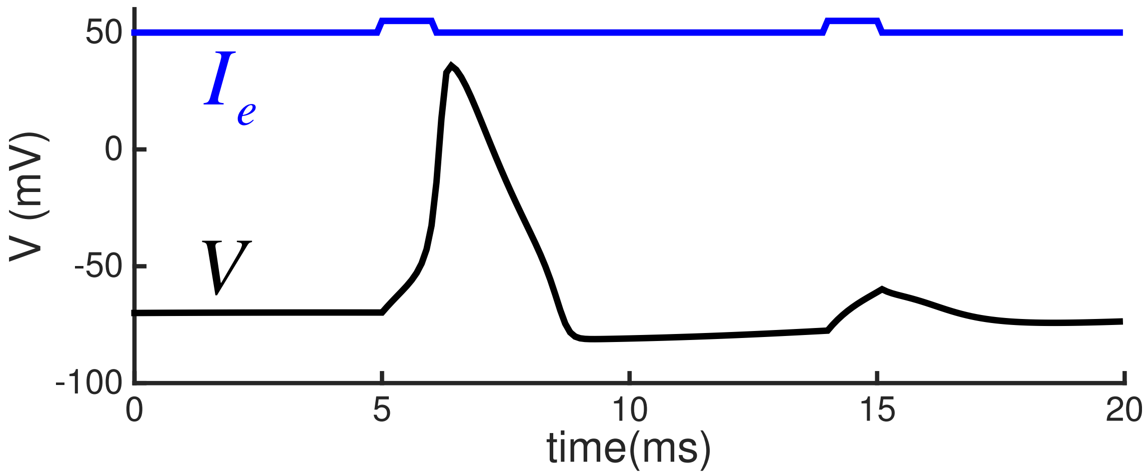
Compute τ_{mem} *and* V_∞

$$V_m(t) = V_m(t-1) + \frac{dV_m}{dt}\Delta t$$

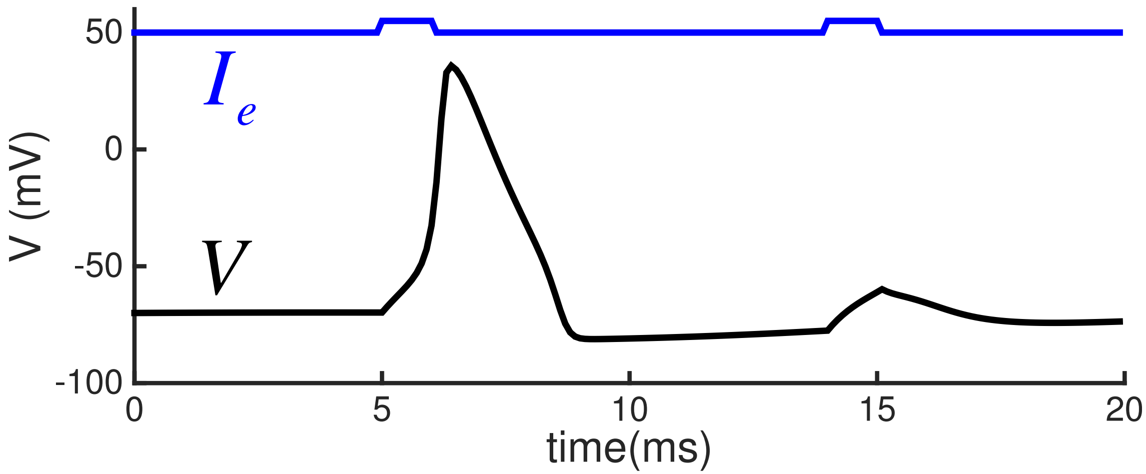
Putting it all together!

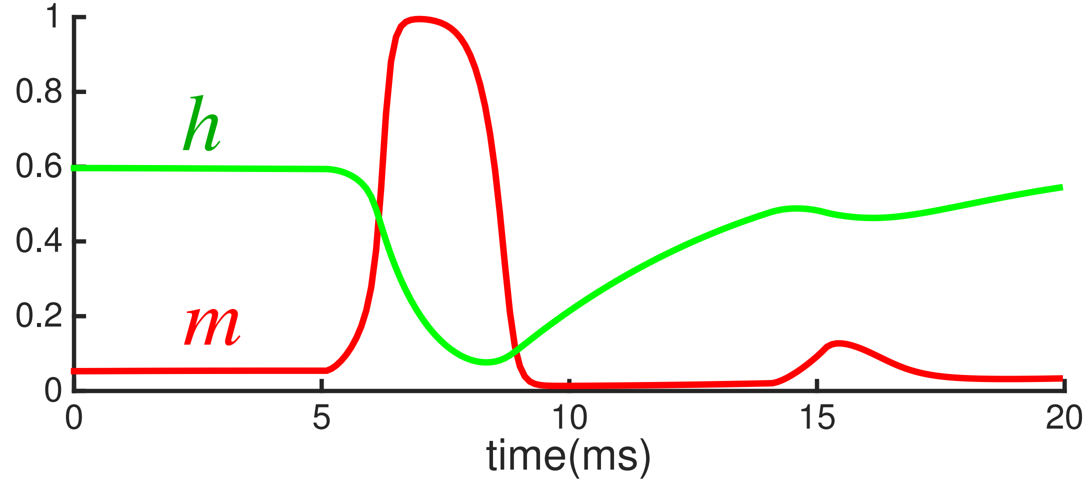


Spike refractory period



Spike refractory period due to sodium channel inactivation





Diseases related to defects in sodium channel inactivation

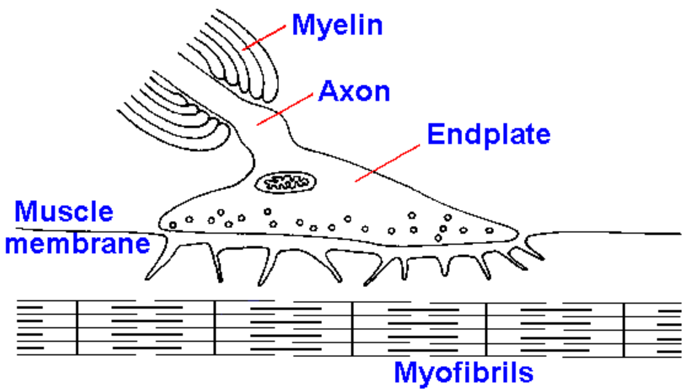
Fainting Goats Video from National Geographic

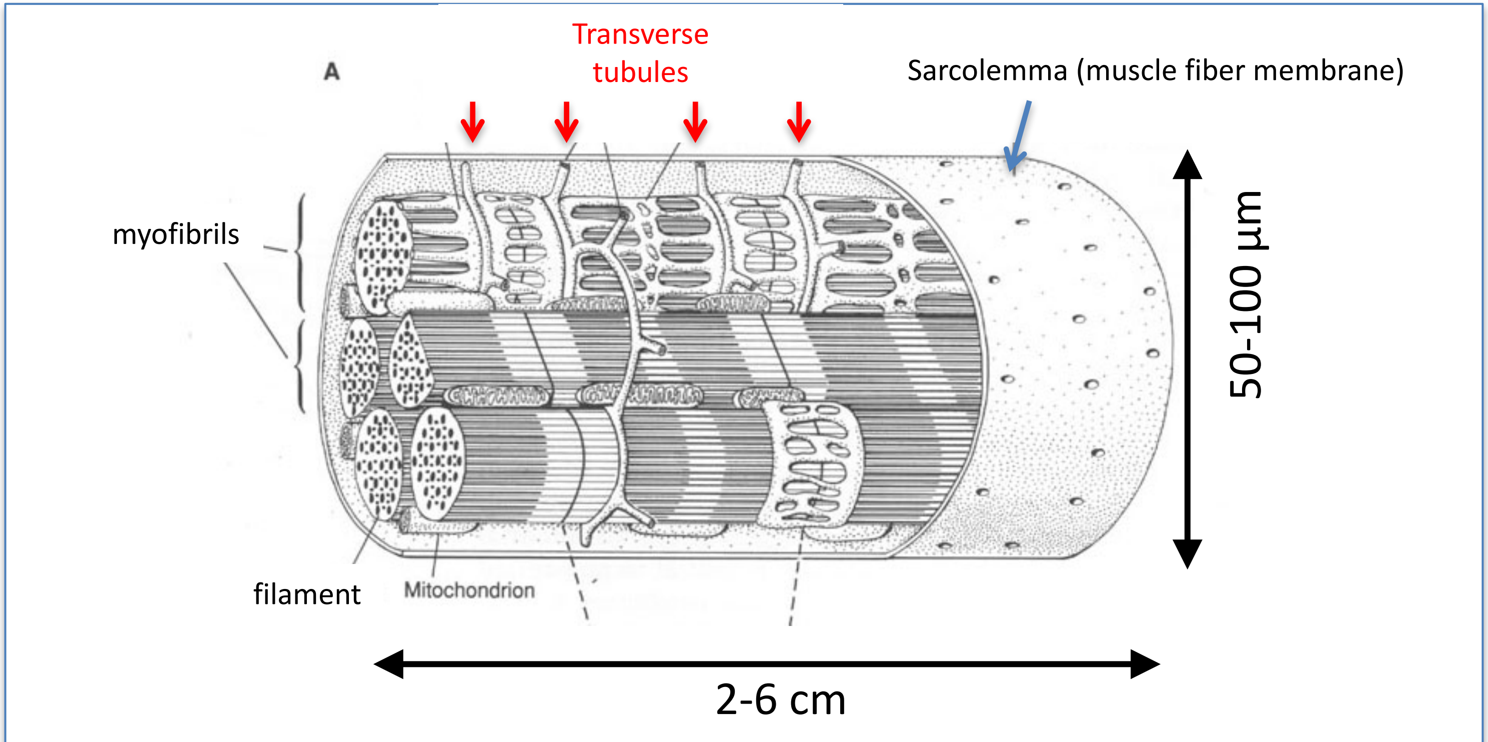
Diseases related to defects in sodium channel inactivation

Hyperkalemic Periodic Paralysis – Hyper PP

See Lecture video to view clip

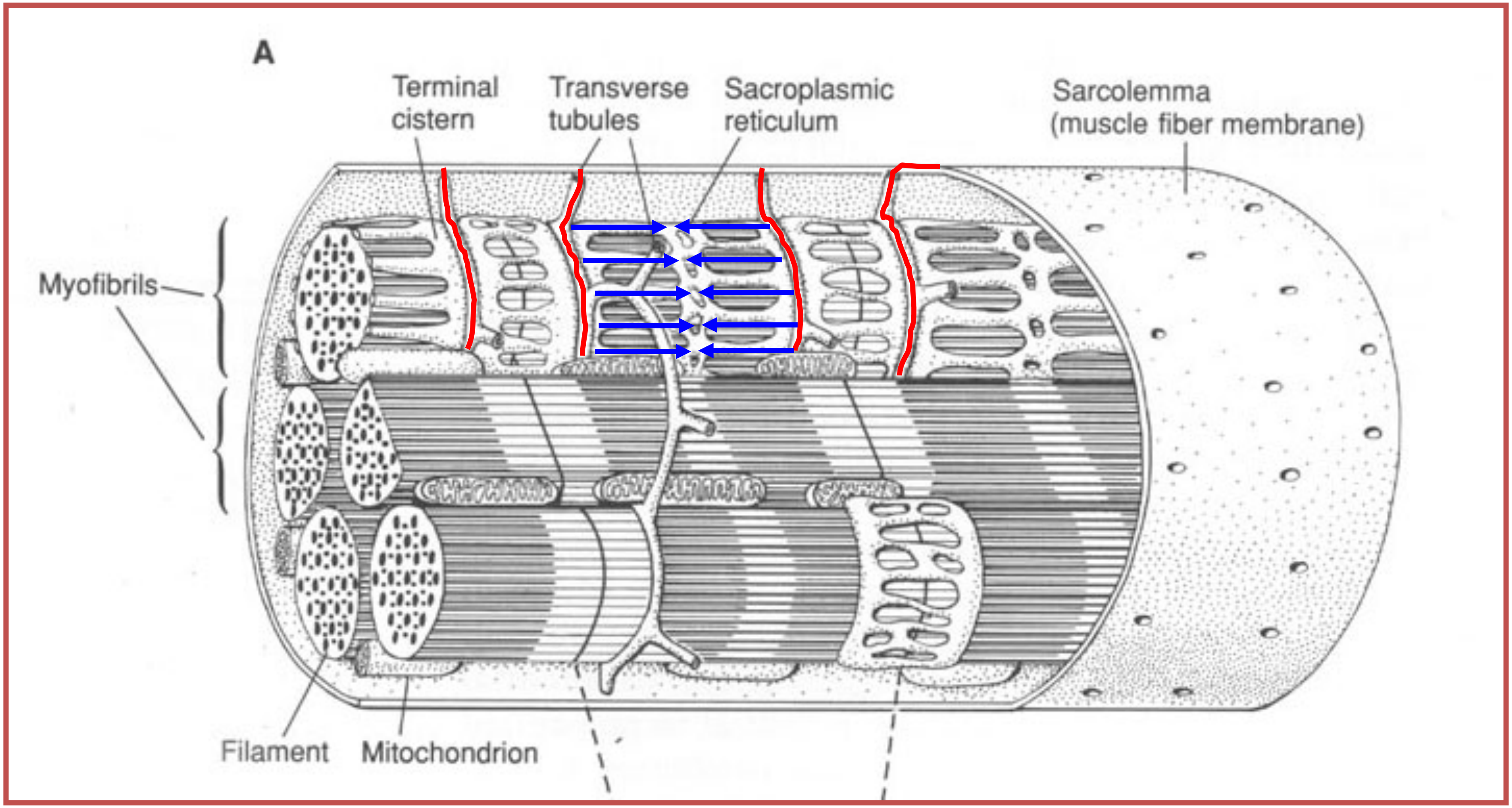
Structure of Muscle Fiber





Annotated Figure © Kandel, E.R, J.H. Schwartz, and T.M Jessell. *Principles of Neural Science* 3rd ed. 1991, McGraw-Hill.

Muscle Fiber AP Leads to Ca Release in Myofibrils



Annotated Figure © Kandel, E.R, J.H. Schwartz, and T.M Jessell. *Principles of Neural Science* 3rd ed. 1991, McGraw-Hill.

Diseases related to defects in sodium channel inactivation

Myotonia and Periodic Paralysis are associated with mutations of the Na channel (skeletal isoform only)

Figure removed due to copyright restrictions. See Figure 2: Cannon, S. "Sodum Channel Defects in Myotonia and Periodic Paralysis." *Annu. Rev. Neurosci.* 19 (1996):141-44.

Sodium channel mutations

wild-type

human M1592V mutation

Figure removed due to copyright restrictions. See Figure 3: Cannon, S. "Sodum Channel Defects in Myotonia and Periodic Paralysis." *Annu. Rev. Neurosci.* 19 (1996):141-44.

Diseases related to defects in sodium channel inactivation

Sea anemone toxin (ATXII, 10uM) partially blocks sodium channel inactivation.

Sea anemone image is in the public domain.
Source: heartypanther on Flickr.

Figure removed due to copyright restrictions. See Figure 5a: Cannon, S. "Sodum Channel Defects in Myotonia and Periodic Paralysis." *Annu. Rev. Neurosci.* 19 (1996):141-64.

Diseases related to defects in sodium channel inactivation

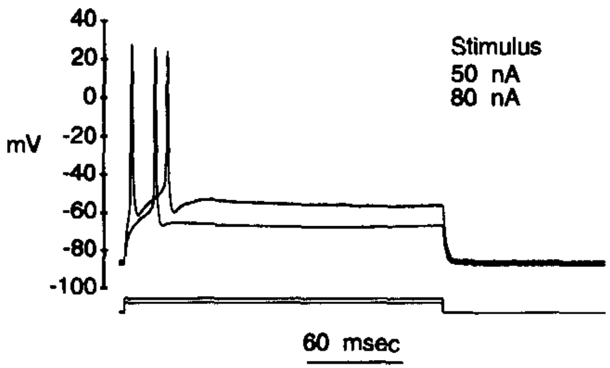
Sea anemone toxin (ATXII) also prolongs muscle fiber twitch duration.

Figure removed due to copyright restrictions. See Figure 5b: Cannon, S. "Sodum Channel Defects in Myotonia and Periodic Paralysis." *Annu. Rev. Neurosci.* 19 (1996):141-64.

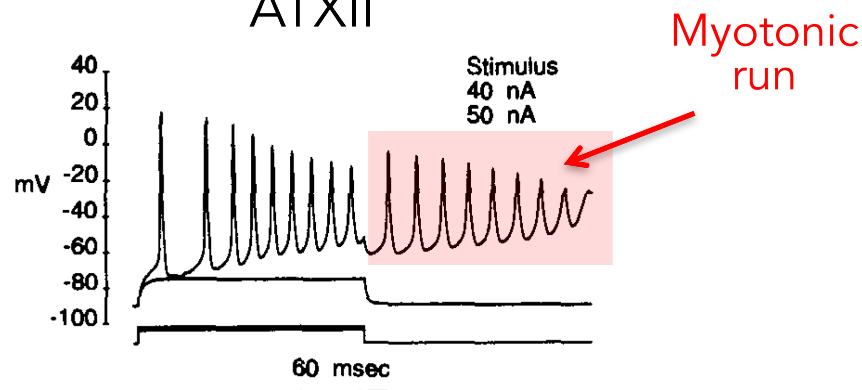
Diseases related to defects in sodium channel inactivation

Sea anemone toxin (ATXII) prolongs spiking in muscle fiber.

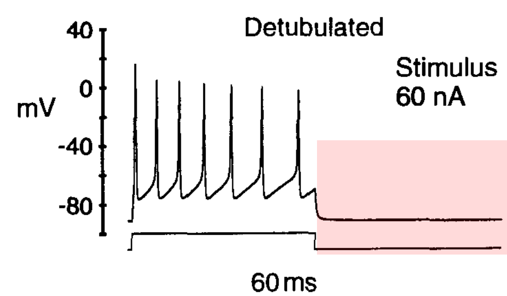
Control



ATXII

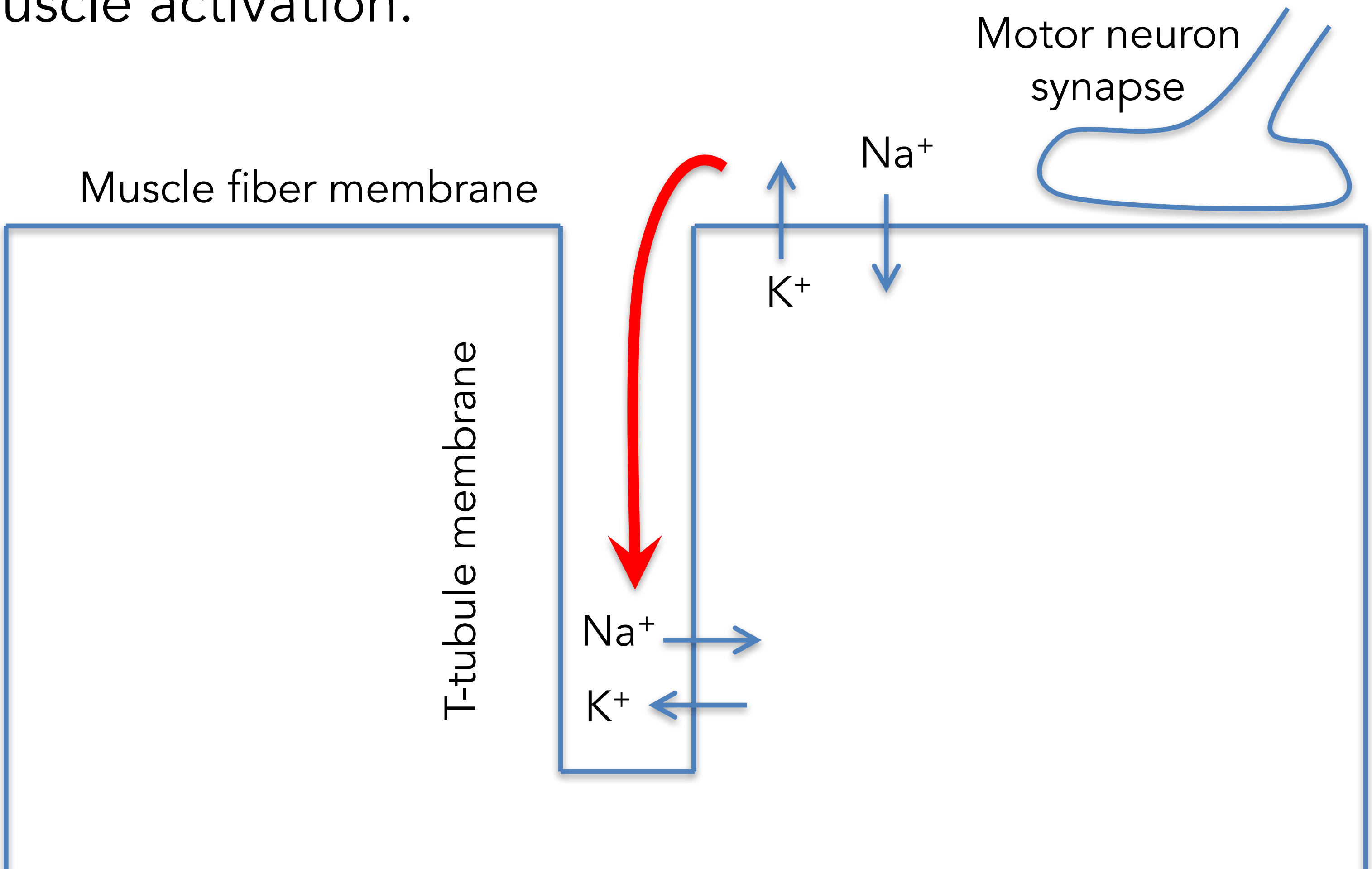


Osmotic shock breaks T-tubules and eliminates myotonic run



Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.



Diseases related to defects in sodium channel inactivation

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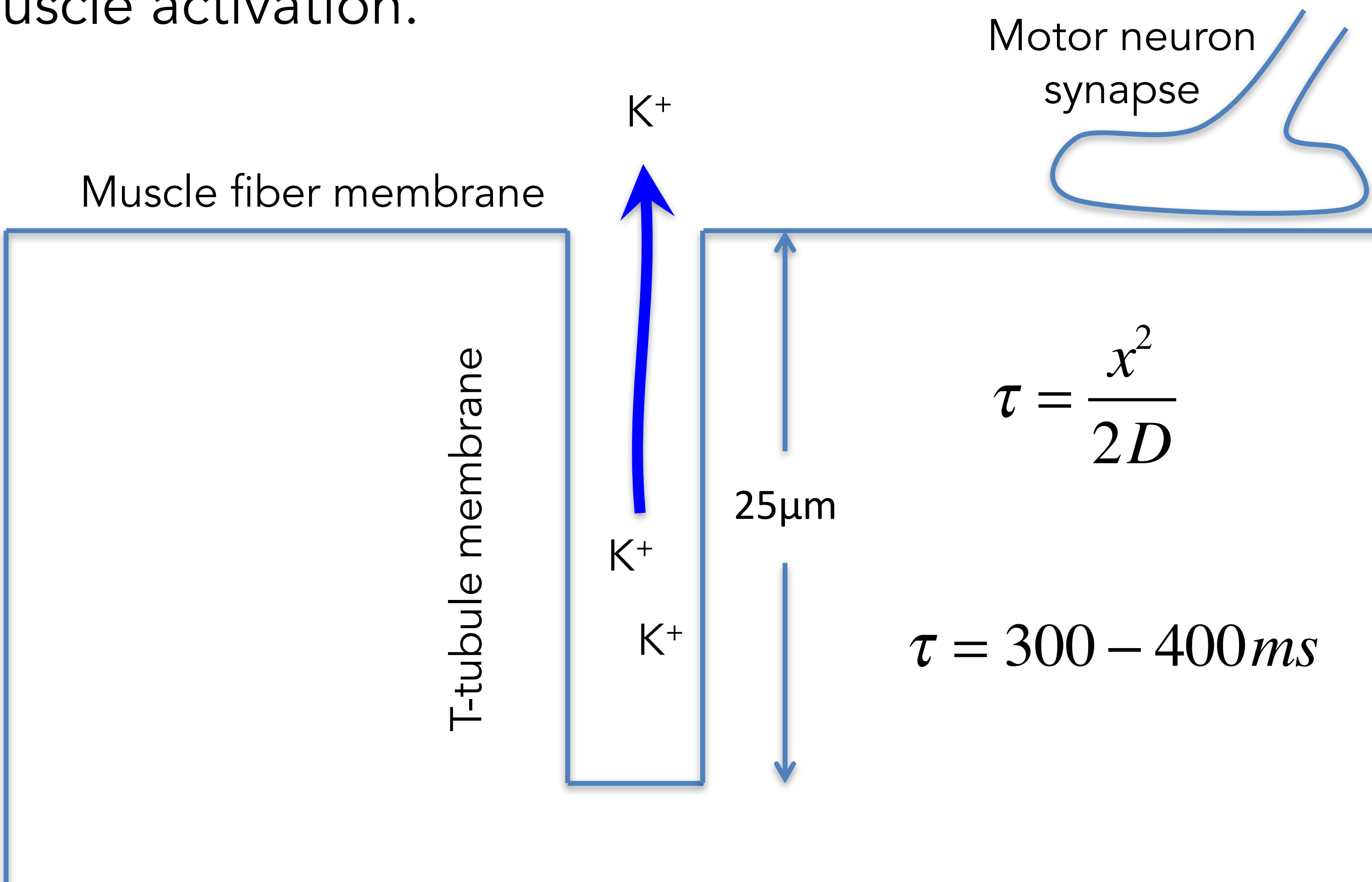
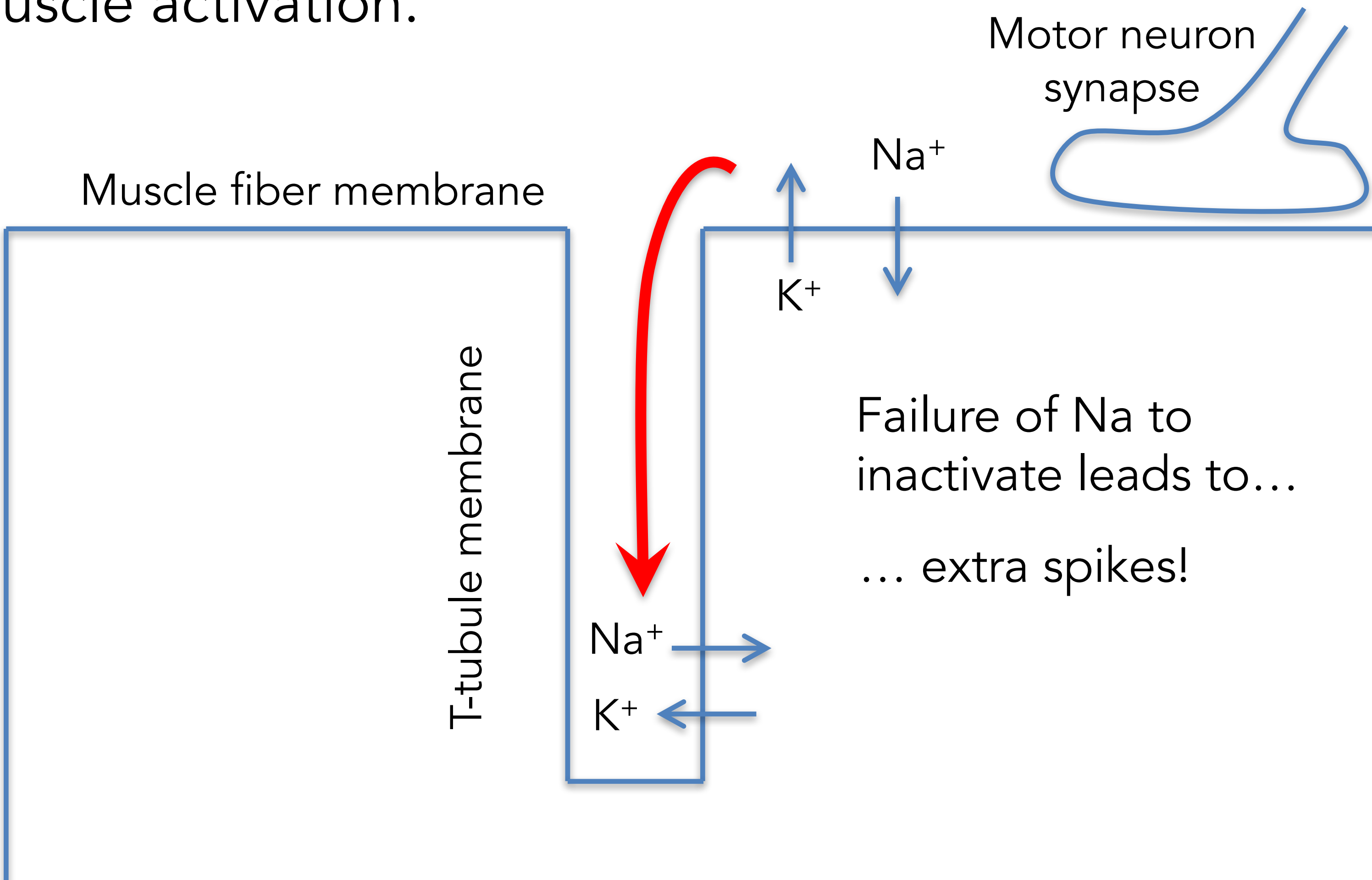


$$\tau = \frac{x^2}{2D}$$

$$\tau = 300 - 400ms$$

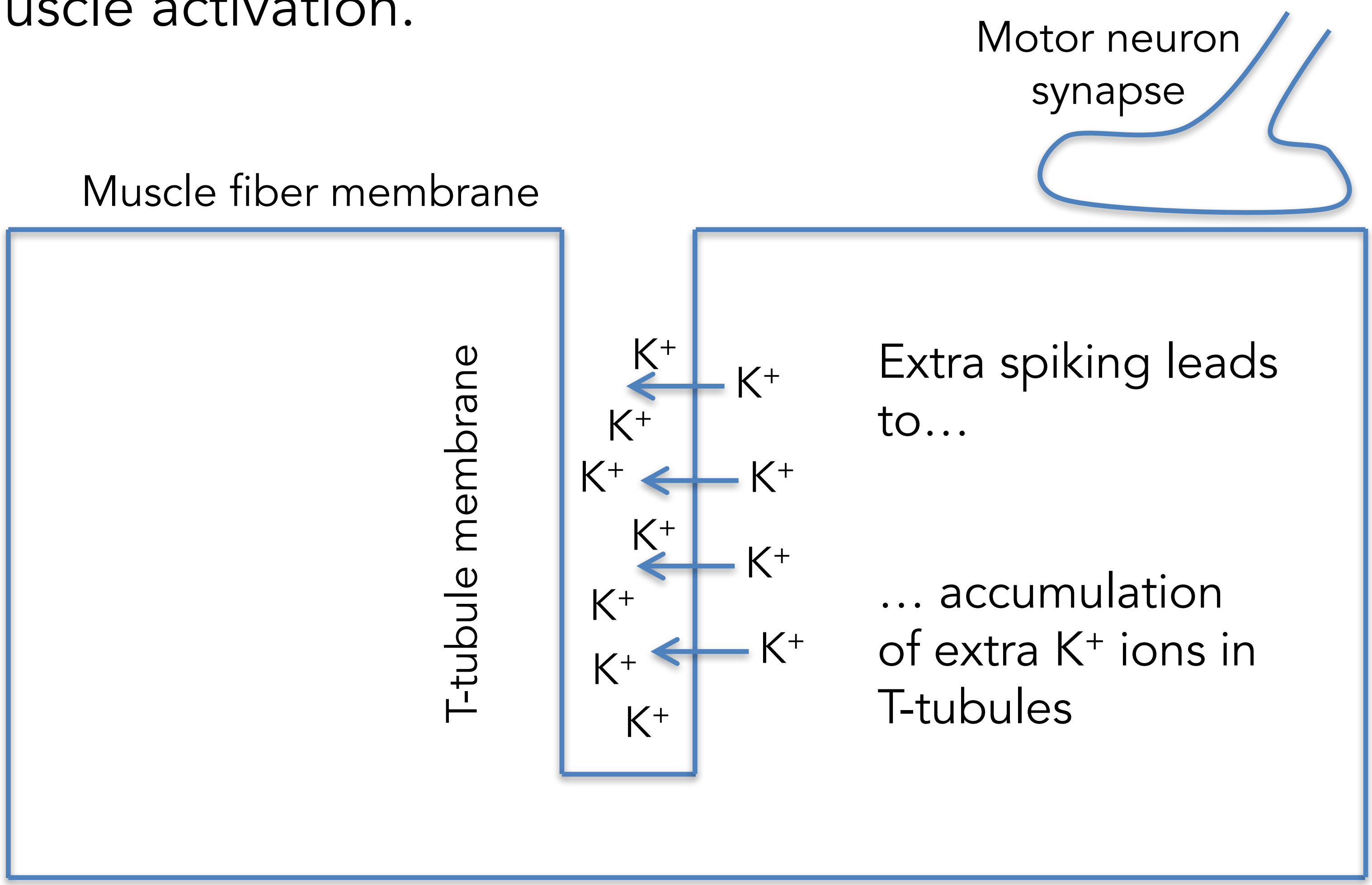
Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.



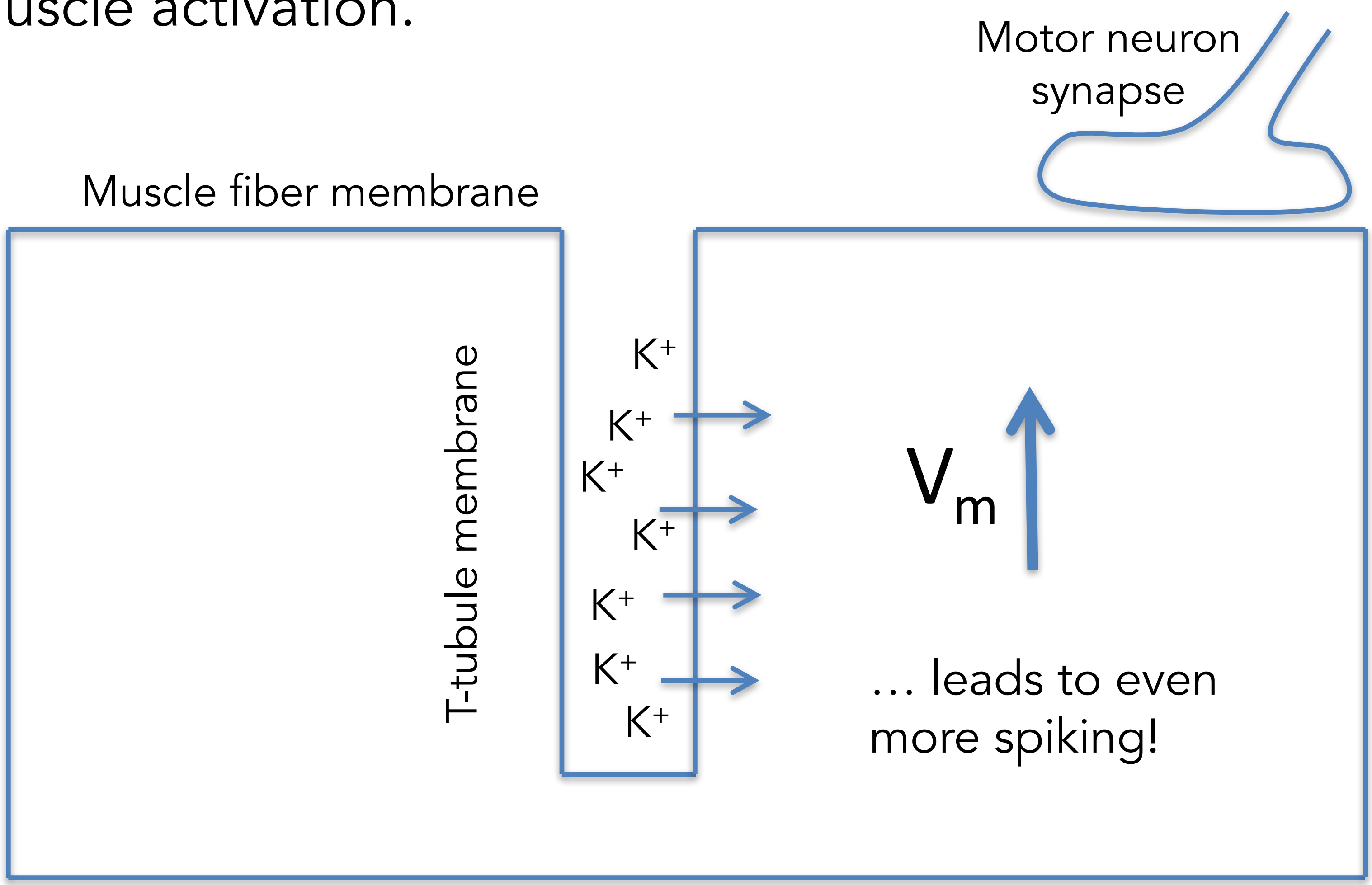
Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.



Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.



Diseases related to defects in sodium channel inactivation

Equivalent circuit model of muscle fiber membrane and T-tubule.

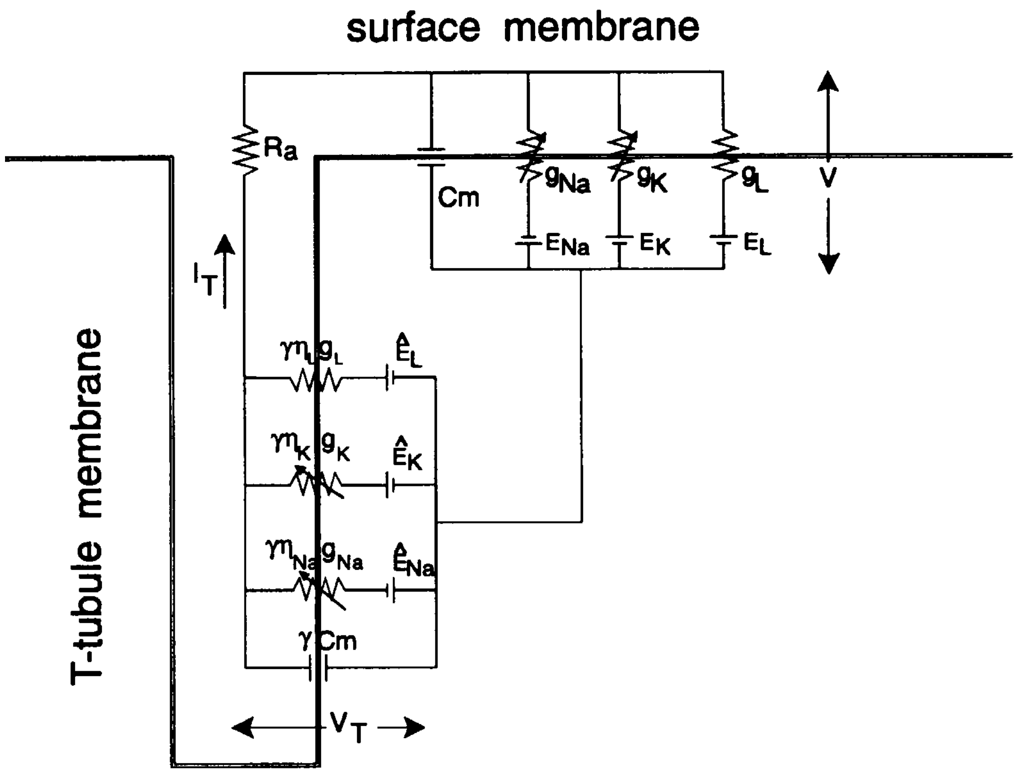
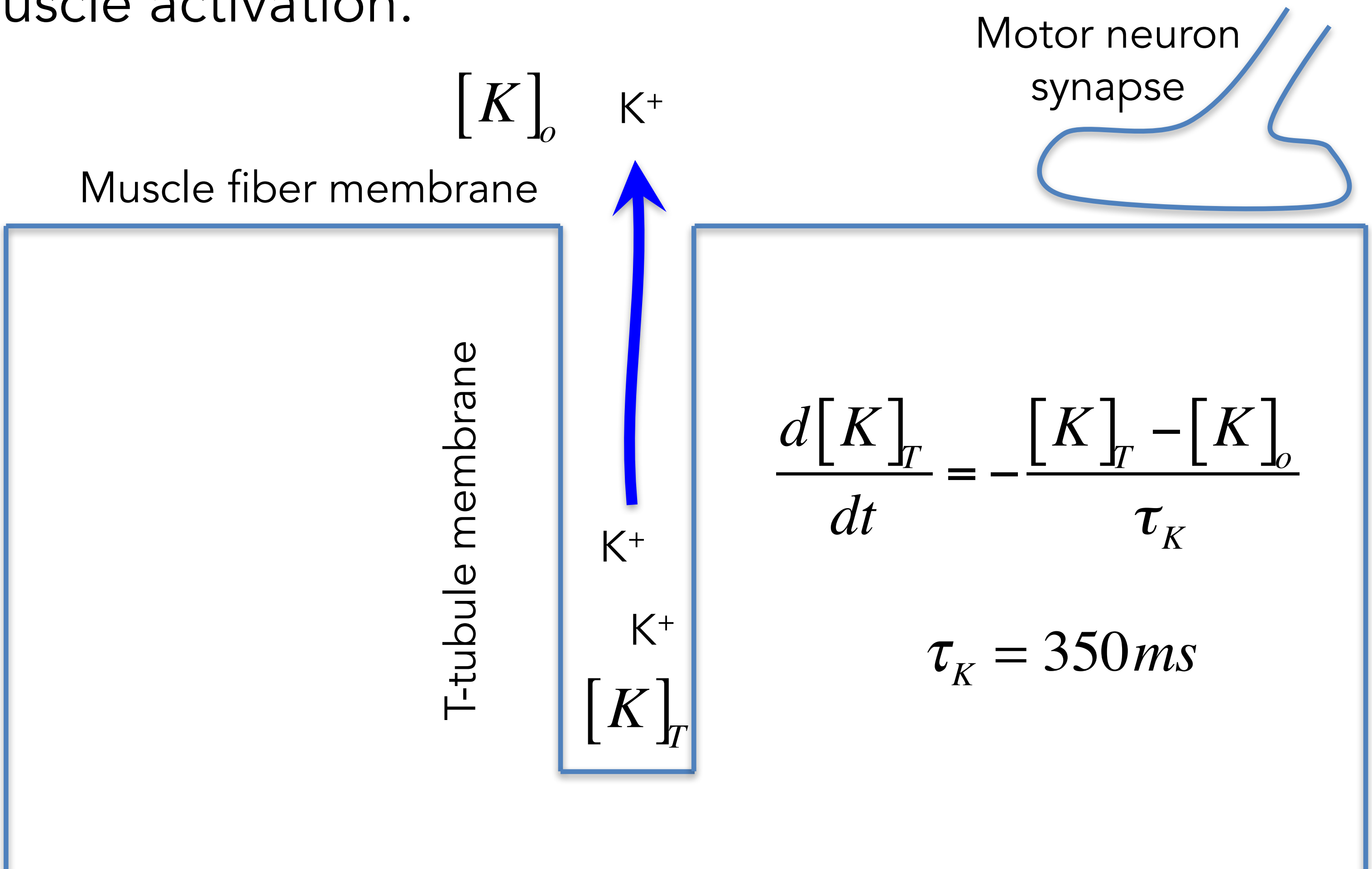


FIGURE 1 Equivalent circuit diagram for the model of the electrical behavior of a muscle fiber. The voltage and time dependence of the variable conductances are given by Eqs. 6, 7, 9, and 10. γ is the ratio of the T-tubular membrane area to surface membrane area. The ηs represent the density of ion channels in the T-tubular membrane relative to that of the surface membrane.

Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.

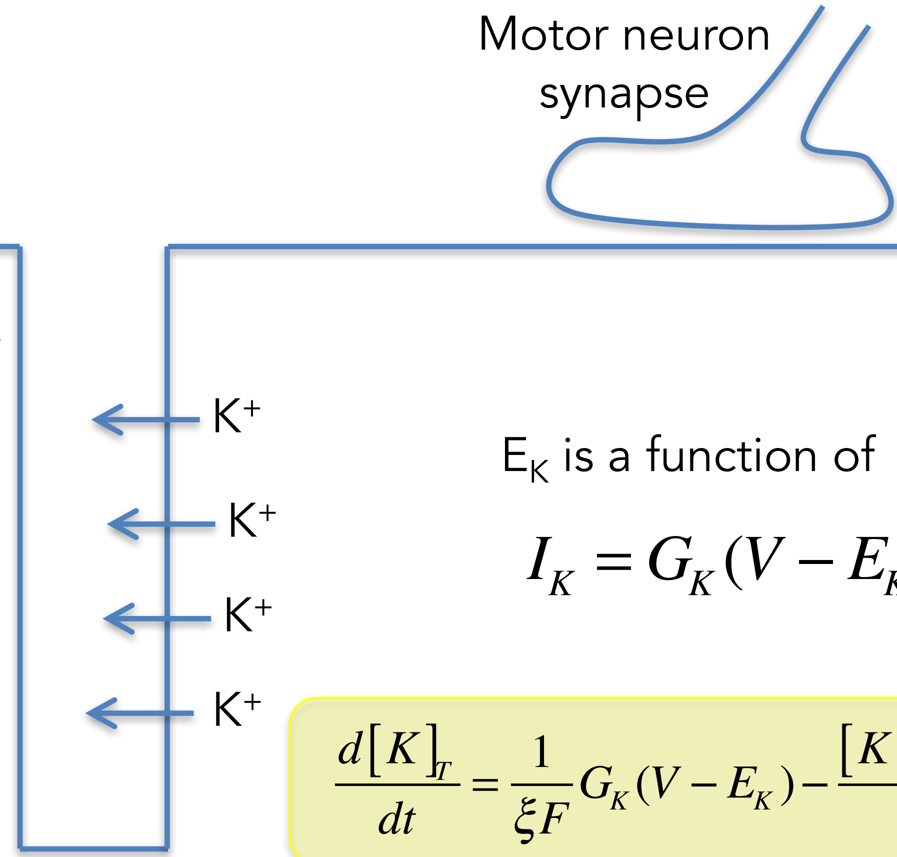


$$\frac{d[K]_T}{dt} = -\frac{[K]_T - [K]_o}{\tau_K}$$

$$\tau_K = 350\,ms$$

Diseases related to defects in sodium channel inactivation

Hypothesis for how persistent sodium leads to persistent muscle activation.



I_K = K-current into T-tubule

$$\frac{d[K]_T}{dt} \propto I_K$$

$$\frac{d[K]_T}{dt} = \frac{1}{\xi}\frac{1}{F} I_K$$

ξ = volume of T-tubule

F = Faraday constant (C/mol)

$= 9.6\text{x}10^4$ (C/mol)

E_K is a function of $[K]_T$!!

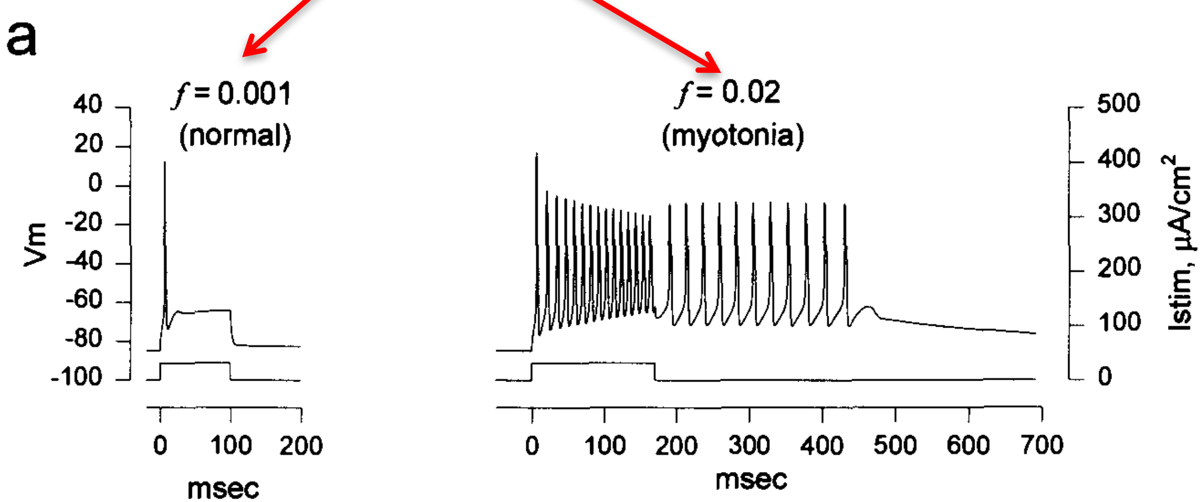
$$I_K = G_K(V - E_K)$$

$$\frac{d[K]_T}{dt} = \frac{1}{\xi F} G_K(V - E_K) - \frac{[K]_T - [K]_o}{\tau_K}$$

Diseases related to defects in sodium channel inactivation

Computer model of effects of defective Na-channel inactivation

Fraction of Na channels that fail to inactivate



Failure to inactivate was modeled by settting h=1 for a fraction of the channels

Diseases related to defects in sodium channel inactivation

Computer model of effects of defective Na-channel inactivation showing transition from myotonia to paralysis

Figure removed due to copyright restrictions. Figue 6: Cannon, S. "Sodum Channel Defects in Myotonia and Periodic Paralysis." *Annu. Rev. Neurosci.* 19 (1996):141-64.