10.52
Mechanics of Fluids
Spring 2006
Problem Set 1

Problem 1

If

$$\underline{\underline{\sigma}} = \begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$$

a) What are the principal values of $\underline{\underline{\sigma}}$?

b) What are the directions in which the principal stresses act?

c) If a new coordinate frame (ξ, η,ζ) is erected such that the new axes are aligned with the directions of the principal axes, what are the relations:

$$\xi = \xi(x, y, z); \eta = \eta(x, y, z); \zeta = \zeta(x, y, z)$$

Problem 2

If

$$\underline{\underline{\sigma}} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Repeat Problem 1a-c

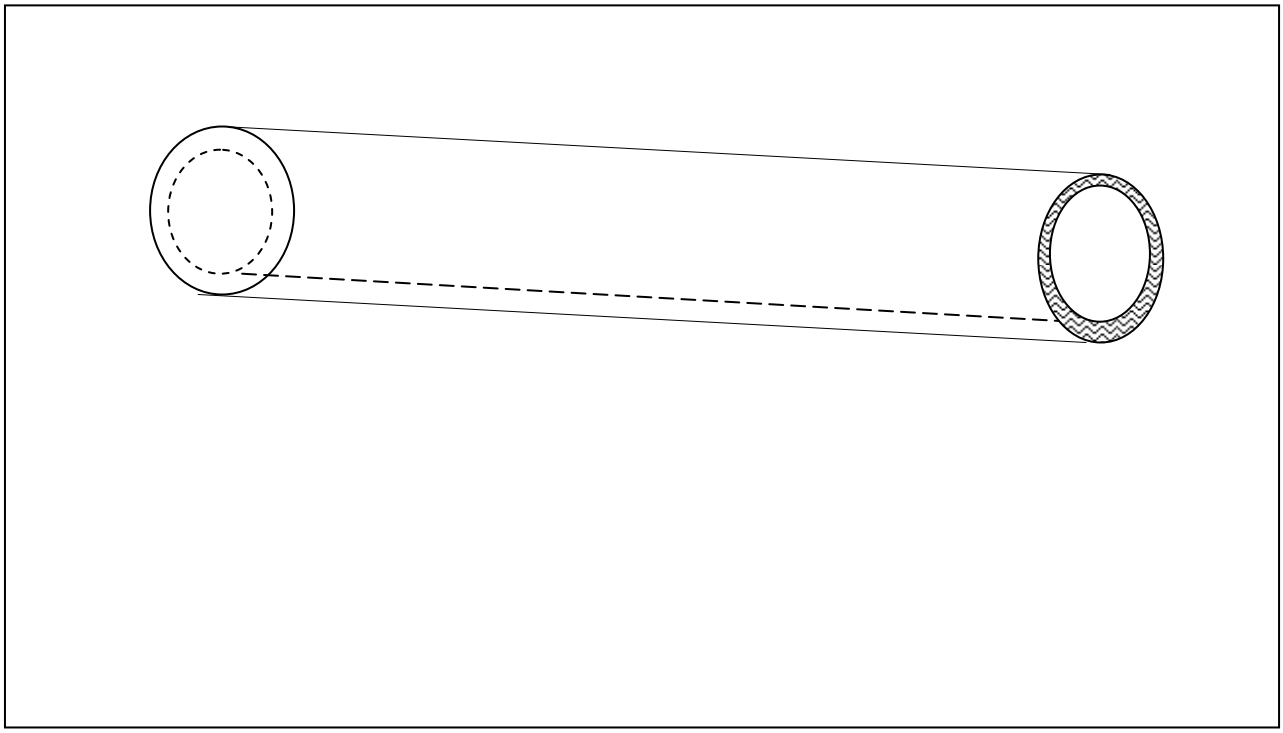
Problem 3

A thin-walled cylinder of inner radius R_i and thickness t ($t/R_i << 1$) is closed at both ends. It contains a fluid with a constant stress tensor $\underline{\underline{\sigma}} = - p_i \underline{\underline{\delta}}$ and is surrounded by a fluid with a constant stress tensor $\underline{\underline{\sigma}} = - p_o \underline{\underline{\delta}}$. Body forces are negligible. Consider the special case in which $p = p_o$. However, a torque $\mathfrak{I}_o \underline{k}$ is applied to one end and $-\mathfrak{I}_o \underline{k}$ to the other end.

a) Assume that the cylinder is very long and, for the region far removed from the ends, evaluate the stress tensor in terms of R_i, t, p_i, p_o etc. Employ cylindrical coordinates.

b) Determine the magnitude and direction of the principal stresses.

c) What is the magnitude of the maximum shear stress?



Repeat Parts (a), (b), and (c) of the preceding problem.