Name

5.73

Quiz 11

1. Consider the Hamiltonian matrix

$$\mathbf{H} = \frac{1}{3}\begin{pmatrix} 4 & 1 & 1 \\ 1 & 7 & -2 \\ 1 & -2 & 7 \end{pmatrix}$$

which has eigenvectors

$$6^{-1/2}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, 3^{-1/2}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, 2^{-1/2}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},$$

and eigenvalues 1, 2, and 3 (not necessarily in the same order as the eigenvectors).

A. Determine the one-to-one correspondence between eigenvectors and eigenvalues.

B. Construct, by assembling eigenvectors in the right way, the matrix $\mathbf{T}$ which you expect will diagonalize $\mathbf{H}$ in the sense $\mathbf{THT}^{\dagger}$ (but do not verify that it actually diagonalizes $\mathbf{H}$).

C. The time-evolution operator is: $\mathbf{U}(t,t_0) = \exp[-i\mathbf{H}(t-t_0)/\hbar]$. The matrix $\mathbf{U}(t,t_0)$, expressed in the same basis set of the original non-diagonal $\mathbf{H}$ is

$$\mathbf{U} = \mathbf{T}^\dagger \exp[-i\mathbf{T}\mathbf{H}\mathbf{T}^\dagger(t-t_0)/\hbar]\mathbf{T}$$

where $\mathbf{T}\mathbf{H}\mathbf{T}^\dagger$ is diagonal. Write the 3×3 diagonal matrix:

$$\exp[-i\mathbf{T}\mathbf{H}\mathbf{T}^\dagger(t-t_0)/\hbar] =$$