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Quiz 31 ANSWERS

1. The sp^2 configuration gives rise to ^2D, ^2P, ^4P, and ^2S L–S states. The degeneracy of an L–S state is (2S + 1)(2L + 1). There are six np spin-orbitals and two ns spin-orbitals. The Pauli principle prohibits putting two electrons into the same spin-orbital.

A. What is the total degeneracy of the sp^2 configuration?

$$2 \cdot \frac{6 \cdot 5}{2} = \frac{60}{2} = 30$$

B. What is the sum of the degeneracies of the L–S states that arise from sp^2?

$^2D = 10$, $^2P = 6$, $^4P = 12$, $^2S = 2$

Sum: $10 + 6 + 12 + 2 = 30$

C. What is the maximum possible value of M_L among all of the L–S states of sp^2?

$L_{MAX} = 2 \qquad M_{L_{MAX}} = 2$

D. Write one of the two 3-electron Slater determinant that corresponds to maximum M_L.

$\| p1\alpha\ p1\beta\ s0\alpha \|$ and $\| p1\alpha\ p1\beta\ s0\beta \|$

E. The maximum M_S value is 3/2. What is the maximum M_L value compatible with $M_S = 3/2$? Write the unique Slater determinant that corresponds to this M_L, M_S pair.

$M_s = 3/2$ comes from a quartet state. There is only one quartet state: ^4P

$M_{s_{MAX}} = 3/2 \qquad M_{L_{MAX}} = 1 \qquad \| p1\alpha\ p0\alpha\ s0\alpha \|$

F. $\mathbf{L}^2 = \frac{1}{2}\left(\mathbf{L}_+\mathbf{L}_- + \mathbf{L}_-\mathbf{L}_+\right) + \mathbf{L}_z^2$.

Is $\|s0\alpha\ p1\alpha\ p1\beta\|$ an eigenstate of $\mathbf{L}^2$? If so, what is its eigenvalue?

$$\mathbf{L}_z^2\|s0\alpha\ p1\alpha\ p1\beta\| = \hbar\mathbf{L}_z(2)\|s0\alpha\ p1\alpha\ p1\beta\| = \hbar^2 4\|s0\alpha\ p1\alpha\ p1\beta\|$$

$$\mathbf{L}_+\mathbf{L}_-\|s0\alpha\ p1\alpha\ p1\beta\| = \hbar\mathbf{L}_+\left(\|s0\alpha\ p0\alpha\ p1\beta\|(2)^{1/2} + \|s0\alpha\ p1\alpha\ p0\beta\|2^{1/2}\right)$$

$$= \hbar^2(2+2)\|s0\alpha\ p1\alpha\ p1\beta\|$$

$$\mathbf{L}_-\mathbf{L}_+\|s0\alpha\ p1\alpha\ p1\beta\| = 0$$

$$\mathbf{L}^2\|s0\alpha\ p1\alpha\ p1\beta\| = \hbar^2\left(4 + \frac{4}{2}\right)\|s0\alpha\ p1\alpha\ p1\beta\|$$

Therefore this Slater determinant belongs to L = 2 because $\mathbf{L}^2|L = 2\rangle = \hbar^2(2)(3) = 6\hbar^2$.

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