Name

5.73

Quiz 3 ANSWERS

1.

$$\hat{H}\psi_n = E\ \psi$$

$$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$\Psi\ (\ ,\ ) = \psi\ ^{-\ \ /\hbar}$ where ψ_n is an eigenstate of $\hat{H}$

$\Psi(x,t) = \sum_n\ \ \psi\ ^{-\ \ /\hbar}$ superposition of eigenstates of $\hat{H}$

$$\int_{-\infty}^{\infty}\psi^*\psi\ \ \ = 0 \text{ if } \ \neq$$

$$= 1 \text{ if } n=m$$

A. What, if any, is the time dependence of $|\Psi_n(x,t)|^2$?

If $\Psi_n(x,t) = \psi_n e^{-iE_n t/\hbar}$

then $|\Psi_n(x,t)|^2 = \psi_n^*\psi_n\left(e^{iE_n t/\hbar}\right)\left(e^{-iE_n t/\hbar}\right) = \psi_n^*\psi_n$

time-independent

B. Let $\Psi(x,t) = 2^{-1/2}\left[\psi_1 e^{-iE_1 t/\hbar} + \psi_2 e^{-iE_2 t/\hbar}\right] = 2^{-1/2}e^{-iE_1 t/\hbar}\left[\psi_1 + \psi_2 e^{+i\omega_{12}t}\right]$ and $= \omega_{12} \equiv (E_1 - E_2)/\hbar$. Assume that ψ_1 and ψ_2 are real, not complex. Solve for $|\Psi_n(x,t)|^2$.

$$|\Psi(x,t)|^2 = \frac{1}{2}\left[|\psi_1|^2 + |\psi_2|^2 + \psi_1^*\psi_2 e^{i\omega_{12}t} + \psi_1\psi_2^* e^{-i\omega_{12}t}\right]$$

$$= \frac{1}{2}\left[|\psi_1|^2 + |\psi_2|^2 + 2\,\mathrm{Re}\left(\psi_1^*\psi_2\right)\cos\omega_{12}t + 2\,\mathrm{Im}\left(\psi_1^*\psi_2\right)\sin\omega_{12}t\right]$$

2. Let $\psi(x) = e^{-ikx}$, $E_{|k|} = \frac{\hbar^2 k^2}{2m} + V_0$, and $\psi(x,t) = e^{i(-kx - E_{|k|}t/\hbar)}$. Think of $\Psi(x,t)$ as a rigid object, $\Psi(x,0)$, moving along the x-axis at a constant velocity. This is the phase velocity, v_ϕ. The motion of the constant phase point is described by

$$x_\phi(t) = x_\phi(0) + v_\phi t.$$

Solve for v_ϕ.

$$\text{constant} = -kx_\phi - E_{|k|}t/\hbar$$

$$x_\phi(t) = -\frac{\text{constant}}{k} - \frac{E_{|k|}t}{\hbar k}$$

$$\text{at } t = 0 \qquad x_\phi(0) = -\frac{\text{constant}}{k}$$

$$x_\phi(t) = x_\phi(0) - \frac{E_{|k|}t}{\hbar k}$$

$$v_\phi = \frac{dx_\phi}{dt} = -\frac{E_{|k|}}{\hbar k} = -\frac{\left[\hbar^2 k^2/2m + V_0\right]}{\hbar k}$$

$$v_\phi = -\frac{\hbar k}{2m} - \frac{V_0}{\hbar k}$$

5.73 Quantum Mechanics I
Fall 2018