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Teaching Statement

Example questions for my physics course

- How long can a Frisbee stay aloft? Make some calculations, and try some experiments. What makes it wobble? Discuss the implications of your results for the design of the Aerobee.
- Why are sunsets orange, particularly in Los Angeles? Why is the sky blue? Why are metals shiny?
- Why do cars get roughly 100 miles per gallon, but not 10,000?
- How fast does grass grow?
- Pick four numbers from the *CRC Handbook of Chemistry and Physics*, such as the boiling points and dielectric constants of water and ammonia. Discuss their relative magnitudes and some everyday consequences of these values.
- How fast can you walk? What limits the speed?
- Why are there no large hummingbirds?
- Give a simple example that shows why the parallel-axis theorem has the form that it does. Discuss similarities with the concept of variance in statistics.
- Find a problem that you solved using conservation of energy and solve it using $F = ma$.
- Find as many ways as you can to evaluate $\sqrt{17}$ without a calculator. How accurate are they? How efficient? What mathematical principles did you use? Which methods are easy to generalize to cube roots?

I want students to learn to observe and explore the world, and to know how to formulate and answer questions on their own. Therefore, I will teach mathematics and physics as one subject, developing mathematics as it is needed to express a physical idea. With an integrated development, the concepts from one subject support the concepts from the other, as threads in a tapestry reinforce one another. For strength and for variety, I will weave in examples from biology and chemistry. Conceptual questions such as the ones above will develop students' scientific flexibility and fluency. Though graduate students with conventional training may find these questions difficult, freshmen with the right education can solve and enjoy them. I will choose the questions to teach mathematical principles, physical concepts, biological and chemical applications, and useful habits of thought.

In collaboration with other faculty, I will organize my physics teaching around the question: What physical principles and mathematical ways of thought do we want students to know ten years later? Instead of covering many topics, I prefer to 'uncover a few' so that there is time to teach thinking patterns and methods of reasoning. Students can learn formal methods from textbooks, where they are well explained, and I will lecture on what is not found in most textbooks, connecting and weaving the concepts into a durable tapestry.

I have made a preliminary list of principles that I would emphasize:

- How to build models, make assumptions, approximate, and estimate
- How to understand and parse an equation; how to explain the physical origin of each term
- How to reason using diagrams and graphs
- How to reason inductively, from observations to hypotheses
- How to break problems into subproblems
- How to use scaling arguments and dimensional analysis
- How to reason in unfamiliar domains
- How to notice examples of physics and mathematics in the world
- How to communicate scientific ideas with clarity

To improve this list, I will discuss with faculty colleagues what quantitative skills their courses require, and what skills are part of a liberal education. I also plan to interview juniors and seniors, asking them which quantitative skills and physical principles they use or wished they could use.

Problems with a normal education

Many students, even at Caltech, retain little of their physics and mathematics education [3]. From my experience teaching freshman physics, I believe the cause is that students develop 'coping skills' instead of understanding [4]. For example, students do a few problems on sliding blocks, where the normal force N equals the weight mg. Mystified by the concept of normal force, students resort to memorizing the incantation '$N = mg$'. They use this potion wherever they see a normal force, even where it is not valid (for example, for a bouncing ball, or in free fall). This lack of comprehension results in poor performance on conceptual exams such as the Hestenes Force Concept Inventory [1].

Furthermore, I found that students have great difficulty making assumptions or approximations to fill in missing information, or in understanding what information is irrelevant. They have learnt to cope with typical textbook problems, which teach poor models of how scientists actually reason [2, pp. 29–31].

Teaching methods

I used various methods to ameliorate these difficulties. To ensure that students learn the important methods of reasoning, I will try new methods and extend the previous ones. Some methods that I will use include:

- **Peer teaching**. Teachers say that they never understood a subject until they had to teach it (I never understood tension—the physics kind—until I had to teach it in freshman physics). We should not be so stingy, reserving the benefits of teaching to ourselves.

  Choose two topics—say, one-dimensional collisions and Kepler's second law. Pair up students randomly, and assign one student topic A, the other topic B. Each student prepares a twenty-minute lecture on her or his topic, and lectures on it to the other student, who asks questions, and gives feedback on confusing points. The student revises the lecture, and presents it again, this time to the teacher. Of course, all the students working on a common topic should discuss methods of presentation, and educate each other—I want to teach students to cooperate. Having students thoroughly understand even one topic is valuable because they realize what a thorough understanding means. This realization will motivate them to understand other topics equally well.
- **Writing**. As I asked my freshmen physics students, I will ask students to explain their reasoning, to draw diagrams, to consider limiting cases, to discuss their solutions, and to explain the origin of each term in an equation (I have attached a revised version of the handout that I gave my students). These written tasks force students to go beyond coping. They learn to interpret equations and to make explicit the concepts and patterns of reasoning that they use. From their own words, students confront and correct their conceptual confusions.
- **Fermi (order-of-magnitude) problems**. How many gas stations are there in the United States? How many piano tuners are there in Chicago (Fermi's original problem)? From such problems, students learn how to approximate and how to make assumptions in unfamiliar domains. They gain confidence in their reasoning ability, a confidence that motivates them throughout their physics and mathematics education.

  Such problems also develop students' intuition for numbers, magnitudes, and equations. Without guidance, none of my freshmen physics students could estimate $\sqrt{1.1}$, let alone $\sqrt{1+x}$; constant calculator use has atrophied their numeric sense. Soon the trouble spreads to the intuition for equations. Along with the Fermi problems, I will have students work out quantities such as $\sin 1$ by hand. Such exercises will strengthen their numeric abilities as well as their knowledge of interpolation and Taylor series.

Part of my doctoral dissertation is a textbook on estimation and dimensional analysis, based on the Order of Magnitude Physics class offered at Caltech. I have included a draft of Chapter 1 with my application, as an example of scholarly work. I will use examples from the text, and invent new ones, to introduce scaling arguments, approximate reasoning, and estimation. Many problems in biology and chemistry are particularly amenable to this approach. For example, I will have the students estimate how fast grass grows (I have attached a solution).

By solving Fermi problems, perhaps related to biology or chemistry, and making calculations without a calculator, students gain confidence in their own ability to understand the world. They become more curious, and spend more energy learning mathematics and physics.

- **Asking students to keep a journal of their physics and mathematics observations and questions**. I have included the handout that I gave my freshmen physics students, which describes the reasons that I felt a journal was important. I suggested possible topics for journal entries, such as finding everyday examples of principles covered in lecture. Students wondered, for example, why waves break, and whether you can study traffic jams using fluid mechanics. I made comments on their observations and suggested new questions to think about or kitchen sink experiments to try. The journal provided a good start for class discussion, and it taught students to see the world with a physicist's eye.

I look forward to extending these techniques, and to experimenting with new ones, as I become a more experienced teacher.

References

[1] David Hestenes and Ibrahim Halloun. Interpreting the force concept inventory. *Physics Teacher*, 33(8):502, 504–506, 1995.

[2] Eric Mazur. *Peer Instruction: A User's Manual.* Prentice–Hall, Upper Saddle River, NJ, 1997.

[3] F. Reif and S. Allen. Cognition for interpreting scientific concepts: A study of acceleration. *Cognition and Instruction*, 9:1–44, 1992.

[4] A. W. Roberts, editor. *Calculus: The Dynamics of Change.* Mathematical Association of America, 1996.

Attachments

[1] Revised handout: 'Explain your reasoning'

[2] Estimation of how fast grass grows

[3] Handout: 'Keeping a journal'

5.95J / 6.982J / 7.59J / 8.395J / 18.094J / 1.95J / 2.978J Teaching College-Level Science and Engineering
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