

## Lectures 13,14 Population Growth: Exponential/Logistic, Chemostats, Life Tables, and Human Population Growth

### READINGS

**Text:** Appendix A, p. 741-744; Appendix B pp. 762-766; Chapter 10: pp. 171 – 180;. Chapter 11: 181-193.

#### Lecture Supplement

Introduction to Chemostat Theory  
Chemostat Lecture Notes

#### Articles:

Smil, Vaclav (1999). How Many Billions To Go? *Nature* 401:429.

Palumbi (2001). Humans as the World's Greatest Evolutionary Force. *Science* 293:1786-1790.

Cohen, J. (1995) Population growth and Earth's human carrying capacity. *Science* 269:341

Tilman et al (2002). Agricultural sustainability and intensive production practices. *Nature* 418:671.

Wackernagel et al (1999) National natural capital accounting with the ecological footprint concept. *Ecol. Econom.* 29:375-390

#### Further Reading

Hindell, M. (1991) Some Life-History Parameters of a Declining population of southern elephant seals, *Mirounga leonina*. *J. Anim. Ecol.* 60: 119-134

### OUTLINE

*Exponential Growth*

Continuous growth in an unlimited environment

*Logistic Growth*

Growth with limits – Carrying Capacity

Density Dependent Growth

*Life Tables and Demography*

Discrete generations and age distributions

*Chemostat Theory*

Growth in nutrient limited environments

*Human Population Growth*

Where have we come from and where are we going?

*Ecological Footprint*

Calculating 'Sustainability'

**NOTE:** The notation used in lecture may not always agree with that used in your text. This should not disturb you. If you understand the concepts with both sets of notation, then you really understand the material.

### STUDY GUIDE

You should be able to solve problems that are similar to those covered in class, in the homework, and on the example quiz that will be available. You don't have to memorize all the formulas, but you should be familiar with them when you see them, and know which to use for what application. (Since there are so few of them, the safest thing to do is to "internalize them". If you understand exponential growth, for example, you can derive the rest from here.)

## SOME QUESTIONS TO THINK ABOUT

- Is the human population in “exponential growth”? Why or why not?
- Considering the growth of a microorganism, what is similar and different between growth in batch culture (Logistic growth) and a chemostat?
- What are the assumptions of the Logistic Growth Model?
- How does age of first reproduction of females influence population growth rates?
- What is the explicit definition of the “Carrying Capacity” of the Earth for humans? Why is it so difficult to estimate?
- How is the “ecological footprint” of a person or country calculated? What is the “ecological capacity” of a country?

## LIFE TABLE TERMS AND EQUATIONS

$x$  = age

$n_x$  = number of individuals surviving to age  $x$

$l_x$  = age-specific survivorship  
= proportion of individuals surviving to age  $x$   
=  $n_x / n_0$

$d_x$  = number dying during the age interval  $x$  to  $x+1$

$q_x$  = age-specific per capita mortality rate  
= proportion of individuals dying between age  $x$  and  $x + 1$   
=  $(l_x - l_{x+1}) / l_x$   
=  $d_x / n_x$

“age class  $x$ ” = all the individuals between age  $x$  and  $x + 1$

$e_x$  = life expectancy of individuals of age  $x = T_x / n_x$

$T_x$  = total years to be lived by individuals of age  $x$  in the population  
=  $\sum_x^{\infty} L_x$

$L_x$  = number of individuals alive on average during the age interval  $x$  to  $x+1$   
=  $(n_x + n_{x+1}) / 2$

$b_x$  = age-specific fecundity  
= average number of offspring produced per individual between ages  $x$  and  $x + 1$

$R_0$  = net replacement per generation  
=  $\sum (l_x b_x)$

$r$  = intrinsic rate of increase  
exact solution:  $\sum (l_x b_x e^{-rx}) = 1$   
approx. solution:  $r \cong (\ln R_0) / G$   
where  $G$  = mean generation time  
 $\cong \{ \sum (x l_x b_x) \} / R_0$

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