1.050 Engineering Mechanics

II. Stresses and Strength

Application in Structural Mechanics

Program 8th Lecture

1-050 CONTENT

I. Dimensional Analysis:

II. Stresses & Strength

2. Stresses and Equilibrium

1. Discrete Model
2. Continuum Model
3. Beam Model

3. Strength Models

III. Deformation and Strain

4. How Strain Gages work?

IV. Elasticity

5. Elastic Model
6. Variational Methods in Elasticity

V. How Things Fail? And How to avoid it.

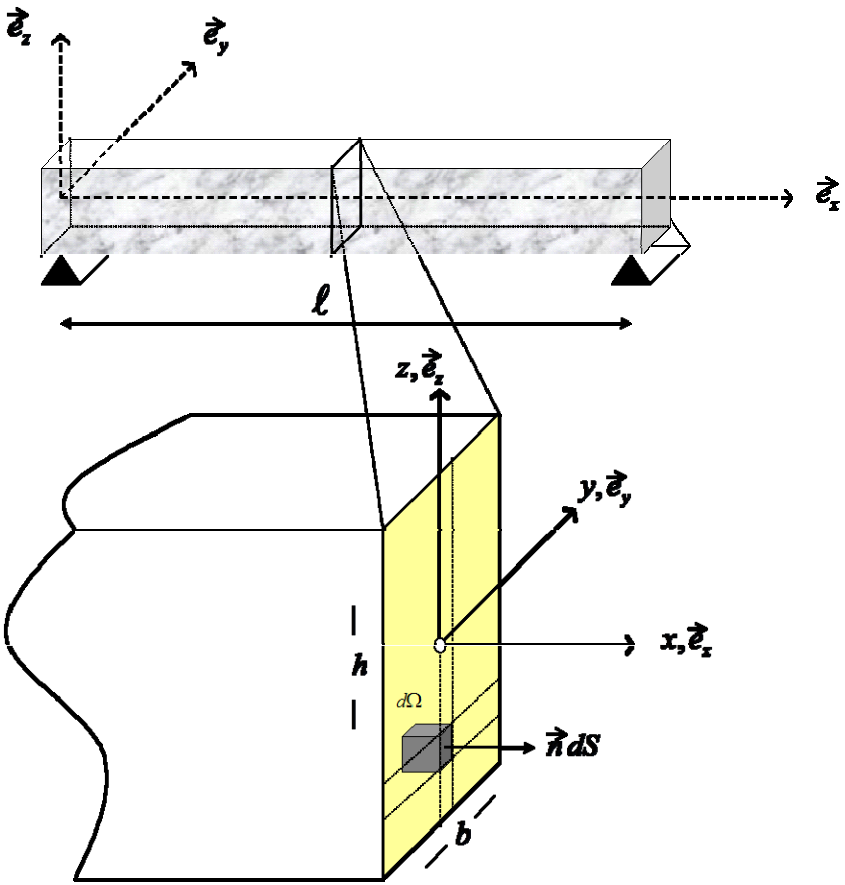
TODAY:

1. Scales of Structural mechanics: Section vs. Beam structure
2. **Link between stresses and forces and moments**
3. **Beam Equilibrium Conditions**
4. **Example**

Goal: Construct a Force-Moment Beam Model

Appreciate the link between Continuum Model and Beam Model

Three Scale Approach



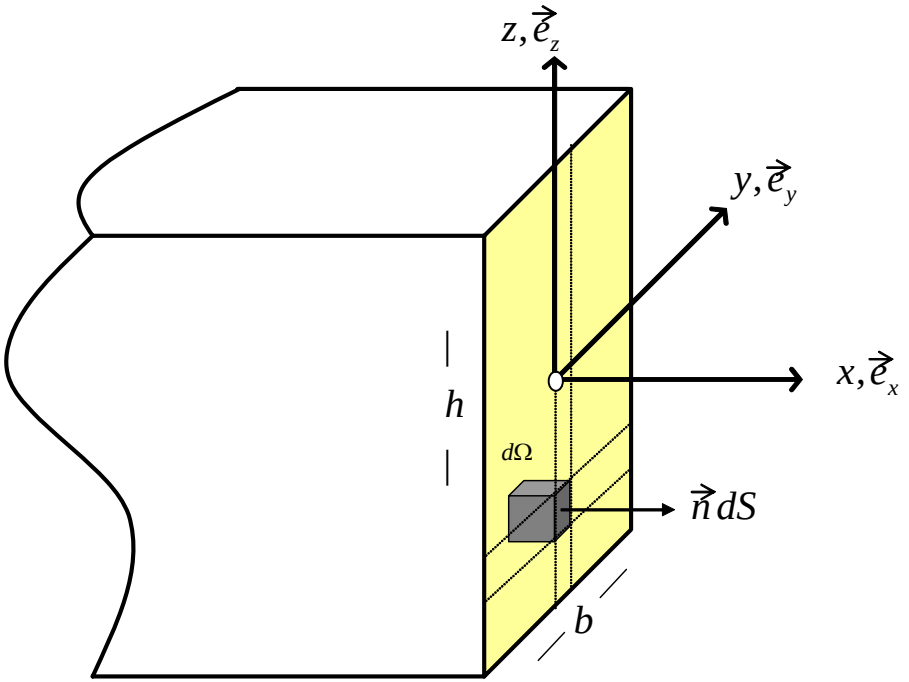
- Beam Scale defined by beam length
- Cross-section scale (height, width)

$$(h, b) << \ell$$

- Continuum scale

$$O\left(d\Omega^{1/3}\right) << (h, b) << \ell$$

From the Continuum Scale to the Cross Section Scale



- Continuum Quantity:
  - Stress vector

$$\vec{T}(\vec{n} = \vec{e}_x) = \boldsymbol{\sigma} \cdot \vec{e}_x$$

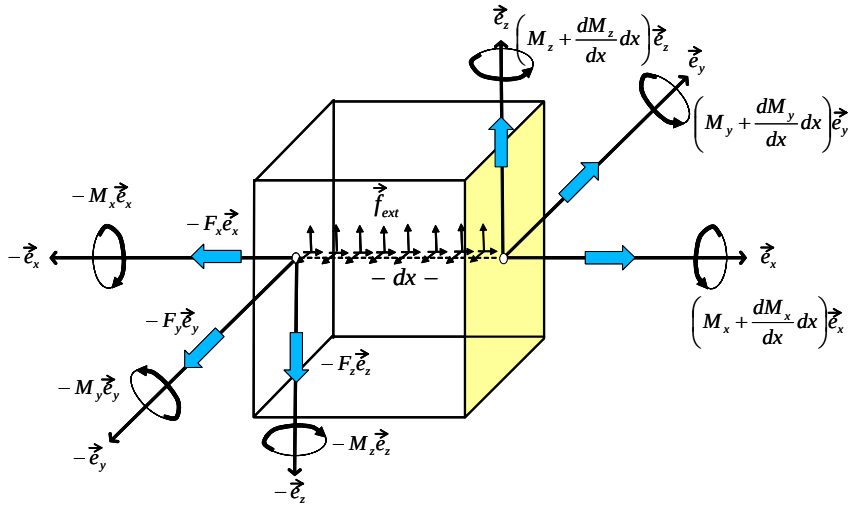
- Section Quantities:
  - Forces

$$\vec{F}_S = \int_S \boldsymbol{\sigma} \cdot \vec{e}_x dS$$

  - Moments

$$\vec{M}_S = \int_S \vec{x} \times (\boldsymbol{\sigma} \cdot \vec{e}_x) dS$$

From the Cross Section Scale to the Beam Length Scale



- Differential Force equilibrium

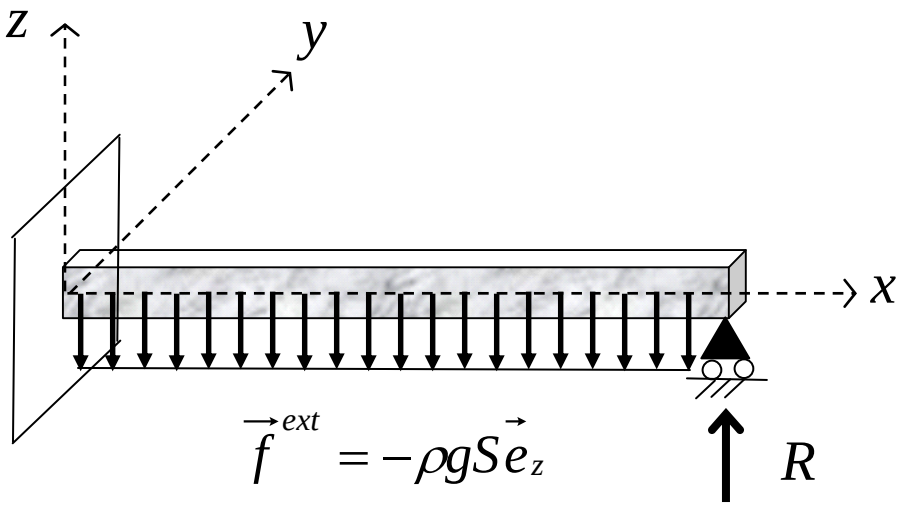
$$\frac{d\vec{F}_S}{dx} + \vec{f}^{ext} = 0$$

- Differential Moment equilibrium

$$\frac{d\vec{M}_S}{dx} + \vec{e}_x \times \vec{F}_S = 0$$

Formulation of a Beam Boundary Value Problem

- Example



- Force and Moment Boundary Conditions
- Sum of all forces and Moments along x is zero
- Differential Equilibrium of
  - Section forces
  - Section moments