

## Problem Set 2

*Due on Monday, February 27th*

**Important note:** Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.

## Problem 1

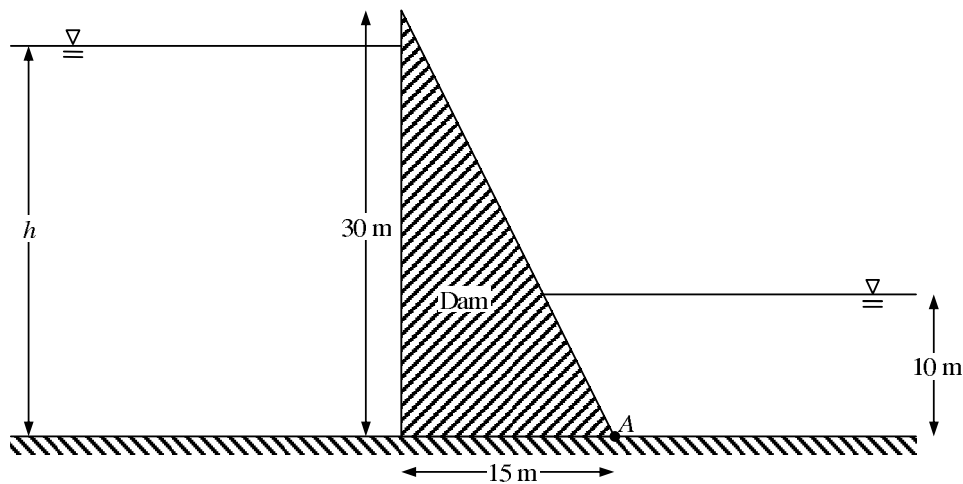


Figure 1: Dam in Problem 1.

You work in the quality control division for an engineering consulting firm. Your company has been hired to design a concrete dam to aid in the creation of a new reservoir. A junior consultant (an MIT grad) has submitted the final plans for the dam (See Figure 1). Your job is to check to make sure that the dam is properly designed and will not topple over about its pivot point  $A$  (it is assumed that the dam will not slide). In order to do this, you need to figure out at what water depth ( $h$ ) the dam will topple over. You are given the following data: The depth of the river downstream of the dam is 10 m and is kept constant, the density of the concrete is  $2300 \text{ kg/m}^3$ , the height of the dam is 30 m, the base of the dam is 15 m long, the dam and the reservoir are 100 m wide (into the page), and water cannot leak under the base of the dam.

- At what water depth  $h$  will the dam topple over about its pivot point  $A$ ?
- Is the dam properly designed?

## Problem 2

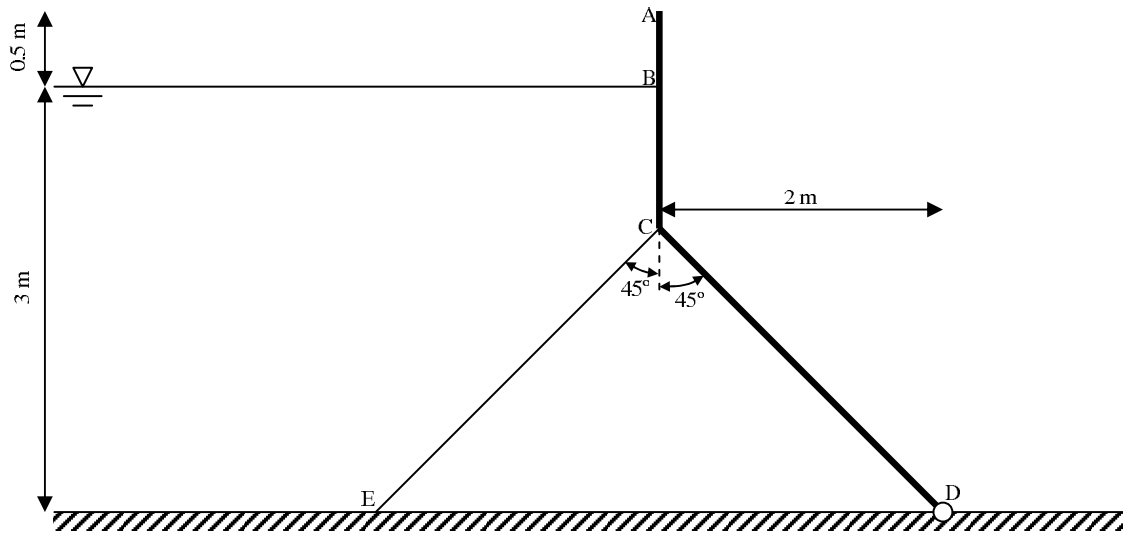


Figure 2: Gate in Problem 4.

A senior engineer (a Harvard grad) designed the gate in Figure 2. The gate consists of a vertical section, AC, and an inclined section, CD, which are rigidly connected to each other at C. The gate is hinged at D, and is kept in place by a single wire EC. Assume the gate to be of 1 meter length into the plane of the paper. The weight of the gate is assumed negligible.

The gate ACD retains water,  $\rho = 1000 \text{ kg/m}^3$ , to its left. On the right of the gate, there is air.

- Sketch the pressure distribution along ACD and specify the values of the pressure at points A, B, C, and D.
- Determine the total pressure force acting on AC,  $F_{AC}$  (magnitude, direction, line of action).
- Determine the total pressure force acting on CD,  $F_{CD}$  (magnitude, direction, line of action).
- Determine the force exerted by the wire EC.
- Determine the horizontal and vertical forces acting on the hinge at D.

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Please see:

Bonillo Martínez, Juan J., Jerónimo Puertas Agudo, and Ricardo Juncosa Rivera. *Problemas de Hidráulica*. Escuela Técnica Superior de Ingenieros de Caminos, Canales y Puertos. Universidad de La Coruña. July, 2002.

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## Problem 4

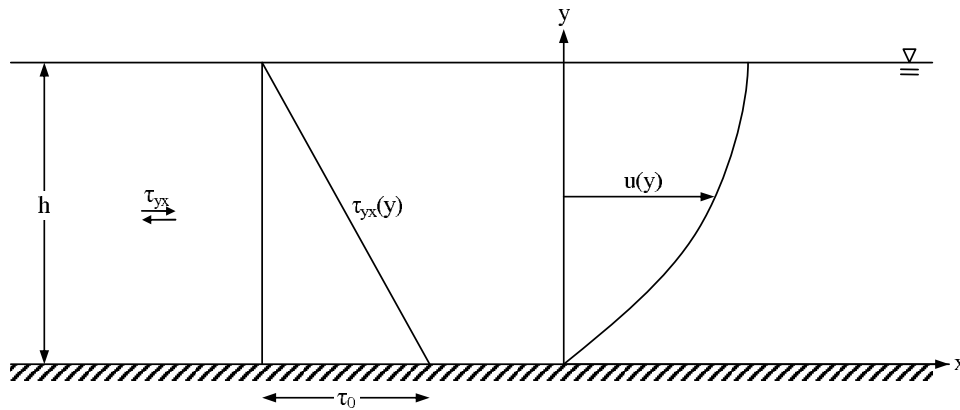


Figure 4: Turbulent flow characteristics in Problem 4.

For a 2-D steady turbulent flow, the shear stress distribution is given by (see Figure 4)

$$\tau_{yx} = \tau = \tau_0 \left(1 - \frac{y}{h}\right). \quad (1)$$

Shear stress is related to velocity gradient, just as in laminar flow, i.e.,

$$\tau = \rho \nu_t \frac{\partial u}{\partial y} \quad (2)$$

but for a turbulent flow the turbulent viscosity,  $\nu_t$ , is **not** a material *constant* but a function of the flow itself. A widely accepted model is

$$\nu_t = \kappa u_* (y + y_0) \left(1 - \frac{y}{h}\right) \quad (3)$$

in which  $\kappa = 0.4$  is a constant (known as von Kármán's constant),  $u_* = \sqrt{\tau_0/\rho}$  is called the shear velocity, and  $y_0$  is the bottom roughness length,  $y_0 \ll h$ .

**a)** Combine (1) through (3) to obtain the equation governing the velocity profile for a turbulent flow,  $u(y)$ , and solve this equation for  $u(y)$  subject to the no-slip boundary condition,  $u = 0$  at  $y = 0$ .

**b)** Show that the depth-averaged velocity,  $U$ , can be expressed as

$$U - u_s = U - u(y = h) = -\frac{u_*}{\kappa}$$

or

$$U = \frac{u_*}{\kappa} \ln \frac{h}{ey_0}$$

when  $y_0 \ll h$  is used.

**c)** For  $u_* = 4 \text{ cm/s}$ ,  $\kappa = 0.4$ ,  $h = 2 \text{ m}$ , and  $y_0 = 0.3 \text{ mm}$ , plot  $u(y)$  and compare it to  $U$ . Comment on how comfortable you would be if you were to approximate  $u(y)$  by  $U$ .

## Problem 5

Water flows through the pipe contraction shown in Figure 5. The fluid in the manometer tube is air. For the given  $0.2\text{ m}$  difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe,  $D$ .

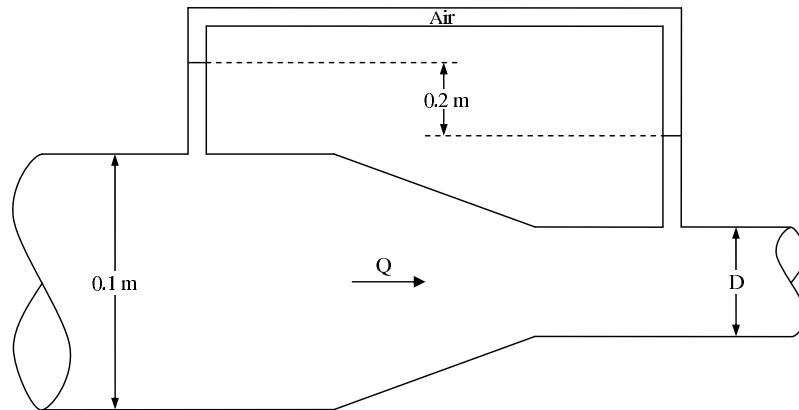


Figure 5: Pipe contraction in Problem 5.

## Problem 6

Water,  $\rho = 1000 \text{ kg/m}^3$ , is flowing in a circular pipe of diameter  $d = 5 \text{ cm}$  (See Figure 6). The pipe is inclined at an angle  $\beta = 10.9^\circ$  to horizontal, and is connected to a mercury manometer ( $\rho_m = 13.6\rho$ ) through two pressure taps ( $A$  and  $B$ ) located at a distance  $l = 2.0 \text{ m}$  apart at the highest point of the perimeter, i.e., a distance of  $2.5 \text{ cm}$  above the centerline of the pipe (denoted by CL in sketch).

- For a manometer reading of  $h_m = 3.0 \text{ cm}$  determine the pressure difference,  $p_A - p_B$ , between the two pressure taps  $A$  and  $B$ .
- Sketch the streamlines for the flow in the pipe and indicate the direction of flows by arrows (justify your choice of direction).
- Is the pressure difference between points  $A'$  and  $B'$  located on the centerline of the pipe the same as the pressure difference obtained in **a** and, if so, why?

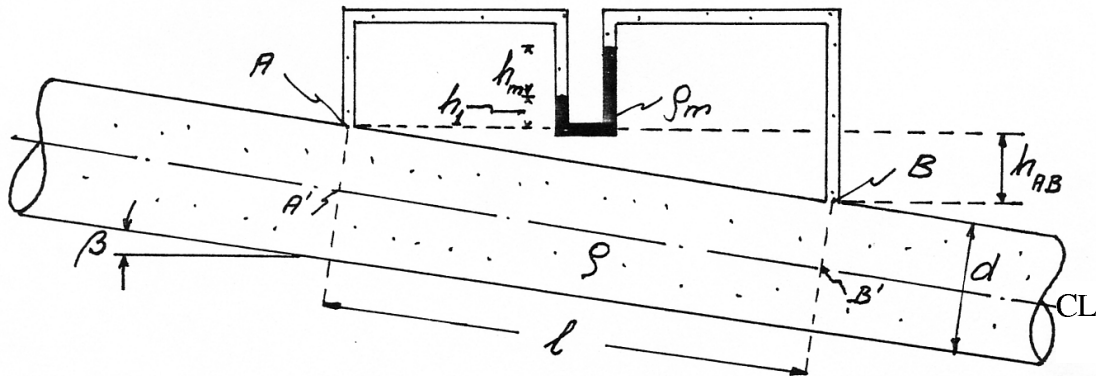


Figure 6: Inclined pipe in Problem 6.