Problem Set 9

Due on Monday, May 8th

Important note: Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.
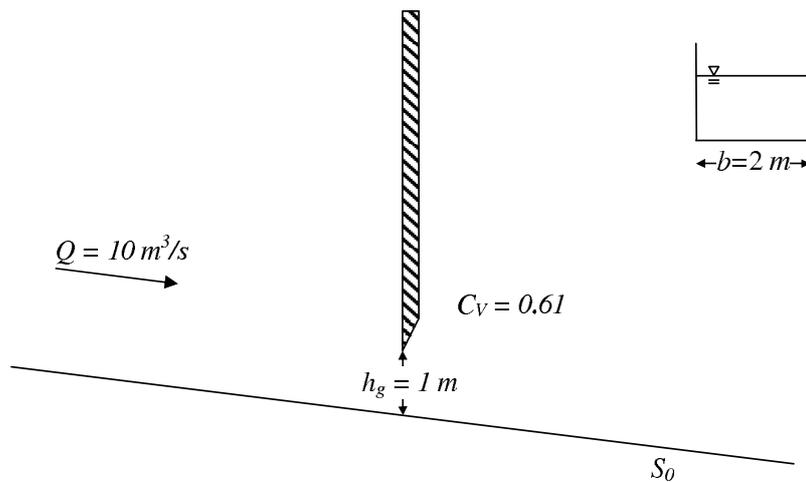
Problem 1

Figure 1: Flow under a gate in Problems 1-3.

A water flowrate of $Q = 10 \ m^3/s$ is being discharged in a very long rectangular channel of width $b = 2 \ m$ and tall sidewalls. The channel is made of concrete, with a finished surface. The bottom slope is $S_0 = 10^{-3}$.

a) Find the normal and critical depths, h_n and h_c.

Halfway down the channel, water has to go under a gate, as shown in Figure 1. The gate is tall enough so that flow cannot spill over it. The opening under the gate is $h_g = 1 \ m$ and spans the entire width of the channel. The contraction coefficient is $C_V = 0.61$.

b) Find the water depth upstream the gate, h_1, necessary to drive the flowrate $Q = 10 \ m^3/s$ under the gate.

c) Draw a sketch showing the free surface shape as the channel approaches the gate from upstream. Name the surface profile(s) that appear in your sketch and specify the relevant values of the water depth.

d) Estimate the distance upstream affected by the conditions imposed by the gate.

Problem 2

Consider the same channel and gate as in the previous problem, but now with slope $S_0 = 5 \cdot 10^{-3}$.

a) Find the normal and critical depths, h_n and h_c.

b) Find the water depth upstream the gate, h_1, necessary to drive the flowrate $Q = 10 \ m^3/s$ under the gate.

c) Draw a sketch showing the free surface shape as the channel approaches the gate from upstream. Name the surface profile(s) that appear in your sketch and specify the relevant values of the water depth.

d) Estimate the distance upstream affected by the conditions imposed by the gate.

Problem 3

I.– Consider the conditions given in Problem 2 ($S_0 = 5 \cdot 10^{-3}$).

a) Draw a sketch showing the free surface shape along the continuation of the channel downstream the gate. Name the surface profile(s) that appear in your sketch and specify the relevant values of the water depth.

b) Estimate the distance downstream affected by the conditions imposed by the gate.

II.– Consider the conditions given in Problem 1 ($S_0 = 10^{-3}$).

c) Draw a sketch showing the free surface shape along the continuation of the channel downstream the gate. Name the surface profile(s) that appear in your sketch and specify the relevant values of the water depth.

d) Estimate the distance downstream affected by the conditions imposed by the gate.

e) Explain qualitatively how the free surface shape downstream the gate would evolve if we were to decrease the channel slope from $S_0 = 10^{-3}$ to a very small value, keeping $Q = 10 \ m^3/s$ constant.

Problem 4

In a wide rectangular channel of **critical slope** and with flow resistance governed by the Chezy equation (with a constant C), show that all gradually varied flow profiles are straight lines. Show in a skecth how each of the possible profiles (from C1 to C3) looks like and give the corresponding values of the free surface slope, dh/dx.

Problem 6

A steady flow of q = discharge per unit width = 3.13 $[m^3/s$ per m] proceeds in a very long, wide, and mildly sloping rectangular channel towards the brink of a drop-structure. As the flow approaches the brink, it passes through critical depth, h_c, a short distance, $\sim 3\text{-}4\,h_c$, upstream of the brink. At the brink the depth of flow is h_b ($< h_c$). The near-brink conditions are shown in Figure 3.

a) Why must the flow pass through critical depth near the brink?

b) At the critical flow section ("c" in the figure), determine h_c and the specific head, E_c.

c) Using the momentum principle between sections "c" and "b", where "b" (see figure) is located right after the flow passes over the brink as a free jet, estimate the depth of flow and the velocity at the brink, h_{bm} and V_{bm}.

d) Using the Bernoulli principle between sections "c" and "b", estimate the depth and velocity at the brink, h_{bB} and V_{bB}.

e) With your best estimate of V_b, the brink velocity, and assuming V_b to be horizontal, estimate the velocity, V_0, thickness of the jet, h_0, and angle of incidence, θ_0, when jet hits the drop-structure's horizontal bottom located 10 m below the brink (neglect friction from the air surrounding the freely falling jet).
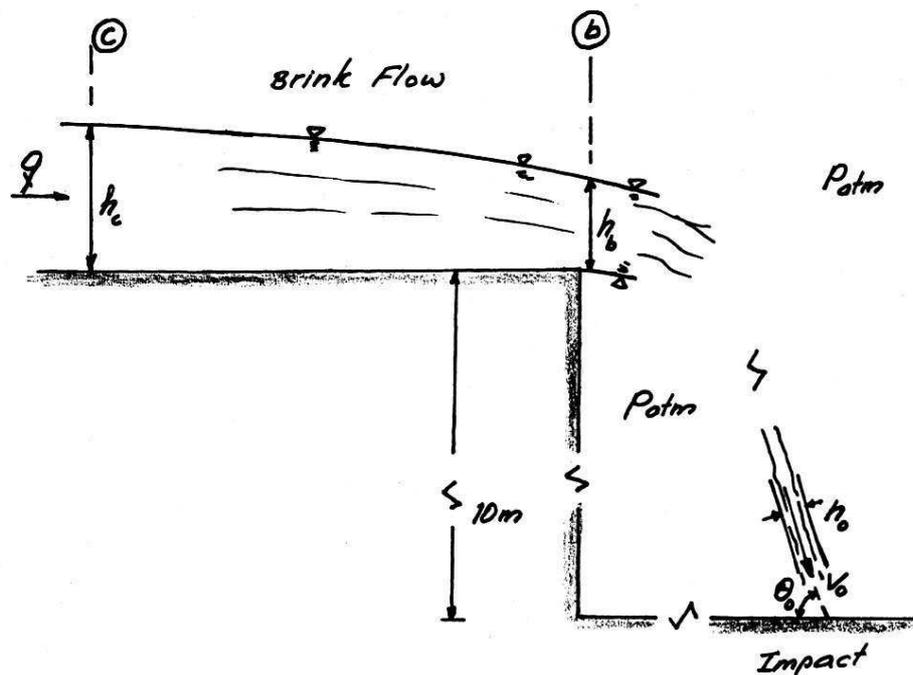

Figure 3: Brink flow in Problem 6.