MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.060 Engineering Mechanics II

Scheduled Final Examination

Monday, May 22, 2006
9 - 12 noon

Prof. Ole S. Madsen

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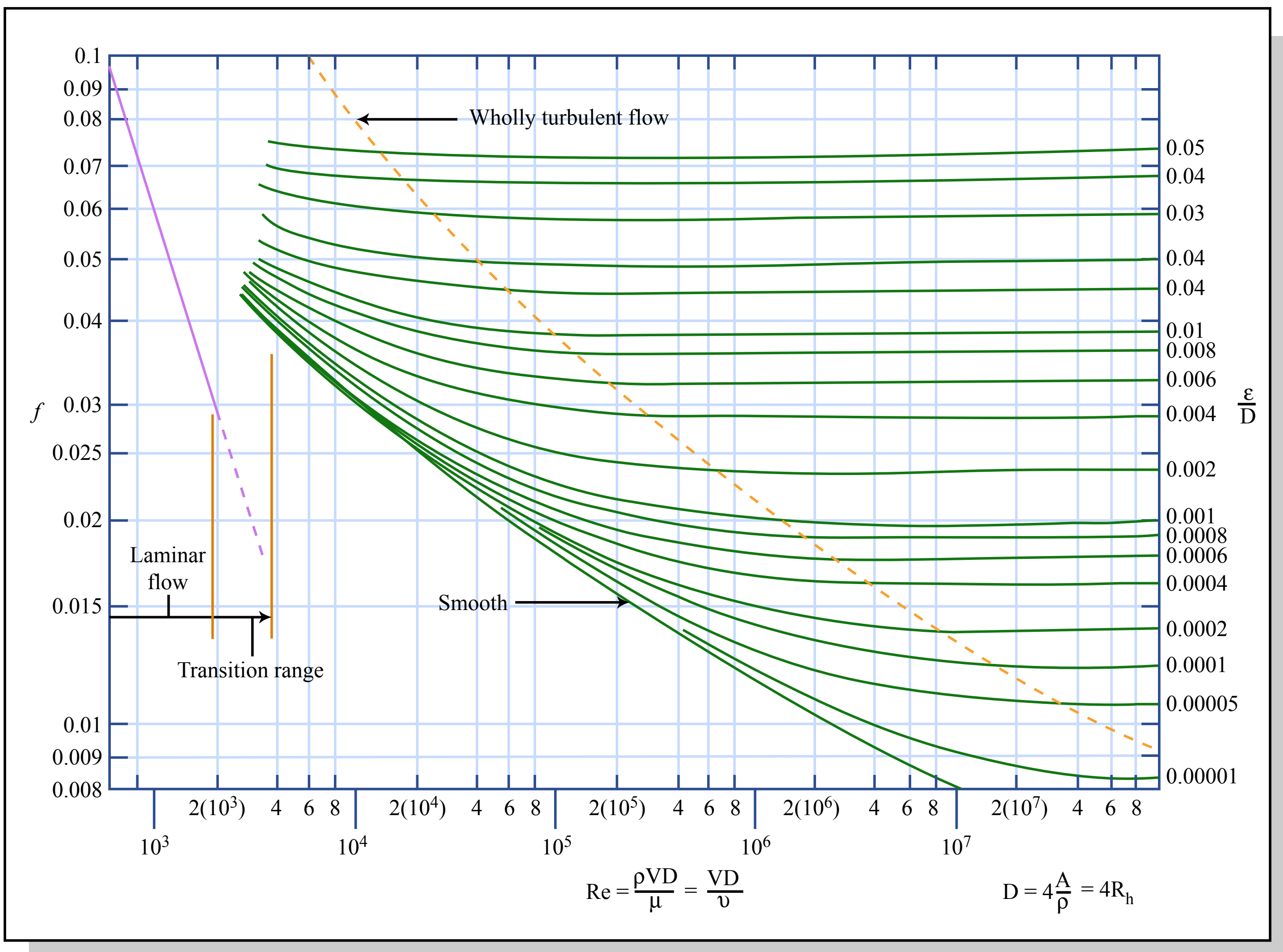
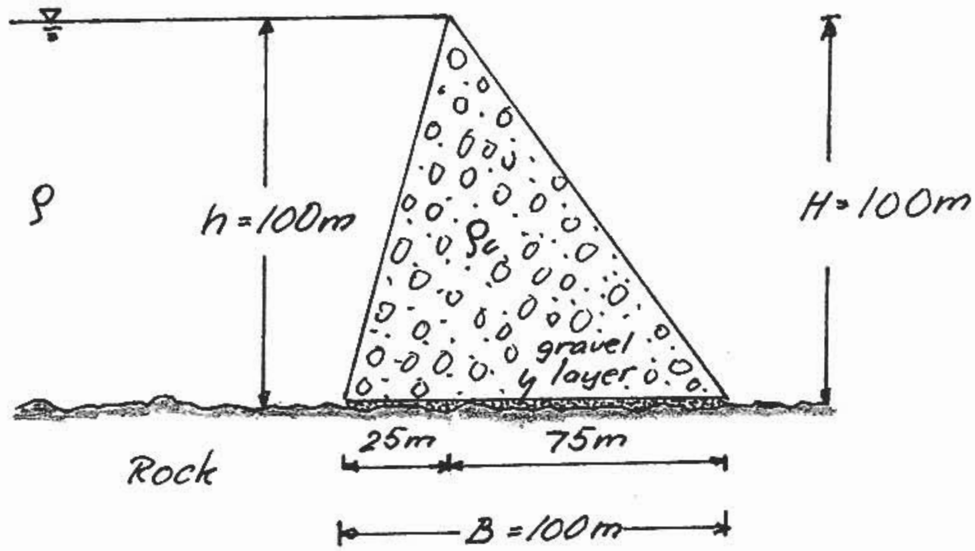
1. There are five problems of equal weight. Be sure to allocate an appropriate amount of time for each.
2. Solutions should be expressed in terms of the problem notation and then the numerical results should be obtained.
3. Please indicate clearly, using sketches when necessary, the assumptions and definitions you are introducing in carrying out your analyses. Do not hesitate to make reasonable assumptions, but state the reason why you make them.
4. Please be as neat as possible and clearly indicate what and where your answer is (only one answer!).
5. Default values are provided in some problems. If your answers differ by more than ~10% from these, continue with default values.
6. Cheat sheets #1, 2, and 3 are provided along with the Moody Diagram below.



Graph by MIT OCW.

Problem No: 1



The sketch shows the cross-section of a very long concrete ($\rho_c = 2{,}300$ kg/m^3) dam, height $H = 100$ m and base width $B = 100$ m, which holds back water ($\rho = 1{,}000$ kg/m^3) of depth $h = H = 100$ m. The dam's foundation is blasted rock with a somewhat uneven surface. To create a level horizontal surface for the dam, a gravel layer of ~50 cm thickness is placed on the blasted rock surface. The gravel layer allows an insignificant amount of water to seep under the dam.

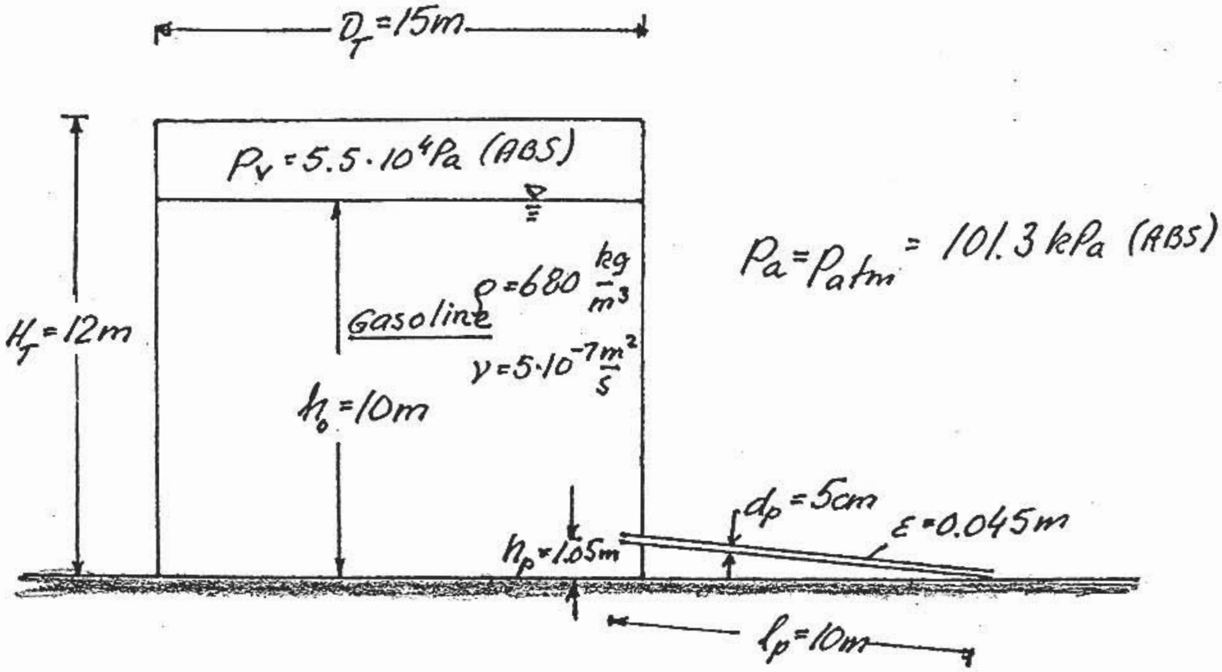
a) Sketch the pressure distribution along the upstream face and the base of the dam.

b) Determine the factor of safety against overturning of the dam (defined as the ratio of stabilizing to overturning moments around the pivot point).

c) With a coefficient of friction for the contact between the base of the dam and the gravel layer μ_f = (Maximum Realizable Frictional Force)/(Normal Force) = 0.7 determine the factor of safety against the dam sliding.

d) Can you suggest a simple way to improve the dam's safety against overturning and sliding.

Problem No: 2



The sketch above shows a large circular-cylindrical fuel tank containing **gasoline** (ρ **= 680 kg/m^3, ν = 5 · 10^{-7} m^2/s**). The "open space" above the free surface of the gasoline contains gasoline fumes at vapor pressure, which is $p_v = 5.5 \cdot 10^4$ *Pa* (ABSOLUTE). The fuel tank is surrounded by air at atmospheric pressure, $p_a = 101.3 kPa$ (ABSOLUTE). The free surface of the gasoline in the tank is $h_0 =$ 10 m above the tank bottom. The total height of the tank is $H_T = 12$ m, and its diameter is $D_T = 15$ m.

a) For the conditions specified above, determine the pressure at the bottom of the fuel tank.

b) Sketch the distribution of net-pressure, i.e., the difference between the pressure inside and outside the tank, along the vertical wall of the fuel tank.

A truck carrying a steel pile (length $l_p = 10$ m, diameter $d_p = 5$ cm, and roughness $\varepsilon = 0.045$ mm) backs into the tank with the pipe penetrating the tank wall a distance of $h_p = 1.05$ m above the tank bottom. The truck driver does not realize what has happened and drives away leaving "the scene of the crime" as sketched above.

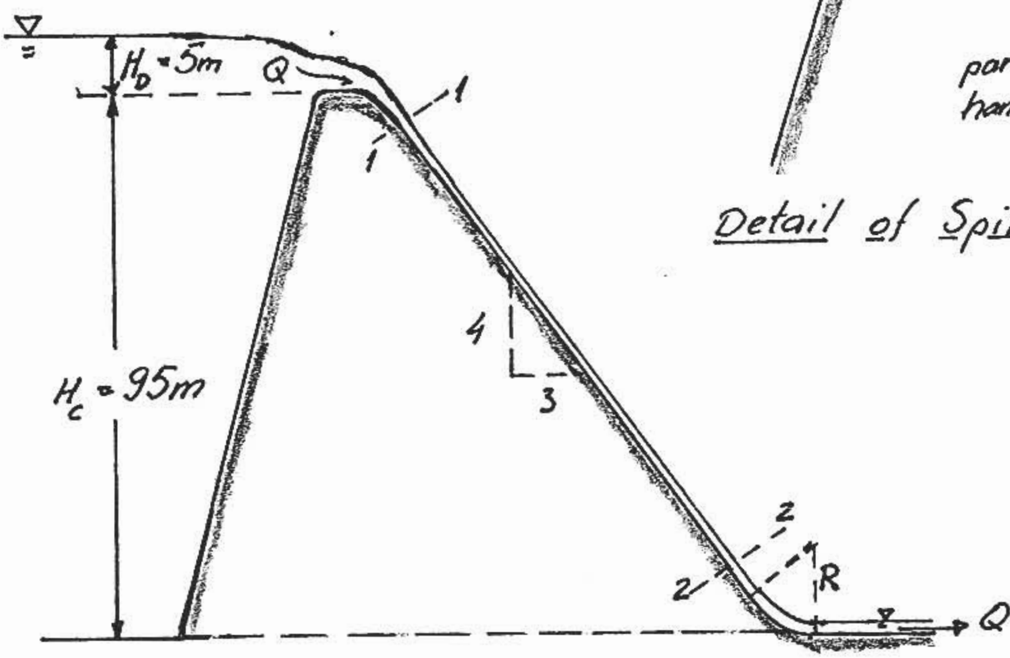
c) *Derive an equation relating* the velocity in the pipe, V_p, and elevation of the free surface in the fuel tank, *h*. (Assume steady flow and that the pressure above the gasoline surface remains at vapor pressure, p_v.)

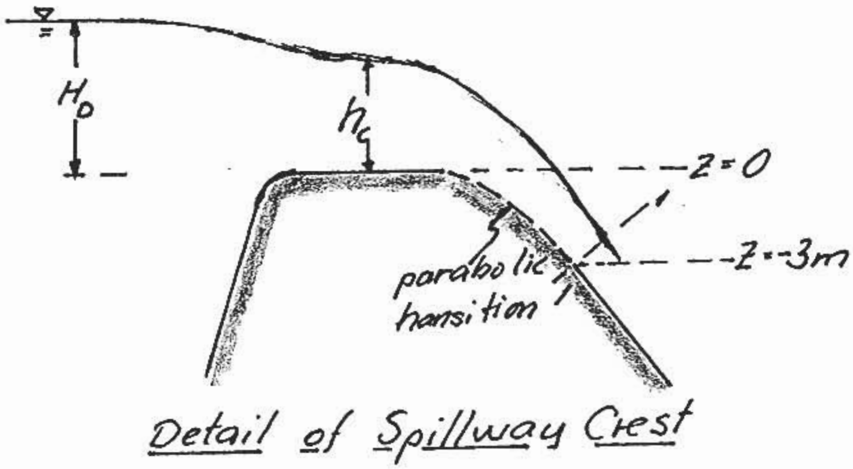
d) Solve the equation derived in (c) for $h = h_0 = 10$ m. (Default value: $V_p = 3.0$ m/s)

e) For the solution obtained in (d) make a realistic sketch of the variation of the Energy Grade Line and the Hydraulic Grade Line along the 10 m length of the pipe. (Include elevations of EGL & HGL)

f) Estimate the value of *h* when outflow essentially stops and the time required to reach this state.

Problem No: 3





An overflow spillway is constructed as sketched. The spillway crown (see detail) acts as a broad-crested weir and has a parabolic transition from horizontal to meet and match the downstream slope of the dam (4 vertical or 3 horizontal) at a level of 3 m below the spillway crest. The spillway channel is concrete (n = 0.014 (SI)), of rectangular cross-section, and b = 10 m wide. The design head, H_D, is the elevation difference between reservoir level and spillway crest. At the toe of the dam the flow direction is changed to horizontal by following a circular transition of radius R.

a) Determine the discharge into the spillway $Q = bq$, and depth of flow on the crest of the spillway, h_c, as a function of design head H_D, and evaluate for $H_D = 5$ m. (Default values: $Q = 190$ m^3/s; $h_c = 3.5$ m)

b) Estimate the depth of flow, h_1, as it enters the constant slope section after passing the short parabolic transition (at section 1-1 in sketch). (Default value: $h_1 = 1.6$ m)

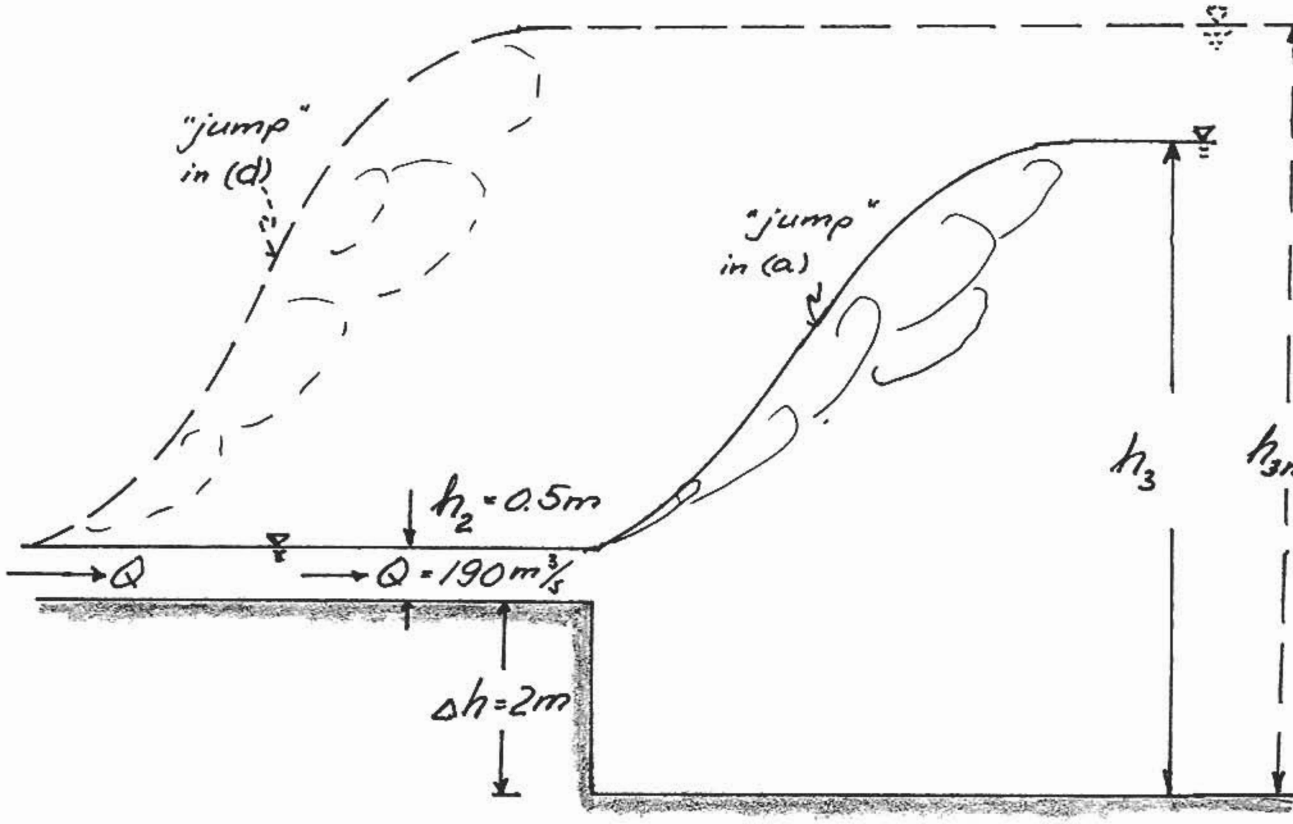
c) Determine the normal depth, h_2, velocity, V_2, and Froude number, Fr_2, assumed to be reached before the circular transition to horizontal at the toe of the dam (at section 2-2 in sketch). Name the profile that describes how the flow goes from h_1 to h_2. (Default value: $h_2 = 0.5$ m)

d) Determine the pressure on the bottom of the spillway channel p_{b2} corresponding to the solution in (c).

e) Estimate the pressure on the bottom of the circular transition at the toe when $R = 10$ m and depth is assumed equal to h_2 along the entire circular transition.

f) Why would you expect the assumption of constant $h = h_2$ along the circular transition to be reasonable?

Problem No: 4



A horizontal rectangular channel of width $b = 10$ m carries a flow of $Q = bq = 190$ m^3/s at a depth of $h_2 =$ 0.5 m and encounters a sudden drop in bottom elevation $\Delta h = 2$ m.

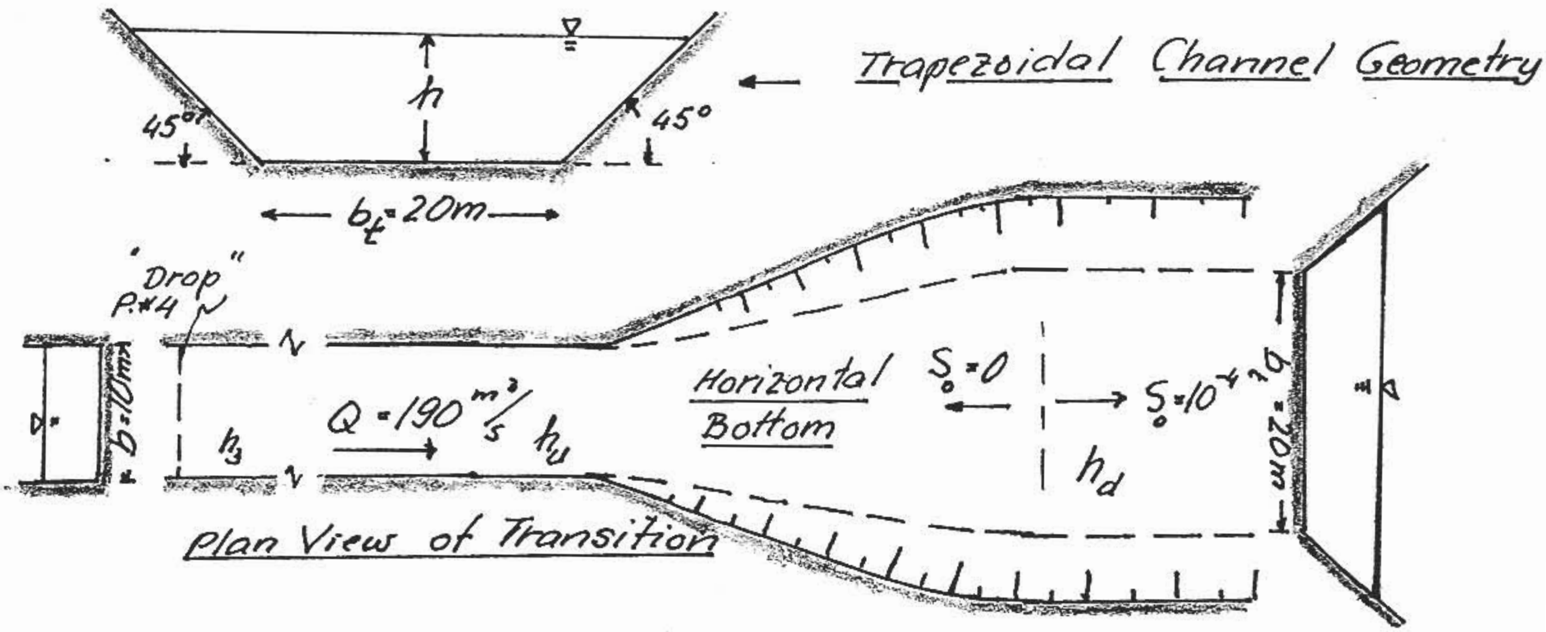
a) Assuming that the depth downstream of the "drop", h_3, is such that a "hydraulic jump" is initiated right at the location of the sudden "drop" (see sketch) determine the depth h_3 required for this to happen. (Default value: $h_3 = 12.0$ m)

b) Corresponding to answer in (a) determine the headloss and rate of energy dissipation in the "hydraulic jump" between h_2 and h_3.

c) If the depth downstream of the jump is larger than the value obtained in (a) the "hydraulic jump" would be initiated (start) upstream of the sudden drop. Explain why this must be so.

d) Determine the value of $h_3 = h_{3m}$ that must be exceeded in order for the hydraulic jump to take place entirely before the "drop" is reached. (see sketch)

Problem No: 5



The trapezoidal channel shown above carries a discharge of $Q = 190\ \text{m}^3/\text{s}$. The channel has $n = 0.02$ (SI) and a slope of $S_0 = 10^{-4}$.

a) Determine depth, velocity, and Froude number corresponding to normal flow (treat channel in its entirety, not as a composite channel). (Default values: $h_n = 5.8$ m; $V_n = 1.3$ m/s; $Fr_n = 0.2$)

The flow enters the natural trapezoidal channel through a short horizontal transition from a 10 m wide horizontal rectangular channel upstream. (see sketch)

b) Assuming that the transition from rectangular to trapezoidal channel causes no headloss, set up an equation that relates the depths upstream and downstream of the transition.

c) Solve the equation established in (b), assuming the downstream depth to be the normal depth obtained in (a), for the depth corresponding to subcritical flow, h_u, in the rectangular channel just upstream of the transition. (Default value: $h_u = 5.0$ m)

d) Do you feel comfortable about the assumptions made in (b) and (c)? Explain your reasoning, e.g. for the no-headloss assumption in (b) suggest a rough estimate of the headloss caused by the transition and argue whether this would be important or not.

"Obviously", this problem is related to the continuation of the flow established after the spillway flow, considered in Problem No. 3, enters the stilling basin and passes the "drop", considered in Problem No. 4a. There we found $h_3 = 12.0$ m (default value), i.e. quite different from the value obtained for $h_u = 5.0$ m (default value) in (c) above.

e) If the 10 m wide rectangular concrete channel after the "drop" (Problem No. 4a) is horizontal would it be possible for the depth to change from the value of h_3 after the "drop" to the depth h_u that is required for the flow to make the transition to the natural trapezoidal channel? and, if so, how long (rough estimate) should the horizontal concrete channel be? (Default value: several km long)

f) Would there be a better solution to the problem of reaching the matching condition for the transition to the natural trapezoidal channel than the one suggested in (e)?

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SOLUTIONS

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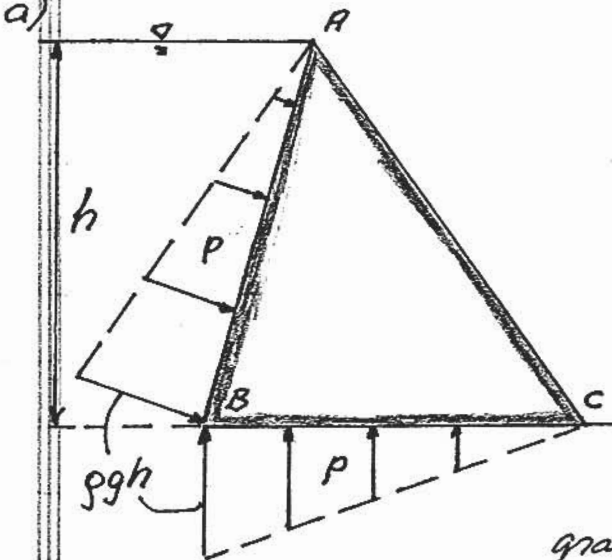
Problem No: 1

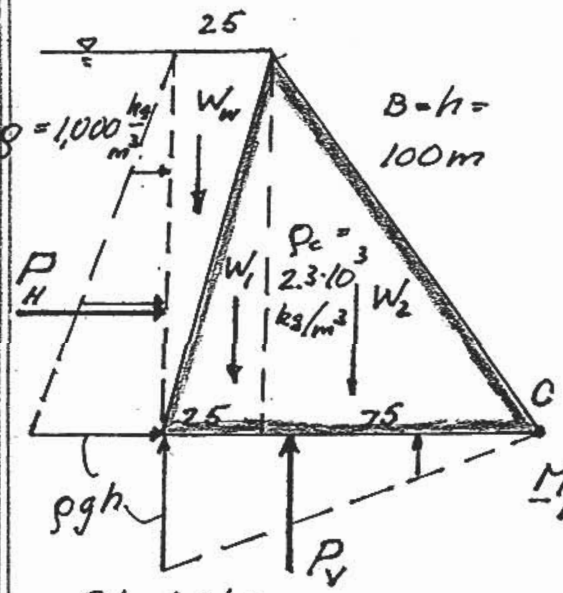
a)



Pressure varies linearly along AB from 0 @ A to $\rho g h$ @ B

Along base pressure is $\rho g h$ @ B and 0 @ C. Since there is a flow – although insignificant – pressure varies linearly between B and C [uneven thickness of gravel layer may change the variation from linear, but if ~uniform it's ~linear]

b)



Pivot Point is C

Overturning Moments

$$P_H \tfrac{1}{3} h = \tfrac{1}{6} \rho g h^3 = 1.63 \cdot 10^6 \ \tfrac{kN\,m}{m}$$

$$P_V \cdot \tfrac{2}{3} B = \left(\tfrac{1}{2} \rho g h B\right) \tfrac{2}{3} B = 3.27 \cdot 10^6 \ \tfrac{kN\,m}{m}$$

M_D = Overturning Moment = $4.9 \cdot 10^6 \ \tfrac{kNm}{m}$

Stabilizing Moments

$$W_w \left(B - \tfrac{1}{3} 25\right) = \tfrac{1}{2} \rho g h \cdot 25 \left(B - \tfrac{25}{3}\right) = 1.12 \cdot 10^6 \ \tfrac{kNm}{m}$$

$$W_1 \left(B - \tfrac{2}{3} 25\right) = \tfrac{1}{2} \rho_c g h \cdot 25 \left(B - \tfrac{2 \cdot 25}{3}\right) = 2.35 \cdot 10^6 \ \tfrac{kNm}{m}$$

$$W_2 \left(75 - \tfrac{1}{3} 75\right) = \tfrac{1}{2} \rho_c g h\, 75 (75 - 25) = 4.23 \cdot 10^6 \ \tfrac{kNm}{m}$$

M_s = Stabilizing Moment = $7.7 \ \tfrac{kNm}{m}$

F.S. against overturning = M_s / M_D = 1.57

c)

<u>Vertical Force</u> (>0 if downwards)

$W_w = \frac{1}{2}\rho g h \cdot 25 = 1.23 \cdot 10^4 \frac{kN}{m}$

$W_1 = \frac{1}{2}\rho_c g h \cdot 25 = 2.82 \cdot 10^4$ "

$W_2 = \frac{1}{2}\rho_c g h \cdot 75 = 8.45 \cdot 10^4$ "

$P_v = -\frac{1}{2}\rho g h B = \underline{-4.9 \cdot 10^4}$ "

Normal force on base = $\underline{7.6 \cdot 10^4} \frac{kN}{m}$ to be carried by gravel = N

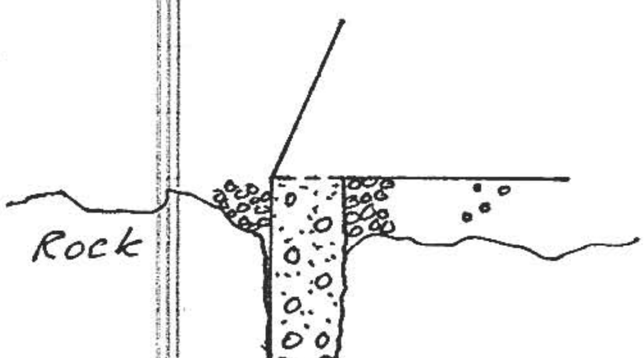
<u>Horizontal Force</u>

$P_H = \frac{1}{2}\rho g h^2 = \underline{4.9 \cdot 10^4 \frac{kN}{m}}$ to be supplied by gravel = F

F_M = Maximum Realizable Friction Force = $N \cdot \mu_f = \underline{5.32 \cdot 10^4} \frac{kN}{m}$

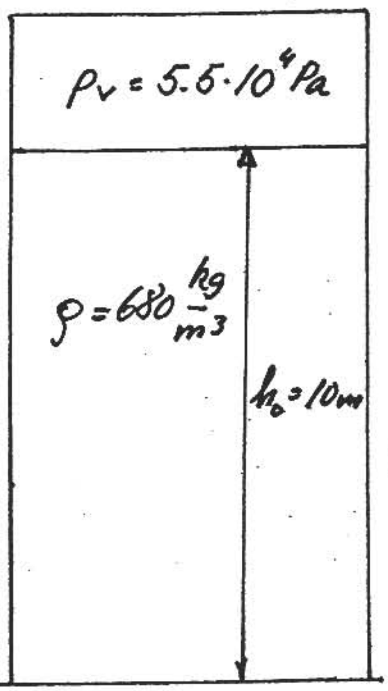
<u>Factor of Safety against Sliding</u> $= \frac{F_M}{P_H} = \underline{\underline{1.09}}$

d)



Blast a trench in the rock along the entire length of the upstream end of the base of the dam. Fill the trench with concrete to level of base. This will prevent water to flow into the gravel layer (nearly), and make the pressure in the gravel layer zero (nearly). Thus, essentially removing the force from the uplift pressure on the base of the dam, $P_v \cong 0$.
F.S. against overturning and sliding become 4.7 (1.57) and 1.8 (1.09), respectively.

Problem No: 2



Patm = 101.3 kPa (absolute)

a) Pressure at bottom of tank =

$p_v + \rho g h_o =$

$5.5\cdot10^4 + 680\cdot9.8\cdot10 =$

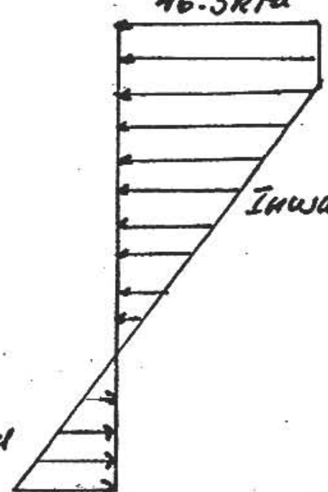
$\underline{1.21.64\ kPa}$ (absolute) =

$\underline{20.34\ kP\ (gauge)}$

b) $p_{net} = p_{atm} - p \Rightarrow$

$p_{net} = 101.3 - 55 = 46.3\,kPa$ (inwards)

for $10 \le z \le 12\,m$



$p_{net} = 101.3 - 55 - \rho g(10 - z) =$

$46.3\,kPa - 6.66(10 - z)\,kPa$

for $0 \le z \le 10$

$\underline{p_{net} = 0 \ @ \ z = 3.05\,m}$

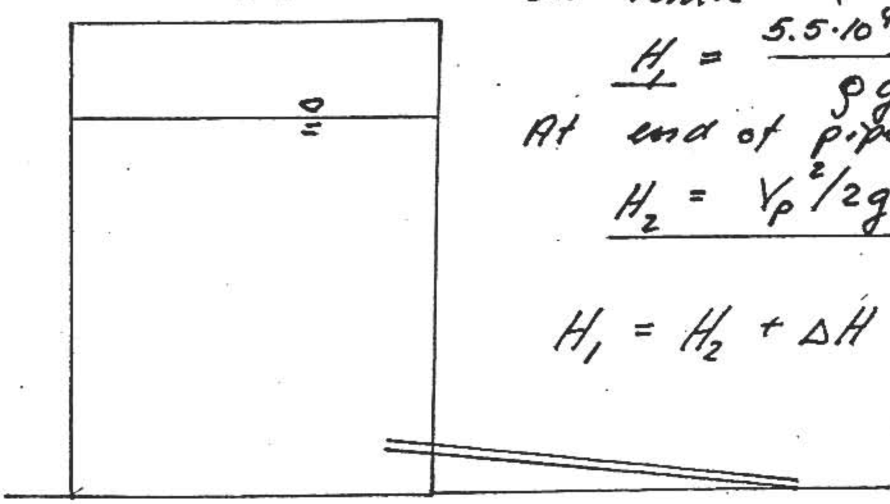
c) In tank: (using gauge pressure)

$\underline{H_1} = \frac{5.5\cdot10^4 - 101.3\cdot10^3}{\rho g} + 10 = \underline{3.05\,m}$

At end of pipe ($p = p_{atm} = 0$; $z = 0$)

$\underline{H_2 = V_p^2/2g}$

$H_1 = H_2 + \Delta H = \frac{V_p^2}{2g} + \left(K_{L,ent} + f\frac{\ell}{D}\right)\frac{V_p^2}{2g}$



$K_{L,ent}$ = Re-entrant flow $= \left(\frac{1}{C_c} - 1\right)^2 = 1$ $(C_c \approx 0.5)$

$l = 10\ m$; $D = 0.05\ m$

$$V_p = \frac{\sqrt{2gH_1}}{\left(1+1+f\frac{1000}{5}\right)^{1/2}} = \frac{\sqrt{2gH_1}}{\sqrt{2+200f}} = \frac{7.73}{\sqrt{2+200f}}\ \frac{m}{s}$$

d)

$\varepsilon = 0.045\ mm \Rightarrow \varepsilon/D = 0.045/50 = 9\cdot 10^{-4} = 0.0009$

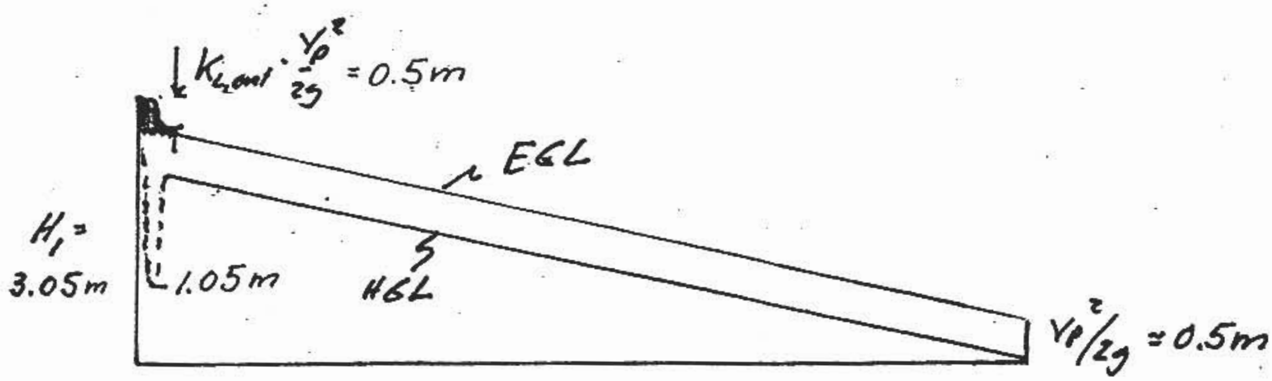
Moody gives $f = 0.0195$ if $Re > 2\cdot 10^6$ - try this

$V_p \approx 3.18\ m/s \Rightarrow Re = V_p D/\nu = 3.18\cdot 0.05/5\cdot 10^{-7} = 3.2\cdot 10^5 \Rightarrow$

$f = 0.0205 \Rightarrow V_p = 3.13\ m/s \Rightarrow Re = 3.1\cdot 10^5 \Rightarrow f = 0.0205$ Done

$\underline{V_p = 3.13\ m/s}$

e)



At vena contracta area is reduced to $\frac{1}{2}A_{pipe}$, i.e. $V_{vc} = 2V_p \Rightarrow V_{vc}^2/2g = 4V_p^2/2g = 2m$

$$H_1 = 3.05 = \frac{p_{vc}}{\rho g} + \frac{V_{vc}^2}{2g} + z_{vc} = \frac{p_{vc}}{\rho g} + 2 + 1$$

$$p_{vc} = \rho g\,(3.05 - 3) = 333\ Pa\ \text{(gauge)} > p_v$$

f)

When $H_1 = 0 = H_2$ outflow - in the sense of a pipe flow - will stop.

$$H_1 = \frac{5.5\cdot 10^4 - 101.3\cdot 10^3}{\rho g} + h_{min} = 0 \Rightarrow \underline{h_{min} = 6.95 m}$$

[Obviously in agreement with $H_1 = 3.05m$ in (c) where $h = h_0 = 10m$]

From (c) we have,

$$V_p = \frac{\sqrt{2g}}{\sqrt{2+200f}}\sqrt{H_1} = 1.79\sqrt{h-6.95} \quad (SI\text{-units})$$

when $f = 0.0205$ is assumed for any "h".

$$Q_p = \frac{\pi}{4}D^2 V_p = 3.5\cdot 10^{-3}\sqrt{h-6.95}$$

Conservation of volume then gives

$$\frac{\partial \forall}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\pi}{4}D_T^2 h\right) = \frac{\pi}{4}D_T^2\frac{\partial h}{\partial t} = -Q_{out} = -Q_p$$

or
$$\frac{dh}{dt} = -1.98\cdot 10^{-5}\sqrt{h-6.95}$$

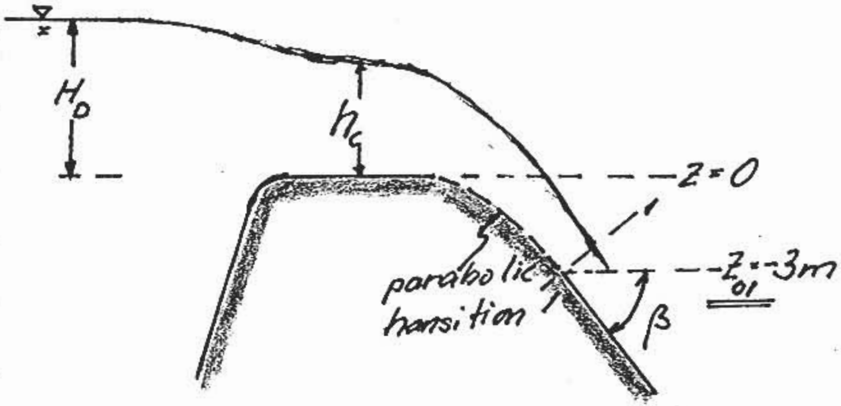
$$\frac{dh}{\sqrt{h-6.95}} = -1.98\cdot 10^{-5} \Rightarrow \left[2\sqrt{h-6.95}\right]_{h_0}^{h_{min}} = -1.98\cdot 10^{-5} t_{stop}$$

$$t_{stop} \cong 10^5\left(\sqrt{h_0 - 6.95} - \sqrt{h_{min} - 6.95}\right) = 10^5\sqrt{3.05} = 1.75\cdot 10^5 s$$

It would take $\sim 1.75\cdot 10^5 s$ = 48.6 hrs. before outflow would stop

Problem No: 3

a)



Detail of Spillway Crest

Flow must be critical (sub to supercritical flow from reservoir to steep channel) over crest of spillway.

h_c = critical depth = $(2/3)E_c = 2/3\ H_D = 3.33m$

$V_c = \sqrt{g h_c} = \sqrt{\frac{2}{3} g H_0}$

$$Q = V_c h_c b = \left(\frac{2}{3}\right)^{3/2} \sqrt{g}\, H_D^{3/2}\, b = 190.5\ m^3/s \quad (\text{for } H_0 = 5m)$$

b)

From $h_c = 3.33m$ on crest to h_1, which is expected to be $< h_c$, we have a short transition of a converging flow $\Rightarrow \Delta H_{c-1} = 0$. Thus,

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = H_c = H_0 \quad ; \quad V_1 h_1 = q = \frac{Q}{b} = 19.0\ \frac{m^2}{s}$$

so, $$\frac{q^2}{2g}\frac{1}{h_1^2} = H_D - \left(\frac{p_1}{\rho g} + z_1\right) = H_D - z_{01} - \frac{p_{b1}}{\rho g}$$

$q = 19\ m^2/s$; $H_0 = 5m$; $z_{01} = -3m$

$$\frac{18.4}{h_1^2} = 8 - \frac{p_{b1}}{\rho g} \qquad [p_{b1} = \rho g h_1 \cos\beta = 0.6 \rho g h_1]$$

$$\frac{18.4}{h_1^2} = 8 - 0.6h_1 \quad \Rightarrow \quad h_1 = \sqrt{\frac{18.4}{8-0.6h_1}} = \sqrt{\frac{2.3}{1-0.075h_1}}$$

$h_1 = 1.62\ m$ $\quad$ [if $p_{b1} = 0$: $h_1 = 1.52m$; if $p_{b1} = \rho g h_1$: $h_1 = 1.71m$]

c)

Normal depth in a rectangular channel of width $b = 10m$, discharge $Q = 190 m^3/s$, $n = 0.014$ (SI) and slope $\underline{S_0 = \sin\beta} = 4/5 = \underline{0.8}$ is obtained from

$$Q/b = q = \frac{1}{n}\frac{h_n^{5/3}}{\left(1+\frac{2h_n}{b}\right)^{2/3}}\sqrt{S_0} \Rightarrow h_n = \left(\frac{qn}{\sqrt{S_0}}\right)^{0.6}\left(1+\frac{2h_n}{b}\right)^{0.4} = 0.483\left(1+\frac{h_n}{5}\right)^{0.4}$$

$$\underline{h_n = 0.502 = 0.50m \; ; \; V_n = \frac{q}{h_n} = 38\frac{m}{s} \; ; \; Fr_n = \frac{V_n}{\sqrt{gh_n}} = 17.2 > 1}$$

assumed in (a) ✓

Transition from $\underline{h_1 \text{ to } h_n \text{ is through an S2-profile}}$

[It is somewhat questionable to assume h_n to be reached before the circular transition to horizontal. Computations (as done in class) suggest $50m < \Delta x < 400m$]

d)

The general formula for pressure variation in a uniform flow in a sloping channel is

$$p = \rho g \underline{\cos\beta}(h-y) \; ; \; \underline{p_{b2} = p_{bn}} = \rho g h_n \underset{}{\overset{3/5}{\cos\beta}} = \underline{2.94\,kPa}$$

e)

Because of the circular path a centripetal force is needed. This force is related to V^2/R.

$$\underline{p_{bR}} = \rho h_2 \frac{V^2}{R} = \rho\frac{q^2}{h_2 R} = \underline{72.2\,kPa} \gg p_{b2}$$

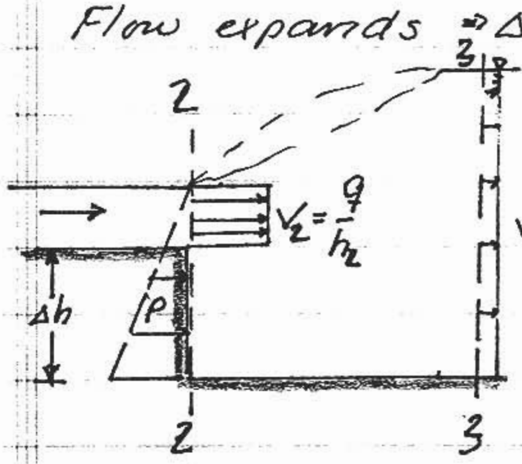
f)

The flow is so supercritical that the depth change would result in a large change in velocity head. No large change in H expected $\Rightarrow h \approx$ constant!

Problem No: 4

a)

Flow expands $\Rightarrow \Delta H \neq 0$. Momentum is all we have!
Short "jump" $\Rightarrow \tau$'s negligible



$$MP_2 = M_2 + P_2 = \rho V_2^2 h_2 b + \frac{1}{2}\rho g (h_2 + \Delta h)^2 b = MP_3 = (\rho V_3^2 + \frac{1}{2}\rho g h_3) h_3 b$$

$$h_3^2 = 2\frac{q^2}{g h_2} + (h_2 + \Delta h)^2 - 2\frac{q^2}{g h_3} = 153.6 - \frac{73.7}{h_3} \Rightarrow \underline{\underline{h_3 = 12.15 m}}$$

b)

$$\underline{\Delta H} = H_2 - H_3 = h_2 + z_{02} + \frac{V_2^2}{2g} - \left(h_3 + z_{03} + \frac{V_3^2}{2g}\right) =$$

$$(h_2 - h_3) + (z_{02} - z_{03}) + \frac{q^2}{2g h_2^2} - \frac{q^2}{2g h_3^2} = \underline{63.9 m}$$

$$\underline{\dot{E}_{diss}} = \rho g Q \Delta H = \underline{119 \cdot 10^6 \text{ Watts}}$$

c)

If $h_3 > 12.15 m$ then MP_3 increases [subcritical - P_3 is more important than M_3]. To balance, MP_2 must increase, but M_2 is given - only way is to increase P_2. If jump starts before "drop" pressure force from "drop" on fluid increases $\Rightarrow$ q.e.d.

d)

This is just a regular hydraulic jump in a rectangular channel. With $h_2 = 0.5 m$, $Fr_2 = 17.2$ and Cheat-Sheet

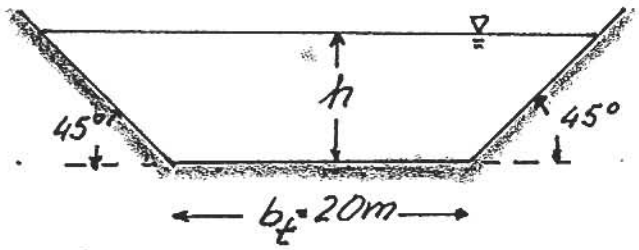
$$h_{2,conj} = \frac{1}{2} h_2 \left(-1 + \sqrt{1 + 8 Fr_2^2}\right) = 11.9 m = h_{3m} - 2$$

[subcritical flow passing drop with negligible change $\frac{V^2}{2g} = 0.1 m$]

$$\underline{h_{3m} = 13.9 m}$$

Problem No: 5

a)



$$A = b_t h + 2\cdot\tfrac{1}{2}h^2 = b_t h\left(1+\frac{h}{b_t}\right)$$

$$P = b_t + 2\sqrt{2}h = b_t\left(1+2\sqrt{2}\frac{h}{b_t}\right)$$

$$Q = \frac{1}{n}\frac{A_n^{5/3}}{P_n^{2/3}}\sqrt{S_0} = \frac{1}{n}\frac{(b_t h_n)^{5/3}}{b_t^{2/3}}\frac{(1+h_n/b_t)^{5/3}}{(1+2\sqrt{2}h_n/b_t)^{2/3}}\sqrt{S_0}$$

$$h_n = \left(\frac{Qn}{b_t\sqrt{S_0}}\right)^{0.6}\frac{(1+2\sqrt{2}h_n/b_t)^{0.4}}{1+h_n/b_t} = 5.85\frac{(1+0.14h_n)^{0.4}}{1+h_n/20}$$

$$\underline{h_n = 5.75\,m}\,;\quad \underline{V_n} = \frac{Q}{A_n} = \frac{190}{148} = \underline{1.28\,m/s}$$

$$\underline{Fr_n} = \frac{V_n}{\sqrt{g h_m}} = \frac{V_n}{\sqrt{g\frac{A_n}{b_s}}} = \frac{V_n}{\sqrt{g\cdot\underbrace{148/(b_t+2h_n)}_{h_m = 4.7m}}} = \underline{0.19}$$

b)

No headloss, $\Delta H = 0$, horizontal transition, $\Delta E = 0$,

$$E_u = h_u + \frac{V_u^2}{2g} = \underline{h_u + \frac{q_u^2}{2g h_u^2}} = E_d = \underline{h_d + \frac{Q^2}{2g A_d^2}} = h_d + \frac{V_d^2}{2g}$$

with $q_u = Q/b = Q/10$, $A_d = b_t h_d(1+h_d/b_t)$, this equation has only h_u and h_d as unknowns

c)

If $h_d = h_n = 5.85\,m$ then $E_d = h_n + \frac{V_n^2}{2g} = 5.84\,m$ and equation from (b) becomes (subcritical form)

$$h_u = E_d - \frac{q_u^2}{2g h_u^2} = 5.84 - \frac{18.4}{h_u^2} \Rightarrow \underline{h_u = 5.14\,m}$$

Corresponding $Fr_u = \sqrt{\frac{q_u^2}{g h_u^3}} = 0.52 < 1$!

d)

$V_u = Q / h_u = 19 / 5.14 = 3.7\ m/s$; $V_d = V_n = 1.28\ m/s$
The transition is an "expansion" since V decreases from upstream to downstream. Conservatively large [since $Fr \ll 1$] we estimate expansion loss from $\Delta H_{exp} = (V_u - V_d)^2/2g = 0.3\ m$. Introducing this in Bernoulli gives

$$E_u = E_d + \Delta H_{exp} = 5.84 + 0.3 = 6.14\ m$$

[This is only an increase of ~5% - expect a similar increase in h_u, so $h_u \approx 5.4\ m \Rightarrow$ Not to worry. Actual solution with ΔH_{exp} included gives $h_u \cong 5.54\ m$]

As for taking $h_d = h_n$ this is not an assumption. Normal flow in trapezoidal channel is SUBCRITICAL, and we must meet normal depth at entrance!

e)

Channel being horizontal, $S_0 = 0$, and gradually varied flow equation becomes

$$\frac{dh}{dx} = -\frac{S_f}{1 - Fr^2} \Rightarrow \Delta x = (-\Delta h)\frac{1 - Fr^2}{S_f}$$

Largest $Fr^2 = Fr_u^2 = 0.52^2 = 0.27$

Largest $S_f = S_{fu} = n^2 Q^2 P_u^{4/3} / A_u^{10/3} = \underbrace{0.014}_{concrete}{}^2\ 190^2 \frac{(10+10)^{4/3}}{(10 \cdot 5)^{10/3}} = 8.3 \cdot 10^{-4}$

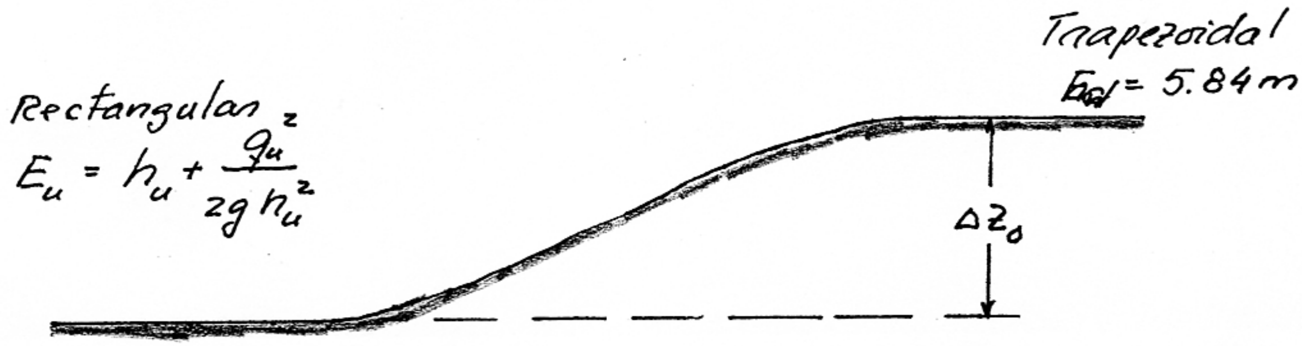
It is possible to go from $h_3 = 12\ m$ to $h_u = 5\ m$ in a horizontal channel w. $Fr < 1$: H2-curve does it

Distance required would be greater than

$$\underline{\Delta x} = (12-5)\frac{1-0.27}{8.3 \cdot 10^{-4}} = \underline{6.2\ km}\ !$$

f)

A much better solution would be to introduce an elevation change in the bottom along the transition from rectangular to trapezoidal cross-section



With $\Delta z_0 = 6.3 m$, we have with $\Delta H = 0$

$$E_u = h_u + \frac{18.4}{h_u^2} = E_d + \Delta z_0 = 12.14 m$$

Solution is $h_u = 12.01 m \approx 12.0 m = h_3$

No transition in depth is necessary!

<u>THE MATCH IS MADE IN HEAVEN</u>