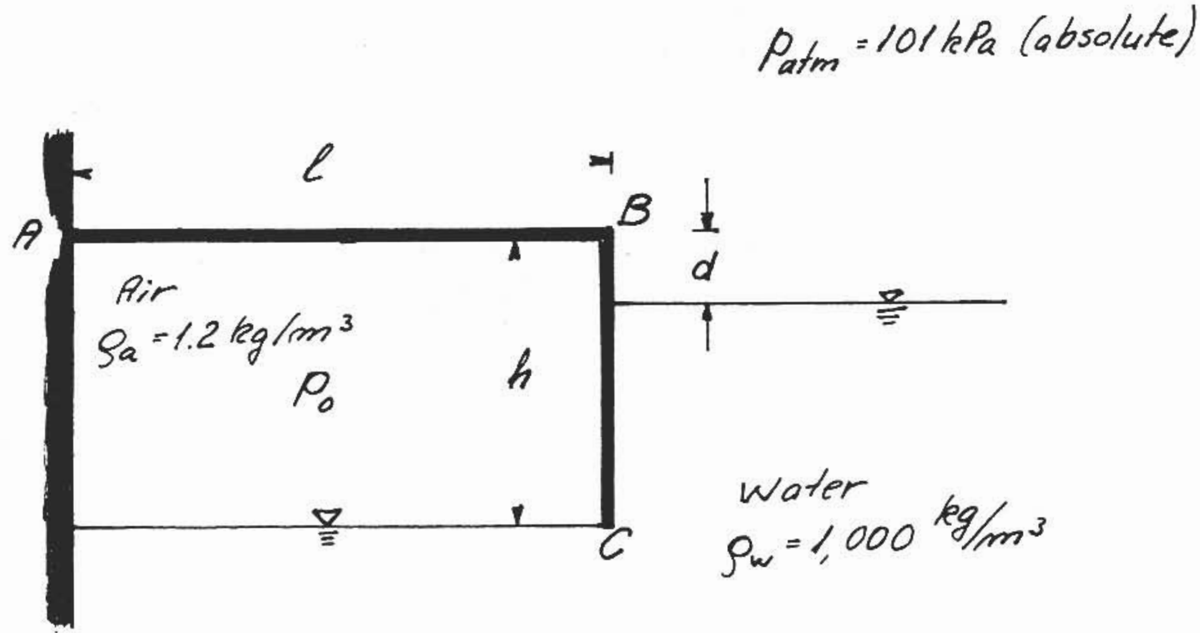
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.060/1.995 Fluid Mechanics

In-class Examination, 4 March , 2005

Problem No. 1: (40%)



A structure of length $\ell = 4.0$ and height h = 2.0 m is cantilevered into a still body of water, $\rho_w = 1000$ kg/m^3, as shown in the accompanying sketch. The entire interior of the structure is filled with trapped air, $\rho_a = 1.2$ kg/m^3 (the ends are closed off) and the free surface of the water body is located a distance of d = 0.5 m below the "lid" of the structure. The width of the structure, b, in the direction into the paper, is so large that it is allowable to treat the problem as two-dimensional, i.e. evaluate quantities depending on the width b in terms of "per unit width".

a) Determine the pressure of the trapped air, p_0.

b) Determine the total pressure force, F_h, on the vertical side of the structure, BC, its direction and its line of action.

c) Determine the total pressure force, F_v, on the horizontal lid of the structure, AB, its direction and its line of action.

d) Determine the total force, F, and moment, M, exerted at the structure's support, A. (Consider the structure itself to be weightless.)

Problem No. 2: (30%)

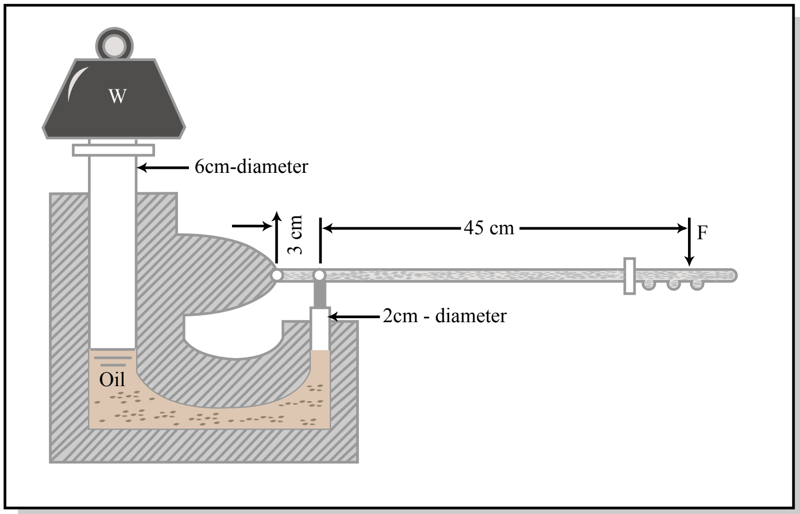


Figure by MIT OCW.

The sketch above illustrates the principle of a hydraulic jack carrying a weight of W = 10 kN (kiloNewton = 10^3 Newton). With the dimensions shown in the sketch determine the force F that must be applied at the handle in order to support the weight W (neglect the weight of pistons, oil and handle assembly).

Problem No. 3: (30%)

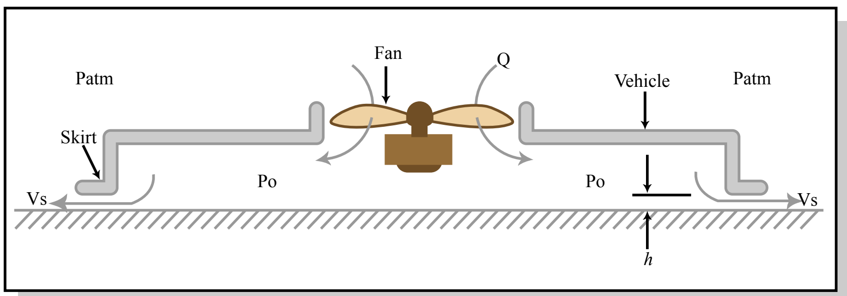


Figure by MIT OCW.

An air cushion vehicle is supported by a fan forcing air into a chamber created by a skirt around the periphery of the vehicle as illustrated in the sketch. The air escapes from the chamber through narrow slots between the skirt and the water or ground surface. Assume air to be an incompressible fluid ($\rho = 1.2\ \mathrm{kg/m^3}$), the vehicle to have a weight of 50 kN and to be essentially of rectangular shape with dimensions 10 by 20 m, and that the air chamber is sufficiently large to justify the neglect of any air velocities in the chamber away from the skirts.

a) Determine the necessary pressure in the air chamber to support the weight of the vehicle.

b) Determine the velocity of the air flow through the slots beneath the skirts of the vehicle.

c) Determine the air flow rate (in $\mathrm{m^3/s}$) to be provided by the fan if the slots are h = 5 cm in height.

1.060 FLUID MECHANICS

Cheat-Sheet No: 1

Pressure

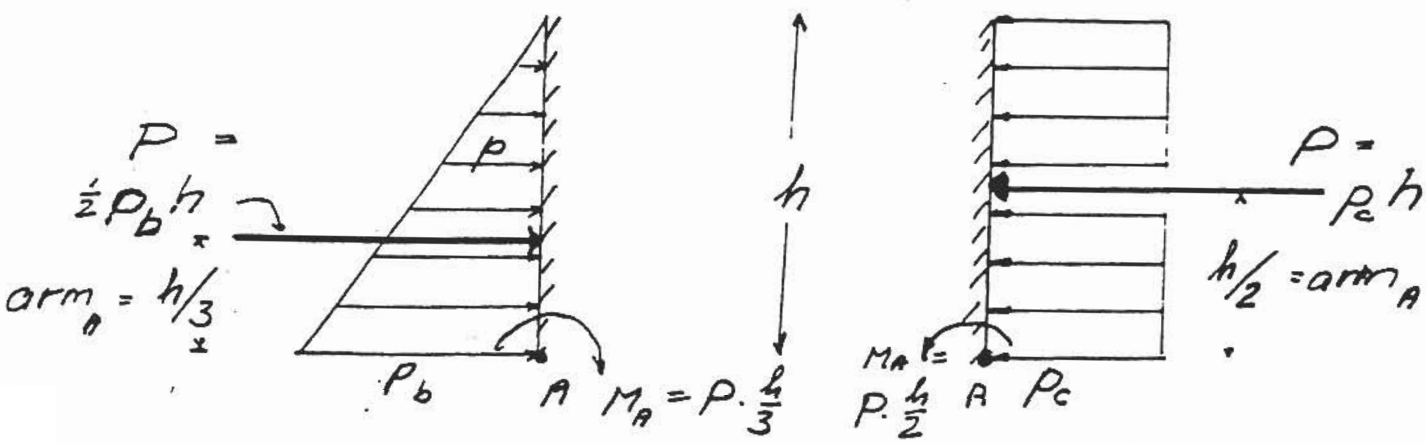
Is Isotropic – same in all directions
Is always perpendicular to surface upon which it acts

Hydrostatics

$$p + \rho g z = \text{CONSTANT} = p_0 + \rho g z_0$$

Valid within fluid of constant density, at rest.
Constant determined by knowledge of pressure p ($= p_0$) at reference elevation $z = z_0$.

Hydrostatic Forces



Horizontal force on surface = Horizontal force on projection of surface onto vertical plane

Vertical force on surface = Weight of fluid "above" surface (or its translation horizontally to a location where there is fluid "above")

Moments

$$M = \text{Force} \times \text{Arm}$$

Bernoulli

Along a Streamline, s

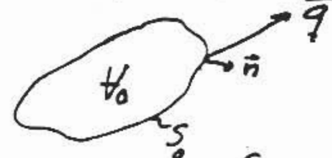
$$\frac{1}{2}\rho V_s^2 + p_s + \rho g z_s = \text{CONSTANT}$$

for steady flow ($\partial/\partial t = 0$). Constant obtained from knowledge of V_s, p_s at reference point of streamline $z = z_s$.

Perpendicular to Streamline, n

$$p_n + \rho g z_n = -\rho \int_{n_0}^{n_1} (V_s^2/R)\,dn + \text{CONSTANT}$$

for steady flow with $n \perp s$ and pointing towards center of curvature of s. R = radius of curvature of streamline. If $R \to \infty \Rightarrow$ straight streamlines $\Rightarrow$ pressure varies hydrostatically normal to straight parallel streamlines.



Conservation of Mass

$$\frac{\partial}{\partial t}\int_{\forall_0} \rho\, d\forall = -\int_{S_0} \rho(\vec{q}\cdot\vec{n})\,dS_0 = \int_{S_{in}} \rho q_\perp\, dS_{in} - \int_{S_{out}} \rho q_\perp\, dS_{out}$$

If ρ = constant over flow areas:

$$\int_A \rho q_\perp\, dA = \rho \int_A q_\perp\, dA = \rho Q = \rho \bar{V} A$$

Q = Discharge, $\bar{V}$ = average velocity = Q/A

If fluid incompressible: Volume Conservation

$$\frac{\partial \forall}{\partial t} = \sum Q_{in} - \sum Q_{out}$$

$\forall$ = volume of fluid within chosen boundaries of fixed control volume

Geometry

Area of circle : πr^2 ; Circumference = $2\pi r$

Volume of sphere : $\frac{4\pi}{3} r^3$; surface area = $4\pi r^2$

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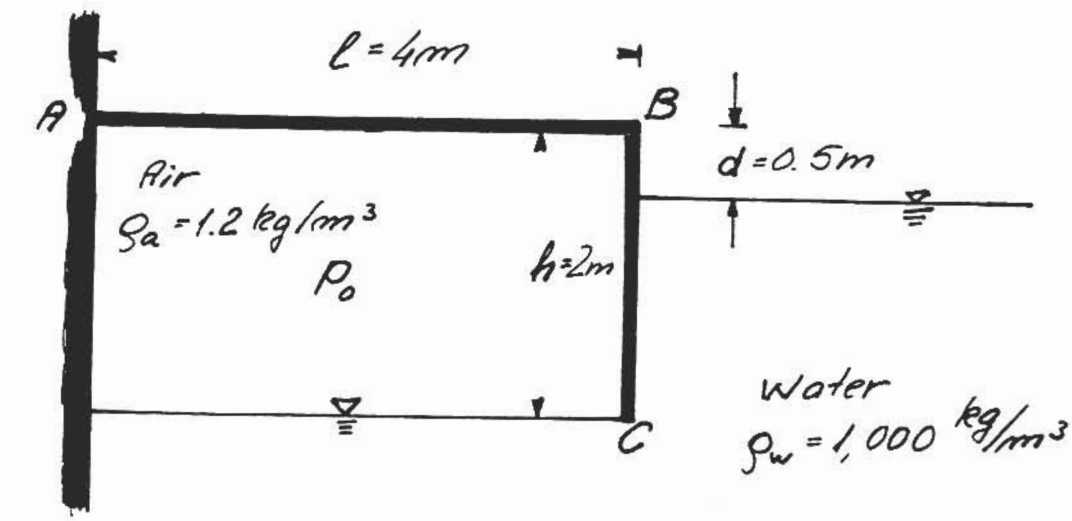
SOLUTIONS

Problem No: 1

$P_{atm} = 101\ kPa$ (absolute)



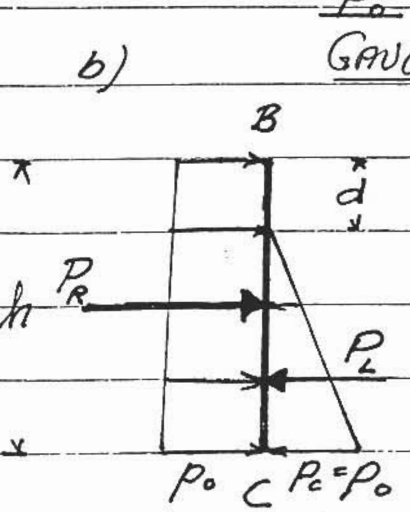
a)

At point C the pressure is $p_C = \rho g (h-d)$
Since pressure must be continuous across free surface inside structure, we have

$$\underline{p_o} = p_C = \rho g (h-d) = 1000 \cdot 9.8 (2.0-0.5) = \underline{14.7\ kPa}$$

GAUGE PRESSURE : OF COURSE !!

b)



$P_R = p_o h = 14.7 \cdot 2 = 29.4\ kN$

$arm_B = \frac{1}{2} h = 1m$. M_R=Moment around B = 29.4 kNm

$P_L = \frac{1}{2} p_o (h-d) = \frac{1}{2} 14.7 (2-0.5) = 11.0\ kNm$

$arm_B = d + \frac{2}{3}(h-d) = 0.5 + \frac{2}{3}(1.5) = 1.5\ m$

M_L=Moment around B = $P_L \cdot arm = 16.5\ kNm$

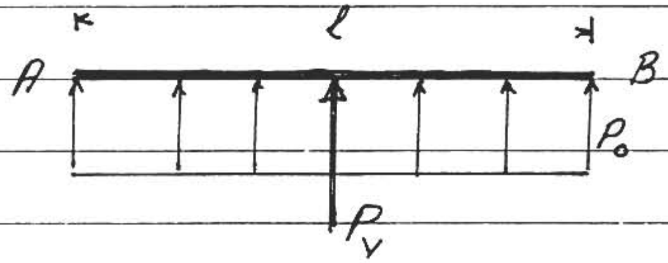
F_H = Total force $= P_R - P_L = 29.4 - 11.0 = \underline{18.4\ kN}$

It is <u>horizontal</u> ($\perp$ BC) and acts <u>to the right</u>

along a line a_h below "lid" (B) given by

$a_h F_H = M_R - M_L \Rightarrow \underline{a_h} = (29.4 - 16.5)/18.4 = \underline{0.70\ m}$

c)



$F_V = P_0 \cdot l = 14.7 \cdot 4 = 58.8\ kN$

arm $= \frac{1}{2} l = 2\ m$

$\underline{F_V}$ = Total force $= \underline{58.8\ kN}$

It is <u>vertical</u> (<u>upwards</u>) and acts $\underline{\frac{1}{2} l = 2m}$

<u>from A</u>

d)

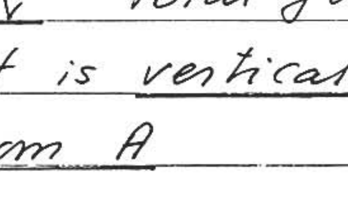
Total force to be supported at A is

$\underline{F_V = 58.8\ kN}$ in vertical (upwards)

$\underline{F_H = 18.4\ kN}$ in horizontal (to right)

and the moment is <u>counter-clockwise</u>

$\underline{M_A} = F_V \cdot (\frac{1}{2} l) + F_H a_h = 117.6 + 12.9 = \underline{130.5\ kNm}$



~~M_A = Turning structure upwards~~

F_V = Force structure upwards

F_H will pull structure out of wall

<u>TOTAL FORCE</u> $= F_T = \sqrt{F_V^2 + F_H^2} = \underline{61.6\ kN}$

at angle of $\tan\theta = F_V / F_H \Rightarrow \theta = 72.6^\circ$ to HORIZONTAL

Problem No: 2

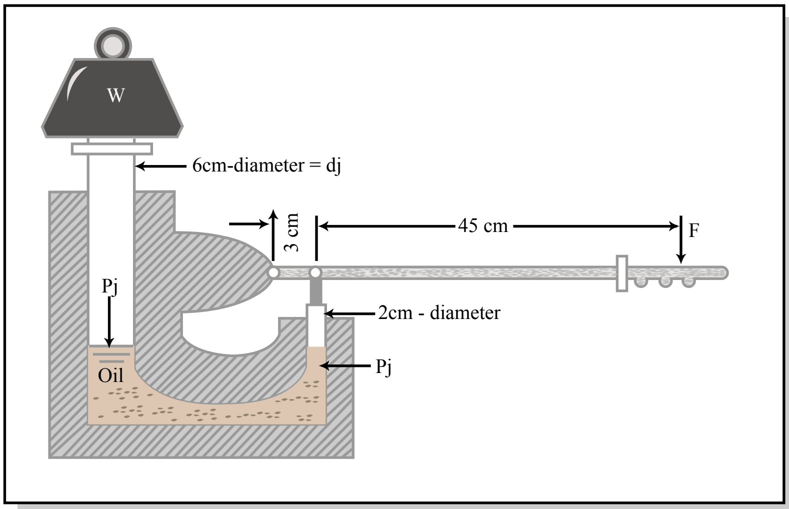


Figure by MIT OCW.

moment around handle pivot:

$$F \cdot (45+3) = \underbrace{\left(p_j \frac{\pi}{4} d_p^2\right)}_{\text{upward force on right piston.}} 3 \qquad (1)$$

p_j is also the pressure under the left piston so

$$p_j \frac{\pi}{4} d_j^2 = W \Rightarrow p_j = \frac{W}{\frac{\pi}{4} d_j^2} \quad \text{into (1)}$$

and

$$F\,(45+3) = \left(W \frac{\frac{\pi}{4} d_p^2}{\frac{\pi}{4} d_j^2}\right) 3 = W\left(\frac{d_p = 2cm}{d_j = 6cm}\right)^2 \cdot 3$$

$$\underline{F} = W\left(\frac{2}{6}\right)^2 \frac{3}{48} = \frac{W}{144} = \frac{10^4 N}{144} = \underline{69.4\ N}$$

Notice $10^4 N$ is $\approx$ weight of 1 ton = 10^3 kg mass
69.4 N is weight of ~ 7 kg mass!
Fluid mechanics makes life easy!

Problem No: 3

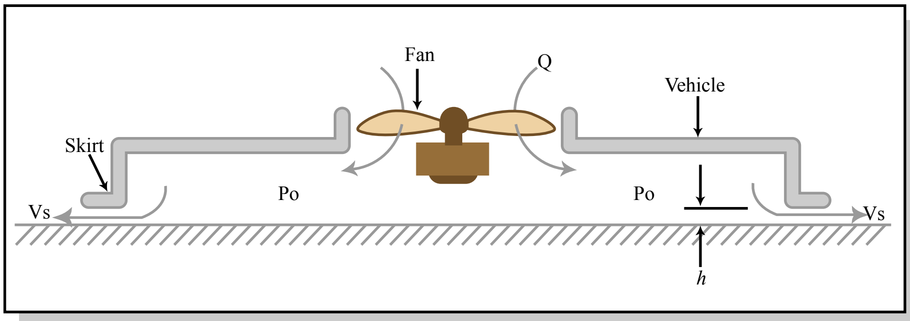


Figure by MIT OCW.

a)

Pressure in air chamber, p_o (gage - of course), carries the vehicle weight, so

$$p_o \times \text{Area of vehicle} = p_o \cdot 10 \cdot 20 = 50\,kN = 5 \cdot 10^4\,N$$

$$\underline{p_o} = 5 \cdot 10^4 / 200 = \underline{250\ Pa}$$

(notice, this is not much of a pressure, only about the pressure corresponding to 25cm water)

b)

Using Bernoulli between chamber (where $V=0$) and exit from gap under skirt (where $p = p_{atm} = 0$ gage) and $V = V_s$) we have

$$p_o + \rho_a g z_o + 0 = 0 + \rho_a g z_s + \frac{1}{2} \rho_a V_s^2$$

negligibly small "pressures" compared to p_o

$$\underline{V_s} = \sqrt{2 p_o / \rho_a} = \sqrt{2 \cdot 250 / 1.2} = \underline{20.4\ m/s}$$

c)

$$\underline{Q_{in} = Q_{fan}} = V_s \cdot \text{Area under skirt} = V_s \cdot h \cdot (2 \times 10 + 2 \times 20) =$$

$$V_s\, 0.05 \cdot 60 = \underline{61.2\ m^3/s}$$