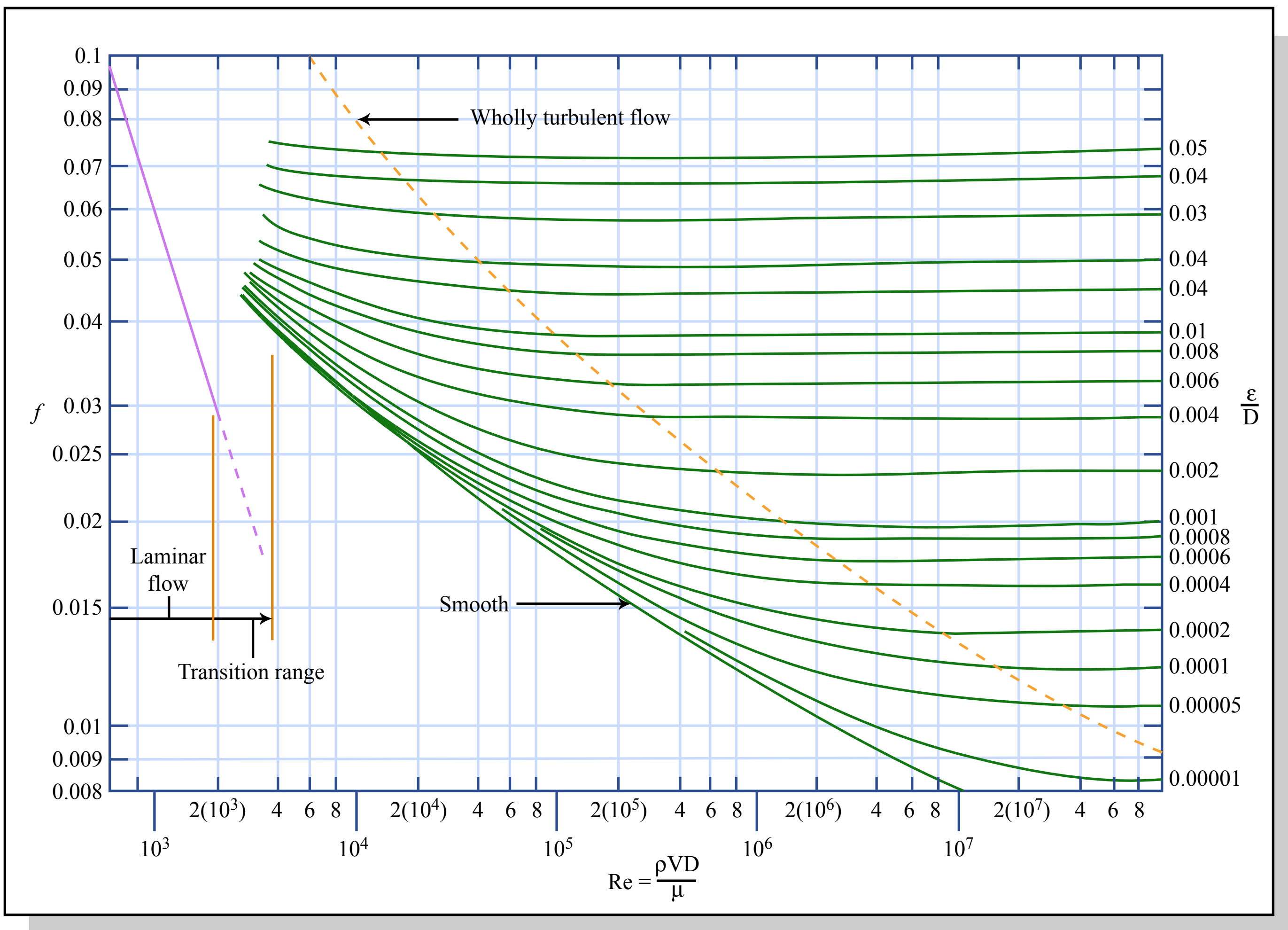
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.060/1.995 FLUID MECHANICS
In-class Test No: 2; 15 April, 2005

General Comment

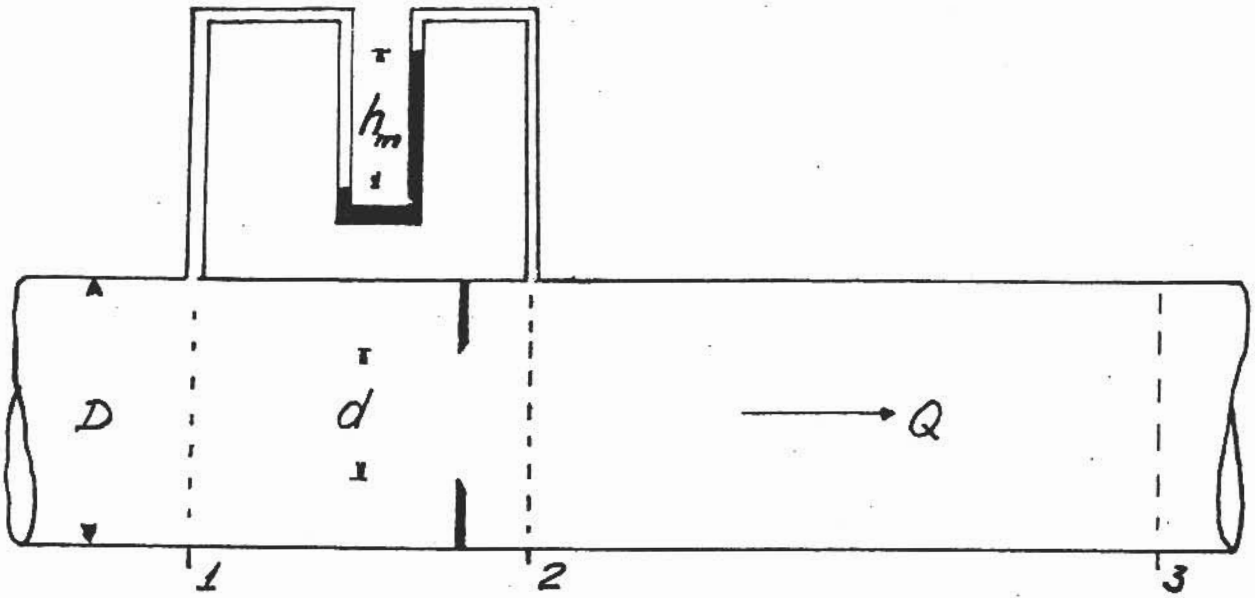
The test consists of three (3) problems. The first two (2) problems consist of questions requiring you to have solved previous questions. For this reason "default" answers are given so that you can proceed. The "default" solutions are not the correct solutions (but they may be close) so continue to use your own solutions unless they differ from the default values by a "lot". In answering some of the questions you may find the figure below helpful.

GOOD LUCK!



Graph by MIT OCW.

Problem No. 1 (45%)



Water, $\rho = 1{,}000$ kg/m^3 and $\nu = 10^{-6}$ m^2/s, flows through a horizontal circular pipe of diameter D = 10.0 cm. The pipe is equipped with an "orifice meter" (see sketch above). An orifice meter consists of an orifice plate in which a sharp-edged circular hole (concentric with the pipe) of diameter $d = 5.0$ cm creates a local flow constriction. The pressure difference across the orifice is measured by a mercury manometer, $\rho_m = 13.6\rho$. The flow is assumed uniform at sections "1" and "3" in the sketch, and flow proceeds from left to right.

a) Visualize the pattern of the flow from "1" to "3" by a rough sketch.

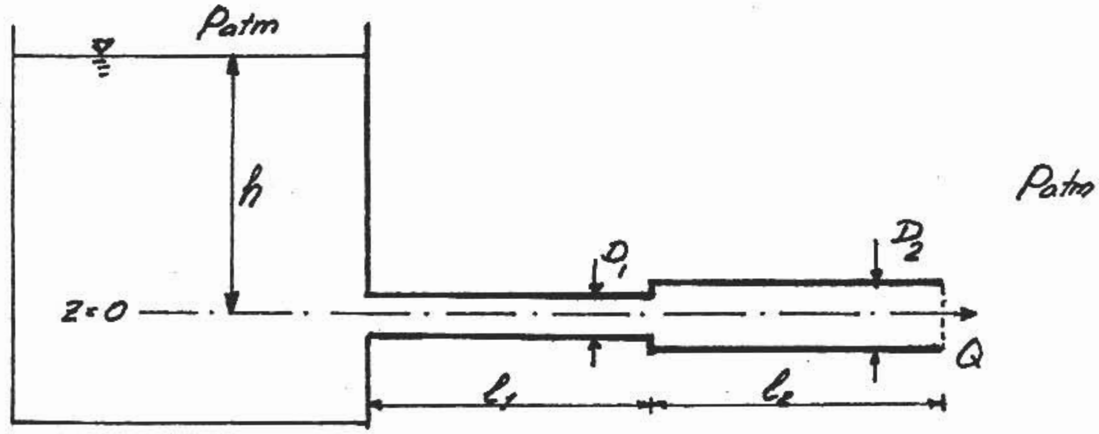
b) For a manometer reading of $h_m = 5.0$ cm determine the pressure difference, $p_1 - p_2$, across the orifice meter. [Default value: $p_1 - p_2 = 6{,}300$ Pa]

c) Determine the flow rate, Q, in the pipe from the pressure difference obtained in (b). [Default value: $Q = 4.25 \cdot 10^{-3}$ m^3/s]

d) Neglecting wall friction, determine the head loss between "2" and "3". [Default value: $\Delta H = 0.45$ m]

e) Use the momentum principle (neglecting wall friction) to determine the force on the orifice plate for the flow condition considered above.

Problem No. 2 (45%)



A very large container is filled with water ($\rho = 1{,}000$ kg/m^3, $\nu = 10^{-6}$ m^2/s) to a level of z = h. The container is connected to a horizontal pipe-system consisting of two circular pipes, both of length $l_1 = l_2 = 7.5$m and both of roughness $\varepsilon = 0.1$mm, one is of diameter $D_1 = 10$cm, the other of diameter $D_2 = 15$cm. All transitions are sharp-edged (see accompanying sketch) and the discharge is $Q = 3.53 \cdot 10^{-2}$ m^3/s.

a) Determine the friction factors f_1 and f_2 for the two pipes. (Default values: $f_1 = f_2 = 0.02$)

b) Determine the level h in the container necessary to generate the specified discharge (Default value: $h = 3$m)

c) Determine the pressure of a point "A" a short distance into the pipe connected to the container (see sketch for location of "A")

d) If you cut-off the pipes and threw them away, so that the container was discharging into the air through a 10-cm-diameter sharp-edged orifice what would be the discharge if h has the same value as determined in (b)?

e) Does your answer in (d) surprise you? Why or why not?

Problem No: 3 (10%)

A pump is specified by its discharge, $Q = 2$m^3/s, the head-increase across it, $H_p = 25$m, and its efficiency, $\eta = 0.85$. The fluid being pumped is water.

Determine the pumping cost per m^3 of water, if electricity is assumed to cost 10 cents per kWhr.

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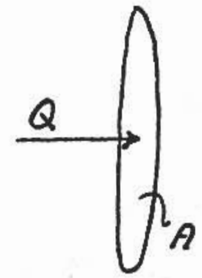
CHEAT-SHEET NO: 2

MOMENTUM

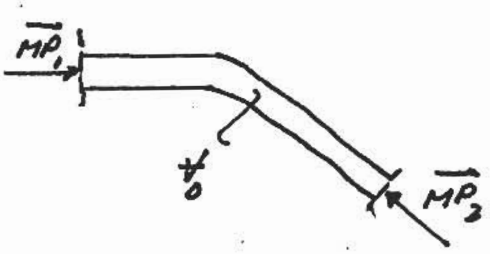
$$Q = VA \quad ; \quad V = Q/A$$



Wellbehaved flow: Streamlines straight & parallel & $\perp A$

$\overrightarrow{MP} = (\rho V^2 + p_{CG})A$, $\perp A$ towards control $\forall_0$

p_{CG} = pressure @ center of gravity of A

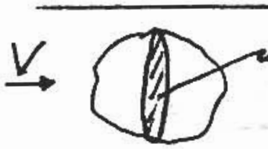


Equilibrium of forces (steady flow):

$\overrightarrow{MP}_1 + \overrightarrow{MP}_2$ + (sum of all other forces on fluid within $\forall_0$) = 0

"Other forces": Shear forces & pressure forces on boundaries of $\forall_0$, gravity, drag forces on objects in $\forall_0$

DRAG FORCE: $F_D = \frac{1}{2} \rho C_D A_\perp V^2$



$A_\perp$ = area of body's projection on plane $\perp V$

C_D = Drag coefficient = $C_D(Re)$

BERNOULLI

$$H = \text{Total Head} = \frac{V^2}{2g} + \frac{p_{CG}}{\rho g} + z_{CG}$$

Piezometric Head $= \frac{p_{CG}}{\rho g} + z_{CG}$; Velocity Head $= \frac{V^2}{2g}$

EGL = Energy Grade Line : $z_{EGL} = H$

HGL = Hydraulic Grade Line : $z_{HGL} = H - \frac{V^2}{2g}$

Flow from ① to ② with wellbehaved flow @ ① & ②

$H_1 = H_2 + \Delta H$; ΔH = head loss between ① & ②

HEAD LOSSES

$\Delta H \simeq 0$ if Short transition with Converging Flow

Pipe Friction Losses

$$\Delta H_f = f \frac{\ell}{D} \frac{V^2}{2g} \quad \left(D = 4\frac{A}{P} = 4 \frac{\text{Area}}{\text{Perimeter}} = 4 \text{ Hyd. Radius} \right)$$

$$f = f\left(\frac{VD}{\nu}, \frac{\varepsilon}{D} \right) \text{ from Moody}$$

ε = pipe roughness
ν = kin. viscosity of fluid

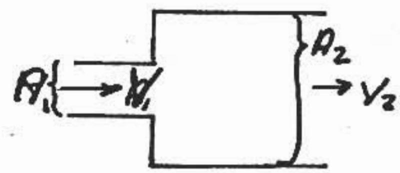
· Wall Shear Stress: $\tau_s = \frac{f}{8} \rho V^2$

Minor Losses

$$\Delta H_m = K_L \frac{V^2}{2g}$$

K_L = Minor Loss Coefficient

Expansion Loss: $\Delta H_{exp} = \dfrac{(V_1 - V_2)^2}{2g}$



Exit loss $(A_2 \gg A_1)$: $K_{L,exit} = 1$

Entry loss (sharp edged orifice): $K_{L,ent} = \left(\frac{1}{C_c} - 1 \right)^2$

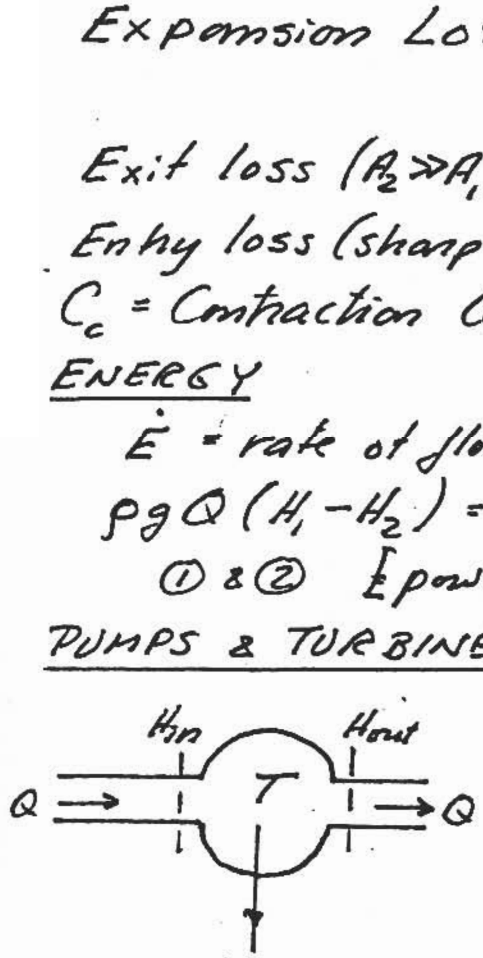
C_c = Contraction Coefficient [$C_c = 0.6 \rightarrow$, $C_c = 0.5 \rightarrow$]

ENERGY

$\dot{E}$ = rate of flow of Mech. Energy $= \rho g Q H$

$\rho g Q (H_1 - H_2)$ = rate of dissipation of Mech. Energy between ① & ② [power loss] = rate of production of internal energy

PUMPS & TURBINES



$BHP = \eta \rho g Q [H_{in} - H_{out}]$ [Turbines]

η = efficiency ≤ 1

For pump flow of energy is reversed

$\eta \, BHP = \rho g Q [H_{out} - H_{in}]$ [Pumps]

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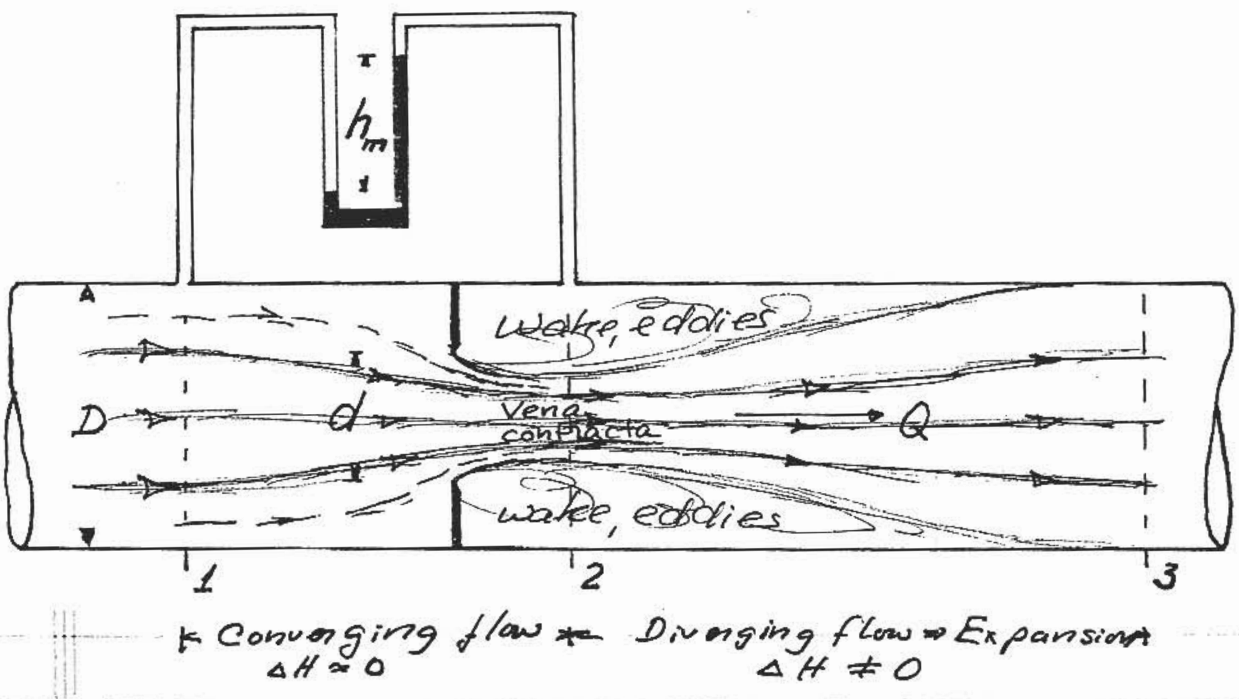
SOLUTIONS

Problem No: 1

a)



(2) assumed to be at vena contracta of orifice flow, i.e. effective flow area $A_2 = C_v \frac{\pi}{4} d^2$ w. $C_v \approx 0.6$

b)

$$p_1 - \rho g \Delta z_1 - \rho_m g h_m + \rho g (h_m + \Delta z_1) = p_2$$

$$\underline{p_1 - p_2} = (\rho_m - \rho) g h_m = 12.6 \cdot \rho g h_m = \underline{6{,}174 \text{ Pa}}$$

c)

Flow from (1) to (2) is a "short transition of a converging flow" ⇒ $\Delta H \cong 0$. So, $H_1 = H_2$ or

$$V_1^2/2g + p_1/\rho g + z_1 = V_2^2/2g + p_2/\rho g + z_2 \; ; (z_1 = z_2)$$

So

$$p_1 - p_2 = \frac{1}{2}\rho (V_2^2 - V_1^2) = \frac{1}{2}\rho \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) Q^2$$

$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4} 0.1^2 = 7.85 \cdot 10^{-3} m^2$

$A_2 = \frac{\pi}{4} d^2 \cdot C_c = 1.96 \cdot 10^{-3} \cdot 0.6 = 1.18 \cdot 10^{-3} m^2$

$$\underline{Q} = \sqrt{\frac{2(p_1 - p_2)}{\rho (A_2^{-2} - A_1^{-2})}} = \underline{4.19 \cdot 10^{-3} \ m^3/s}$$

d)

From (2) to (3) we have an expansion, so there is a head-loss

$\Delta H_{2-3} = \Delta H_{exp} = (V_2 - V_3)^2 / 2g$

$V_2 = Q / A_2 = 4.19 \cdot 10^{-3} / 1.18 \cdot 10^{-3} = 3.55 \, m/s$

$V_1 = Q / A_1 = 4.19 \cdot 10^{-3} / 7.85 \cdot 10^{-3} = 0.53$

$\underline{\Delta H_{exp}} = \underline{\Delta H_{2 \to 3}} = (3.55 - 0.53)^2 / 2 \cdot 9.8 = \underline{0.47 \ m}$

e)

Using momentum from (1) to (3) we have

$MP_1 = MP_3 + F_{op} \Rightarrow V_1 = V_3$ since $A_1 = A_3$

So

$F_{op} = (p_1 - p_3) A_1$

Using Bernoulli from (1) to (3) we have

$H_1 = H_3 + \Delta H_{1 \to 3} = H_3 + \Delta H_{2 \to 3}$

Again $V_1 = V_3$, and $z_1 = z_3$, so

$\frac{p_1 - p_3}{\rho g} = \Delta H_{2 \to 3} = 0.47 m \Rightarrow p_1 - p_3 = \rho g \, 0.47 = 4{,}600 Pa$

$\underline{F_{op}}$ = Force on orifice plate $= (p_1 - p_3) A_1 = \underline{36.2 \ N}$

Same result could be obtained if applying Momentum Principle between (2) and (3), but it would be much more involved!!

At 2-2 flow is over the small area of Vena Contracta:

i.e. $M_2 = \rho V_{vc}^2 A_{vc}$ whereas p_2 acts over entire area, i.e. $P_2 = p_2 \cdot A_2 =$

$A_{vc} = C_c (\pi/4 d^2)$, $A_2 = \pi/4 \, D^2$

Problem No:2

a)

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.1)^2 = 7.85 \cdot 10^{-3} m^2 \; ; \; A_2 = \left(\frac{D_2}{D_1}\right)^2 A_1 = 2.25 \cdot A_1 = 1.77 \cdot 10^{-2} m^2$$

$$V_1 = Q/A_1 = 3.53 \cdot 10^{-2} / 7.85 \cdot 10^{-3} = 4.5 \, m/s \; ; \; V_2 = 2.25^{-1} V_1 = 2.0 \, m/s$$

$$\underline{Re_1} = V_1 D_1 / \nu = 4.5 \cdot 0.1 / 10^{-6} = \underline{4.5 \cdot 10^5} \; , \; \underline{\varepsilon / D_1} = 0.1 \cdot 10^{-1} / 10 = \underline{0.001}$$

MOODY : $\underline{f_1 = 0.021}$

$$\underline{Re_2} = V_2 D_2 / \nu = 2 \cdot 0.15 / 10^{-6} = \underline{3 \cdot 10^5} \; ; \; \underline{\varepsilon / D_2} = 0.1 \cdot 10^{-1} / 15 = \underline{0.00067}$$

MOODY : $\underline{\underline{f_2 = 0.019(5)}}$

b)

$$H_1 = h = H_{end} + \Delta H = V_{end}^2/2g + p_{end}/\rho g + \sum \Delta H_m + \sum \Delta H_f$$

Open ended pipe $\Rightarrow p_{end} = 0 \; ; \; V_{end} = V_2 = V_1 / 2.25$

$$\underline{V_{end}^2 / 2g} = (2.25)^{-2} V_1^2 / 2g = \underline{0.198 \, V_1^2 / 2g}$$

$$\underline{\sum \Delta H_m} = K_{L,ent} \, V_1^2 / 2g + \Delta H_{exp} = \left(\frac{1}{C_v} - 1\right)^2 \frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} =$$

$$\left[\left(\frac{1}{C_c} - 1\right)^2 + (1 - V_2/V_1)^2\right] V_1^2 / 2g = \left[\left(\frac{1}{0.6} - 1\right)^2 + \left(1 - \frac{1}{2.25}\right)^2\right] \frac{V_1^2}{2g} =$$

$$(0.44 + 0.31) V_1^2 / 2g = \underline{0.75 \, V_1^2 / 2g}$$

$$\underline{\sum \Delta H_f} = f_1 (l_1 / D_1) V_1^2 / 2g + f_2 (l_2 / D_2) V_2^2 / 2g =$$

$$\left[f_1 (l_1 / D_1) + f_2 (l_2 / D_2)(2.25)^{-2}\right] V_1^2 / 2g =$$

$$\left[0.021 (7.5/0.1) + 0.019(5) \cdot 7.5/0.15 \, (2.25)^{-2}\right] V_1^2 / 2g =$$

$$\left[1.58 + 0.19\right] V_1^2 / 2g = \underline{1.77 \, V_1^2 / 2g}$$

$$\underline{h} = (0.2 + 0.75 + 1.77) \frac{V_1^2}{2g} = 2.72 \cdot \frac{V_1^2}{2g} = 2.72 \frac{4.5^2}{2 \cdot 9.8} = \underline{2.81 m}$$

c)

$$H_A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = H_{end} + \Delta H = \frac{V_{end}^2}{2g} + 0 + z_A + \Delta H$$

$$\underline{p_A} = \rho g \left(0.2 + 2.52 - \left(\frac{V_A}{V_1}\right)^2\right) \frac{V_1^2}{2g} = \frac{1}{2} \rho V_1^2 \left(2.72 - \frac{1}{(0.6)^2}\right) =$$

$$\frac{1}{2} \rho V_1^2 (2.72 - 2.78) = -0.03 \cdot \rho \cdot V_1^2 = \underline{-608 \, Pa}$$

d)

For free outflow we have, since $h >> D_1 = 10cm$, that

$$V_j = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 2.81} = 7.42\, m/s$$

and

$$A_j = C_c A_{hole} = C_c A_1 = 0.6 \cdot 7.85 \cdot 10^{-3} = 4.71 \cdot 10^{-3}\, m^2$$

$$\underline{Q_{free}} = V_j A_j = \underline{3.49 \cdot 10^{-3}\, m^3/s}$$

e)

This is less than discharge when there is a pipe (in fact a rather <u>long</u> pipe) through which flow has to pass before discharging into the air. One "should be" surprised that adding the pipe increases(!) the discharge. Reason, of course, is that the pipe allows for a negative pressure at vena contracta for the outflow from the container. So, in this case the pipe's presence help to "suck out" water from the container.

Problem No: 3

For pumps we have

$$\eta \cdot BHP = \rho g Q H_p \Rightarrow \underline{BHP} = \frac{\rho g Q H_p}{\eta} = \underline{576.5 kW}$$

Since $Q = 2 m^3/s = 7.2 \cdot 10^3 \ m^3/hr$ the cost of running the pump for 1hr would be
$576.5 \cdot 10$ cents/~~hr~~ $= 5765$ cents/hr.
Since $7.2 \cdot 10^3 \ m^3/hr$ is being pumped, price per m^3 is $= 5765/7200 = \underline{\underline{0.8 \text{ cents}/m^3}}$

Think about this for a second. $H_p = 25m$ corresponds approximately to the "eighth floor" of an appartment complex. $1 m^3$ corresponds to approximately ~ 250 gallons. A person carrying 2 1-gallon bottles of water would have to climb 8 flights of stairs 125 times each to do the work done by the pump for a cost of less than 1 cent!!
<u>This is pretty cheap labor, indeed!</u>