1.060 Engineering Mechanics II Spring 2006

Recitation 6 - Problems

March 23rd and 24th

Problem 1

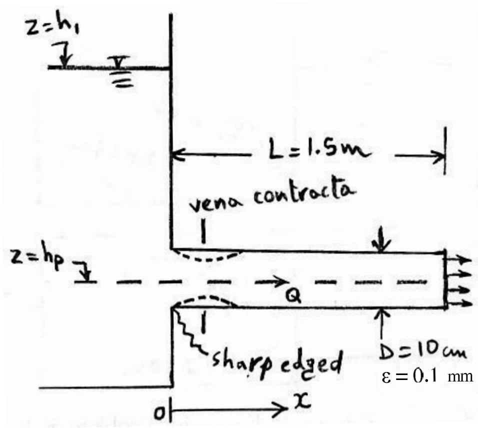


Figure 1: Reservoir discharging through a pipe in Problem 1.

A constant level reservoir ($h_1 = 10\ m$) of large surface area discharges water into the atmosphere through a pipe ($L = 1.5\ m$, $D = 10\ cm$, $\epsilon = 0.1\ mm$) which has its centerline at $z = h_p = 8.5\ m$ (see Figure 1). The sharp edged inflow corresponds to a contraction coefficient $C_v = 0.61$.

a) Determine the pipe discharge, Q, and the velocity after the flow expands from *vena contracta* to the full pipe, V_p.

b) Determine the velocity and pressure at *vena contracta*, V_v and p_v.

c) Compare V_v with the velocity at *vena contracta* of a free outflow (i.e., a flow out of the reservoir through an orifice of diameter $D = 10\ cm$, with no pipe). Why do these two velocities differ?

d) Carefully draw the energy grade line (EGL) and hydraulic grade line (HGL) for the flow along the pipe, from the sharp edged entrance to the pipe ($x = 0$) to the end ($x = L$).

Problem 2

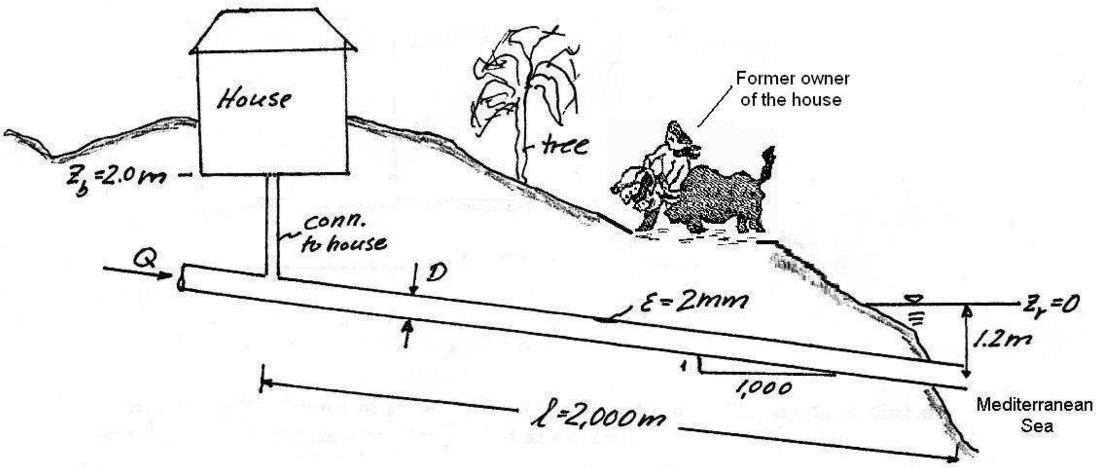


Figure 2: Your Spanish house in Problem 2.

Inspired by your TA, you decide to move to Spain and live in a town next to the Mediterranean Sea. After buying a cozy house from a local bullfighter (admire it in Figure 2), you find out that there is no sewage system. You complain to the mayor of the town, who tells you to design the sewage pipe yourself.

The pipe will be made of concrete ($\epsilon = 2\ mm$) and will have a length of $l = 2000\ m$ and a slope $S_0 = 10^{-3}$. It will discharge into the Mediterranean Sea (no tide, density $\rho_{sw} = 1030\ kg/m^3$), whose free surface ($z_r = 0$) is located 1.2 m above the centerline axis of the sewer pipe. The elevation of the basement floor in the house is $z_b = 2.0\ m$ above the seawater level. The sewage can be assumed to have the characteristics of water ($\rho = 1000\ kg/m^3$, $\nu = 10^{-6}\ m^2/s$). The available pipe diameters are 40 cm, 45 cm, 50 cm, 55 cm, 60 cm, 65 cm, 70 cm, 75 cm, and 80 cm.

For a discharge in the sewer of $Q = 0.20\ m^3/s$, determine the optimal pipe diameter to avoid flooding in the basement of your house.

Problem 3

For each of the three pipes in the system sketched in Figure 3 determine the flow rate and the direction of flow. The connections between the pipes and the large tanks are all very well-rounded. Neglect the minor headloss at the junction E.

The lengths and diameters of the pipes are: $L_1 = L_2 = 5\ km$, $L_3 = 3\ km$, $D_1 = 800\ mm$, $D_2 = 400\ mm$, $D_3 = 500\ mm$. The roughness of all pipes is $1\ mm$.

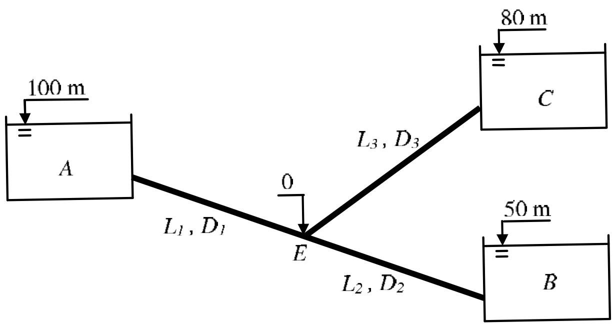


Figure 3: System of pipes in Problem 3.

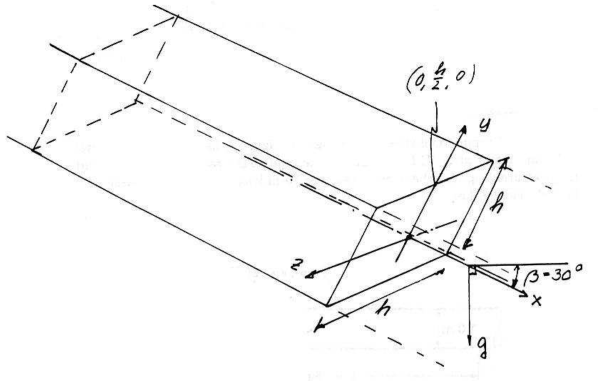


Figure 4: Square conduit in Problem 4.

Problem 4

Figure 4 shows a piece of very long square conduit, of side length $h = 0.30\ m$, carrying water (through the whole cross-section) at a flow rate $Q = 0.045\ m^3/s$. The centerline of the conduit, the x-axis, is inclined at an angle of $\beta = 30°$ to horizontal in the direction of the flow. Gravity acts in the x-y plane.

a) Determine the average velocity in the conduit.

b) Determine the hydraulic radius of the conduit.

c) Is the flow in the conduit laminar or turbulent? (Justify your answer).

d) We want to determine the roughness of this conduit. To this end, we place two piezometers separated by a distance of $10\ m$ along the conduit. For the given flowrate, the difference of water elevation between the two piezometers is $8.5\ mm$. With this information, estimate the roughness ϵ of the conduit.

e) For the conditions described in **d**, what is the average shear stress in the conduit walls?

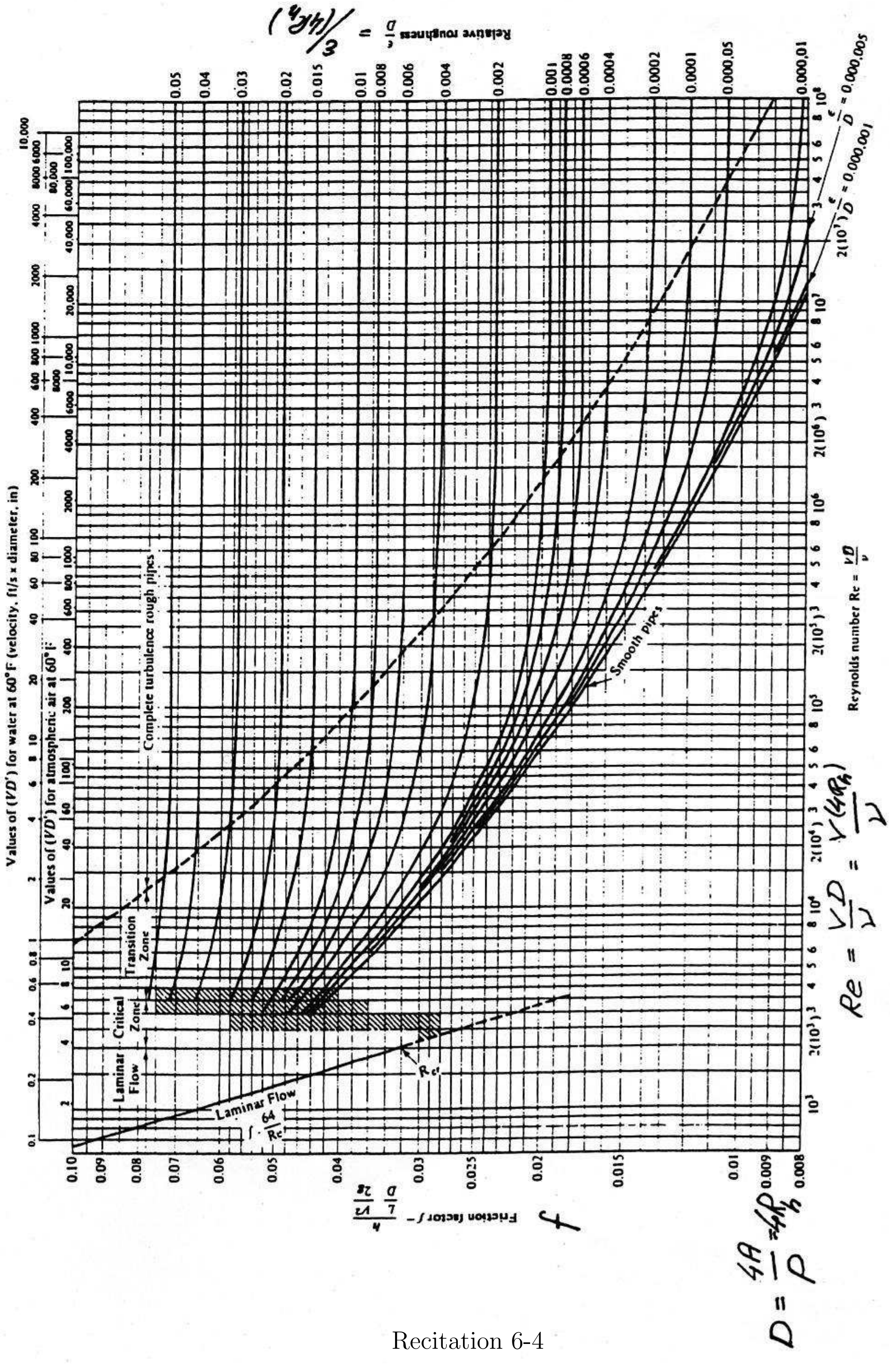
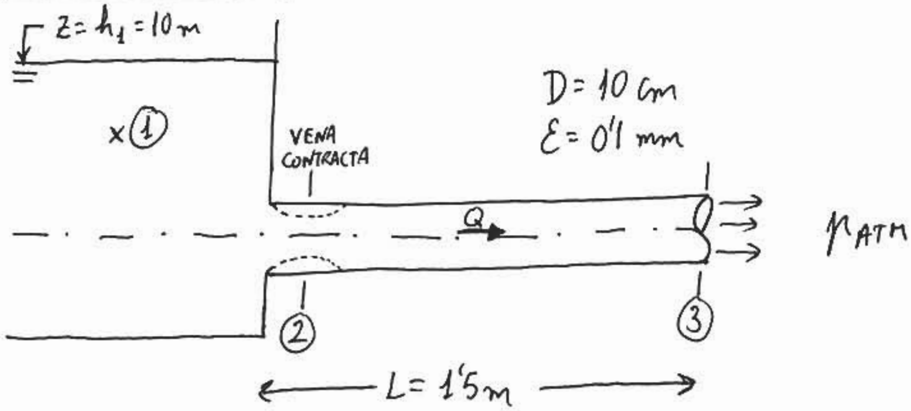


Fig. 6.13 The Moody chart for pipe friction with smooth and rough walls. This chart is identical to Eq. (6.64) for turbulent flow. (*From Ref. 8, by permission of the ASME.*)

RECITATION 6 - SOLUTIONS

- PROBLEM N° 1:



To find $Q = V_p A_p$, we apply conservation of energy between the reservoir (1) and the pipe section right before the outflow (3):

$$H_1 = H_3 + \sum \Delta H_L$$

$$H_1 = \underbrace{z_1 + \frac{p_1}{\rho g}}_{= h_1} + \underbrace{\frac{V_1^2}{2g}}_{= 0 \text{ (large reservoir)}} = h_1$$

$$H_3 = z_3 + \frac{p_3}{\rho g} + \frac{V_3^2}{2g} = h_p + 0 + \frac{V_p^2}{2g}, \text{ since } p_3 = p_{ATM} = 0 \text{ (gauge)}$$

$$\sum \Delta H_L = \underbrace{\Delta H_f}_{\text{FRICTION LOSSES}} + \underbrace{\sum \Delta H_m}_{\text{MINOR LOSSES}}$$

- Friction losses: Re is unknown → Assume rough turbulent flow; $\varepsilon / D = \frac{10^{-4}}{0'1} = 10^{-3}$ } MOODY ⟹ $f = 0'0197$

$$\Delta H_f = f \frac{L}{D} \frac{V_p^2}{2g}$$

- Minor losses: $\sum \Delta H_m = K_{L_2} \frac{V_p^2}{2g}$; $K_{L_2} = 0'5$ (SHARP EDGED INFLOW)

Therefore, energy conservation yields:

$$\underbrace{h_1}_{H_1} = \underbrace{h_p + \frac{V_p^2}{2g}}_{H_3} + \Big(\underset{\uparrow \atop \Delta H_f}{f \frac{L}{D}} + \underset{\uparrow \atop \Delta H_m}{K_{L_2}} \Big) \frac{V_p^2}{2g} \qquad (1)$$

where the only unknown, V_p, is determined to be $V_p = 4'05$ m/s.

But we need to check our assumption:

$$Re = \frac{V_p D}{\nu} = \frac{4'05 \cdot 0'1}{10^{-6}} = 4'05 \cdot 10^5 \quad \varepsilon/D = 10^{-3} \Big\} \xrightarrow{\text{SEE MOODY}} \text{IN THE TRANSITION ZONE!} \quad f = 0'0205$$

Since our hypothesis of rough turbulent flow was not correct, we should iterate to refine the result. We go back to (1) with the new value of f and get $V_p = 4'03\ m/s$, which is almost the same as before, so we don't need to keep iterating.

Thus, $V_p = 4'03\ m/s$, $Q = A_p V_p = \frac{\pi D^2}{4} V_p = 3'17 \cdot 10^{-2}\ m^3/s$

b) From continuity, $V_\upsilon = \frac{A_p}{A_\upsilon} V_p = \frac{A_p}{C_\upsilon A_p} V_p = \frac{V_p}{C_\upsilon} = \frac{V_p}{0'61} = 6'61\ m/s$

Since there is no headloss between 1 and 2,

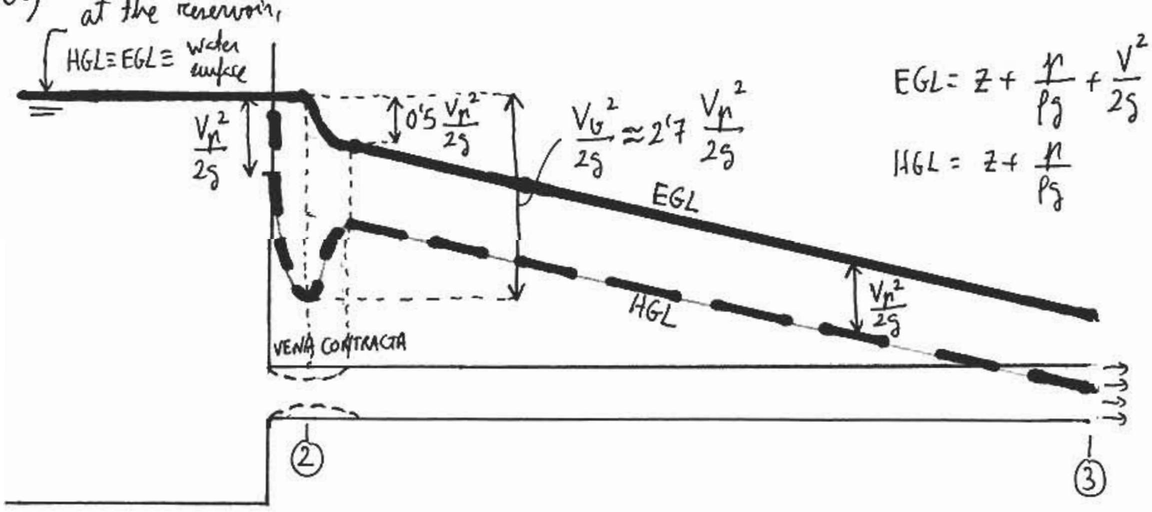
$$H_1 = H_2 \Rightarrow h_1 = h_p + \frac{p_\upsilon}{\rho g} + \frac{V_\upsilon^2}{2g} \Rightarrow p_\upsilon = -7'1\ kPa \text{ (gauge)}$$

Should we worry about cavitation? No, because $p_{\upsilon,ABS} > p_{VAPOR}$ (check it yourself)

c) $V_{orifice} = \sqrt{2g(h_1 - h_p)} = 5'42\ m/s$ (at vena contracta) $< 6'61\ m/s$, because

$p_{\upsilon,ORIFICE} = p_{ATM} = 0 > p_\upsilon = -7'1\ kPa$.

d)

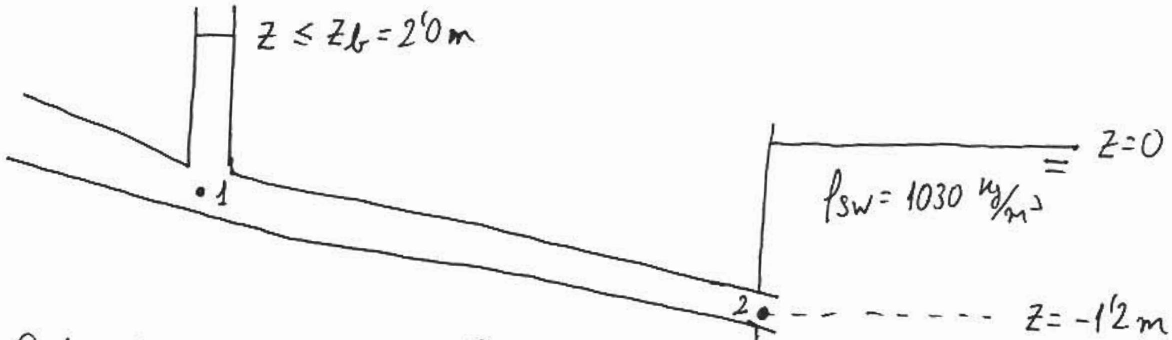


NOTES:

EGL must always be decreasing in the direction of the flow. Except at singular points (where there are minor losses), its slope is given by $-\Delta H_f / L$

HGL is always $\frac{V^2}{2g}$ below EGL, where V is the local average velocity.

- PROBLEM N.º 2:



Continuity: $V_1 = V_2 = V = \dfrac{Q}{A}$

Energy conservation between 1 and 2:

$$H_1 = H_2 + \sum \Delta H_\ell$$

$$H_1 \leq \underbrace{z_1 + \frac{p_1}{\rho g}}_{\leq z_b \text{ to avoid flooding}} + \frac{V_1^2}{2g} \leq 2 + \frac{V^2}{2g} \quad (\text{S.I.})$$

$$H_2 = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} = -1'2 + \frac{\rho_{sw}\, g\, 1'2}{\rho g} + \frac{V^2}{2g} =$$

$$= -1'2 + 1'03 \cdot 1'2 + \frac{V^2}{2g} = 0'036 + \frac{V^2}{2g} \quad (\text{S.I.})$$

$$\sum \Delta H_\ell = \underbrace{\cancel{\sum \Delta H_m}}_{\substack{0 \\ \text{(No minor losses} \\ \text{between 1 and 2)}}} + \Delta H_f = f \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{Q^2}{2g\left(\frac{\pi D^2}{4}\right)^2} =$$

$$= 6'617 \frac{f}{D^5} \quad (\text{S.I.})$$

Therefore, to avoid flooding:

$$2 + \frac{V^2}{2g} \geq H_1 = H_2 + \sum \Delta H_\ell = 0'036 + \frac{V^2}{2g} + \underbrace{6'617 \frac{f}{D^5}}_{\Delta H_f} \Rightarrow$$

$$\Rightarrow \Delta H_f = 6'617 \frac{f}{D^5} \leq 1'964 \quad (\text{S.I.}) \qquad (1)$$

We have to determine the optimal (smaller, i.e., cheaper) D that satisfies (1). Since in (1) $D \propto f^{-1/5}$ (i.e., D is not very sensitive to the value of f), we can obtain a reasonably good guess by taking $f \simeq 0'02$. With this,

$$\left.\begin{array}{l} 6'617 \dfrac{f}{D^5} = 1'964 \\ f \simeq 0'02 \end{array}\right\} \Rightarrow D \simeq 0'58 \text{ m}$$

<u>Try D = 60 cm</u>

$$D = 0'60 \text{ m} \Rightarrow V = 0'707 \text{ m/s} \Rightarrow \left.\begin{array}{l} Re = 4'2 \cdot 10^5 \\ \varepsilon/D = 3'3 \cdot 10^{-3} \end{array}\right\} \xrightarrow{\text{MOODY}} f = 0'0274 \Rightarrow$$

$$\Rightarrow \Delta H_f = 6'617 \frac{f}{D^5} = 2'33 \text{ m} > 1'964 \text{ m} \Rightarrow \text{TOO MUCH HEADLOSS} \Rightarrow$$

$\Rightarrow$ NEED A LARGER D

<u>Try D = 65 cm</u>

$$D = 0'65 \text{ m} \Rightarrow V = 0'603 \text{ m/s} \Rightarrow \left.\begin{array}{l} Re = 3'9 \cdot 10^5 \\ \varepsilon/D = 3'1 \cdot 10^{-3} \end{array}\right\} \xrightarrow{\text{MOODY}} f = 0'0268 \Rightarrow$$

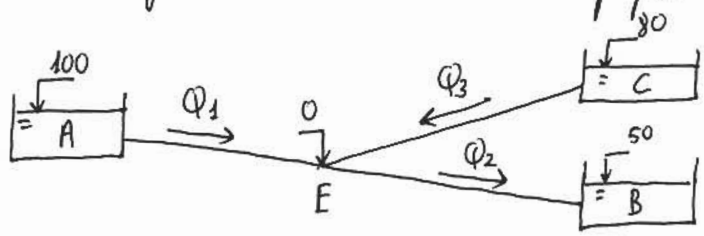
$$\Rightarrow \Delta H_f = 6'617 \frac{f}{D^5} = 1'53 \text{ m} < 1'964 \text{ m} \Rightarrow \text{OK}$$

Therefore, <u>D = 65 cm</u> is the smallest diameter that does the job.

Note: Given the elevation of the pipe outflow, $z = -1'2$ m, (which is relevant because we are discharging into seawater and would not be if we were discharging into fresh water ... think why), the value of S_0 is irrelevant.

- PROBLEM N° 3 :

Assume a flow direction in each pipe:



Assuming rough turbulent flow in all pipes, we calculate the friction factor for each pipe:

	L (m)	D (m)	A (m²)	ε (mm)	ε/D	f (R.T. flow)
PIPE 1	5000	0'8	0'502	1	0'00125	0'021
PIPE 2	5000	0'4	0'126	1	0'0025	0'025
PIPE 3	5000	0'5	0'196	1	0'002	0'0235

Cons. energy between A and E: $100 = H_E + f_1 \frac{L_1}{D_1} \frac{Q_1^2}{2gA_1^2}$ (s.i.) (1)

" " " E and B: $H_E = 50 + \left(f_2 \frac{L_2}{D_2} + 1\right) \frac{Q_2^2}{2gA_2^2}$ (s.i.) (2)

" " " C and E: $80 = H_E + f_3 \frac{L_3}{D_3} \frac{Q_3^2}{2gA_3^2}$ (s.i.) (3)

Continuity at E : $Q_1 + Q_3 = Q_2$ (4)

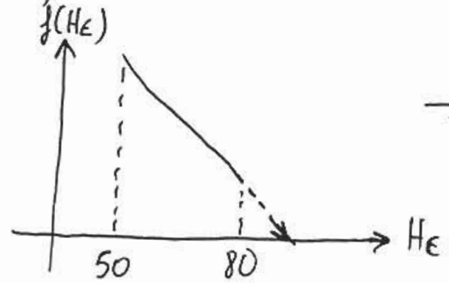
Note: $\Delta H_{m,OUTFLOW} = 0'5 \frac{V^2}{2g}$, $\Delta H_{m,INFLOW} \approx 0$ b/c "very well-rounded".

(1) $\Rightarrow Q_1 = \sqrt{\frac{100 - H_E}{26'57}}$; (2) $\Rightarrow Q_2 = \sqrt{\frac{H_E - 50}{1007'5}}$; (3) $\Rightarrow Q_3 = \sqrt{\frac{80 - H_E}{312'1}}$

Substituting in (4)

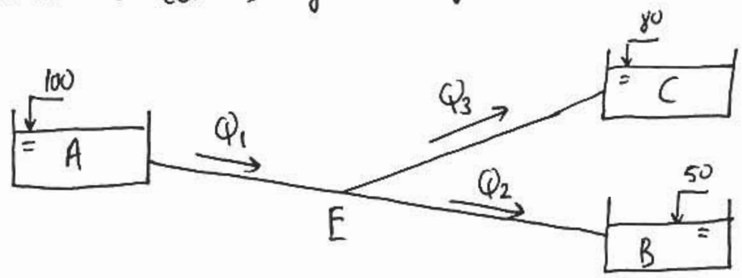
$$\underbrace{\sqrt{\frac{100 - H_E}{26'57}} + \sqrt{\frac{80 - H_E}{312'1}} - \sqrt{\frac{H_E - 50}{1007'5}}}_{f(H_E)} = 0$$

Represent $f(H_E)$:
(use calculator or computer)



$\rightarrow$ Suggesting $H_E > 80 \Rightarrow$
$\Rightarrow$ flow in Q_3 is in the opposite direction.

Correct directions of the flow:



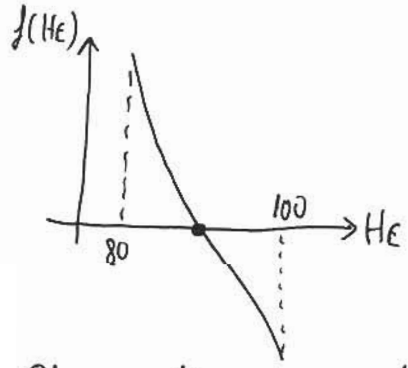
Assuming rough turbulent flow, the values of f remain the same. Equations (1) and (2) remain unchanged, while equations (3) and (4) now read:

$$H_E = 80 + \left(f_3 \frac{L_3}{D_3} + 1\right) \frac{Q_3^2}{2gA_3^2} \qquad (3) \Rightarrow Q_3 = \sqrt{\frac{H_E - 80}{313'4}}$$

$$Q_1 = Q_2 + Q_3 \qquad (4)$$

Therefore

$$(4) \rightarrow \underbrace{\sqrt{\frac{100 - H_E}{26'57}} - \sqrt{\frac{H_E - 50}{1007'5}} - \sqrt{\frac{H_E - 80}{313'4}}}_{f(H_E)} = 0$$



$$\Rightarrow H_E = 95'07\ m \rightarrow \begin{cases} Q_1 = 0'431\ m^3/s \\ Q_2 = 0'212\ m^3/s \\ Q_3 = 0'219\ m^3/s \end{cases}$$

Check the assumption of R.T. flow: (Use MOODY DIAGRAM)

$Re_1 = 6'9 \cdot 10^5$, $\frac{\varepsilon_1}{D_1} = 0'00125$ → Close enough to R.T. flow ✓

$Re_2 = 6'7 \cdot 10^5$, $\frac{\varepsilon_2}{D_2} = 0'0025$ → R.T. flow ✓

$Re_3 = 5'6 \cdot 10^5$, $\frac{\varepsilon_3}{D_3} = 0'002$ → Close enough to R.T. flow ✓

- PROBLEM N° 4:

a) A = cross-sectional area $= h^2 = 0'09\ m^2$

$V = Q/A = 0'045/0'09 = \underline{\underline{0'5\ m/s}}$

b) P = wetted perimeter $= 4h = 1'20\ m$

$R_h = A/P = 0'09/1'20 = \underline{\underline{0'075\ m}}$

c) $Re = \dfrac{V\cdot(4R_h)}{\nu} = \dfrac{0'5\cdot(4\cdot 0'3)}{10^{-6}} = 1'5\cdot 10^5 \gg Re_{critical} \approx 2\cdot 10^3 \Rightarrow$

$\Rightarrow$ TURBULENT FLOW

d) Since the conduit has a constant cross-sectional area, V is constant (by continuity). Therefore, the difference in piezometric head between two points, $\Delta\left(z+\frac{p}{\rho g}\right)$, is equal to the difference in total head, $\Delta\left(z+\frac{p}{\rho g}+\frac{V^2}{2g}\right)$, i.e., the headloss:

$$\underset{\substack{\uparrow \\ \text{no minor losses}}}{\Delta H = \Delta H_f} = 8'5\cdot 10^{-3}\ m = f\,\frac{L}{(4R_h)}\,\frac{V^2}{2g} = f\cdot\frac{10}{4\cdot 0'075}\cdot\frac{0'5^2}{2\cdot 9'8} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} f \approx 0'02 \\ Re = 1'5\cdot 10^5 \end{array}\right\} \xrightarrow{\text{MOODY}} \frac{\varepsilon}{4R_h} \approx 6\cdot 10^{-4} \Rightarrow \underline{\underline{\varepsilon \approx 1'8\cdot 10^{-4}\ m}}$$

(0'18 mm)

e) $\underline{\underline{\tau_0}} = \frac{1}{8}\,\rho f V^2 = \frac{1}{8}\cdot 1000\cdot 0'02\cdot 0'5^2 = \underline{\underline{0'625\ N/m^2}}$