1.061 / 1.61 Transport Processes in the Environment
Fall 2008

Solution 1

From Fick's Law (Eq 3 in Chap 1)

$$q_z = -DA\frac{\partial C}{\partial Z} \approx -DA\frac{C\,(z=10\,cm) - C\,(z=0\,cm)}{(10-0)\,\,cm}$$

$$= -\left(10^{-5}\,cm^2 s^{-1}\right)\left(1\,cm^2\right)\left(\frac{0 - 100\,gcm^{-3}}{10\,cm}\right)$$

$$= +10^{-4}\,g/s$$

"+" z-direction is upward

Solution 2

Temperature:

As temperature of the fluid increases, the water molecules move about more vigorously. The collision of water molecules with molecules of the diffusing species causes the species molecules to move about as well. As the strength and number of collisions increase with temperature, the mean distance (Δx) moved per time (Δt) increases. Therefore, based on the random walk model, we expect D to increase with temperature. $D \sim \frac{\Delta x^2}{\Delta t}$.

Molecule size:

Bigger molecules will be less easily moved by collisions from surrounding water molecules, especially if species molecule is much bigger than water molecule, and $\therefore$ will experience multiple, simultaneous, uncoordinated collisions. Thus, the mean step size, Δx, per time is <u>smaller</u> for bigger molecules. And we expect $D \sim \frac{\Delta x^2}{\Delta t}$ to be smaller too.

Solution 3

Question 1

t(day)	t(sec)	C_{max}	$0.6\,C_{max}$	$\sigma(cm)$
1	86400	13.5	8.1	3.0
4	345600	6.8	4.1	6.0

Assuming you released the dye in a very thin horizontal layer, such that initially $\sigma = 0$, then:

$$\sigma = \sqrt{2Dt} \Rightarrow D = \frac{\sigma^2}{2t}$$

Thus, our two estimates of D are:

$$D_1 = (3.0\,cm)^2\,/2\,(86400\,s) = 5.2 \times 10^{-5}\,cm^2 s^{-1}.$$

and

$$D_2 = (6.0\,cm)^2 / 2\,(345600\,s) = 5.2 \times 10^{-5}\,cm^2 s^{-1}.$$

Therefore, the estimated diffusion coefficient of the dye is $\mathbf{5.2 \times 10^{-5}\,cm^2 s^{-1}}$.

Question 2

We must consider the boundaries of the tank when the cloud width (4σ) is 40 cm. That is, when:

$$4\,(2Dt)^{1/2} = 40\,cm$$

(i.e. when $Dt = 50\,cm^2$)

Using our value of D above, the boundaries of the tank become important when

$$t = 50\,cm^2 / 5.2 \times 10^{-5}\,cm^2 s^{-1} = 9.6 \times 10^5\,s = 11.1\,days.$$

Solution 4

(i) **Air**

The concentration we are interested in is given by:

$$\frac{C}{\rho_{air}} = 1\,ppm \Rightarrow C = \frac{1.23\,kgm^{-3}}{10^6} = 1.23 \times 10^{-9}\,gcm^{-3}$$

The normal distribution of concentration is:

$$C = \frac{M}{A\sqrt{4\pi Dt}} exp\left(-\frac{x^2}{4Dt}\right)$$

Therefore, for a mass fraction of 1 ppm,

$$1.23 \times 10^{-9} = \frac{9.599 \times 10^{-4}}{\sqrt{t}} exp\left(-\frac{4.464 \times 10^3}{t}\right) \Rightarrow exp\left(\frac{4.464 \times 10^3}{t}\right) = \frac{7.804 \times 10^4}{\sqrt{t}}$$

Solving this equation by trial and error gives us the solution that $C = 1\,ppm$ at $x = 50\,cm$ when $\mathbf{t = 423\ s}$.

(ii) **Water**

Following the same solution method,

$$\frac{C}{\rho_{water}} = 1\,ppm \Rightarrow C = \frac{1000\,kgm^{-3}}{10^6} = 1.00 \times 10^{-6}\,gcm^{-3}$$

Therefore, for a mass fraction of 1 ppm,

$$1.00 \times 10^{-6}\, gcm^{-3} = \frac{0.1\,g}{\left(\pi\,(5\,cm)^2\right)\sqrt{4\pi\,(1.71 \times 10^{-5}\,cm^2 s^{-1})\,t}}\,exp\left(-\frac{(50\,cm)^2}{4\,(1.71 \times 10^{-5}\,cm^2 s^{-1})\,t}\right)$$

With t in seconds, the dimensions cancel, and we are left with:

$$1.00 \times 10^{-6} = \frac{8.686 \times 10^{-2}}{\sqrt{t}}\,exp\left(-\frac{3.655 \times 10^7}{t}\right) \Rightarrow exp\left(\frac{3.655 \times 10^7}{t}\right) = \frac{8.686 \times 10^4}{\sqrt{t}}$$

Solving this equation by trial and error gives us the solution that C = 1 ppm at x = 50 cm when $\mathbf{t = 1.12 \times 10^7\,s}$(= **130 days**), much longer than when the fluid is air.

Solution 5

A) To answer this question, you must define the edge of the diffusing patch of fuel. By convention, this is typically taken as $\pm 2\sigma$ from the center of the patch. By this definition, the edge of the fuel patch will reach your house

$$when\ 2\sigma = 50\,m$$
$$or\ 2\sqrt{2Dt} = 50\,m.$$

Solving for $t = 31,250\,s = 8.7\,hr$.

B) At the above time (31, 250 s), the concentration at your house is, from EQ. 7,

$$C(x = -2\sigma, t = 31250\,s) = \frac{M}{A\sqrt{4\pi Dt}}\,exp\left\{-\frac{(-2\sigma)^2}{2\sigma^2}\right\}$$
$$= \frac{1\,kg}{(5\,m \cdot 1\,m)\sqrt{4\pi(0.01\,m^2/s)(31250\,s)}}\,exp\left\{-2\right\}$$
$$= 4.3 \times 10^{-4}\,kgm^{-3} = 0.43\,gm^{-3}$$

C) The maximum concentration at $x = -50\,m$ can be found by setting $\frac{\partial}{\partial t}\,[C(x = -50\,m, t)] = 0$. As this is a difficult derivative, a graphical solution is easier and more instructive.

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C (x = -50 m) vs t

MAXIMUM CONCENTRATION
OCCURS ~ 120000 s (≡ 33 HOURS)
AFTER SPILL