1.101 Structures Lab.
Fall 2005
Week 2 - Beam Bending

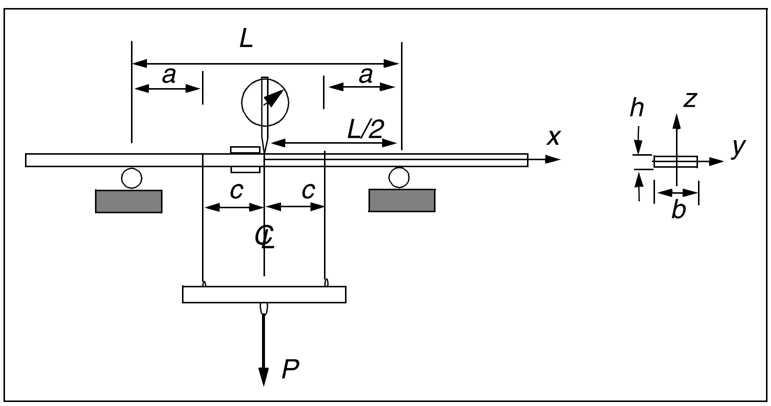
Our objective is to compare engineering beam theory with experiment.

We will subject a “simply supported” beam to “pure bending” (over a portion of its span) and measure the strain (stress) at the top and bottom surfaces and the mid-span, vertical displacement. Once again strain gages will be used to measure the former and a dial gage indicator employed to measure the latter. We will restrict our loading to the elastic range.

You will use the pages that follow to report your results.

The test set-up and procedure.

We use the same specimen used in last week’s tension test only now we apply the load transversely. Y**ou will discover that it takes very little load *P*, on the order of a pound or two to produce a significant and visible, vertical displacement.** We will also test a second specimen of double the thickness.



The span *L* should be no more than 10 inches. The dial gage is to measure the displacement at mid-span. The strain gages, on the other hand can be located off the center line as shown for the region $-c < x < +c$ will experience a constant bending moment. (See Appendix A for the proof of this statement). Hence the strains at any point within that region at the top and bottom of the beam will not change with position x.

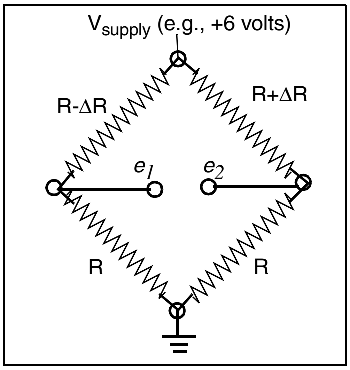
Locate the dial gage at mid-span, making sure it is supported as rigidly as possible. You will note that the dial will turn “backwards” as the beam deflects downward. Make sure that the dial gage will not run out of play as the beam deflects.

The load is applied using the light-weight yoke suspended (by kevlar string) from two points located symmetrical at x = +/- c. **Make sure that the strings are located symmetrically with respect to the center line.** The distance a should be as small as feasible but you are not to interfere with the strain gages. You will again use a gaged specimen for temperature compensation. This should rest on the roller supports, parallel and next to the loaded specimen. (Not shown in the figure).

In making a comparison with engineering beam theory, you will need to compute the value of the geometric parameter, ***I***, the “second moment of area” of the cross section-often called the moment of inertia of the cross section. (See Appendix A). Measure the width, *b*, and thickness, *h*.

b = ____________. ***h =*** _______________

I $= bh^3/12$ = ________________ *(units)*

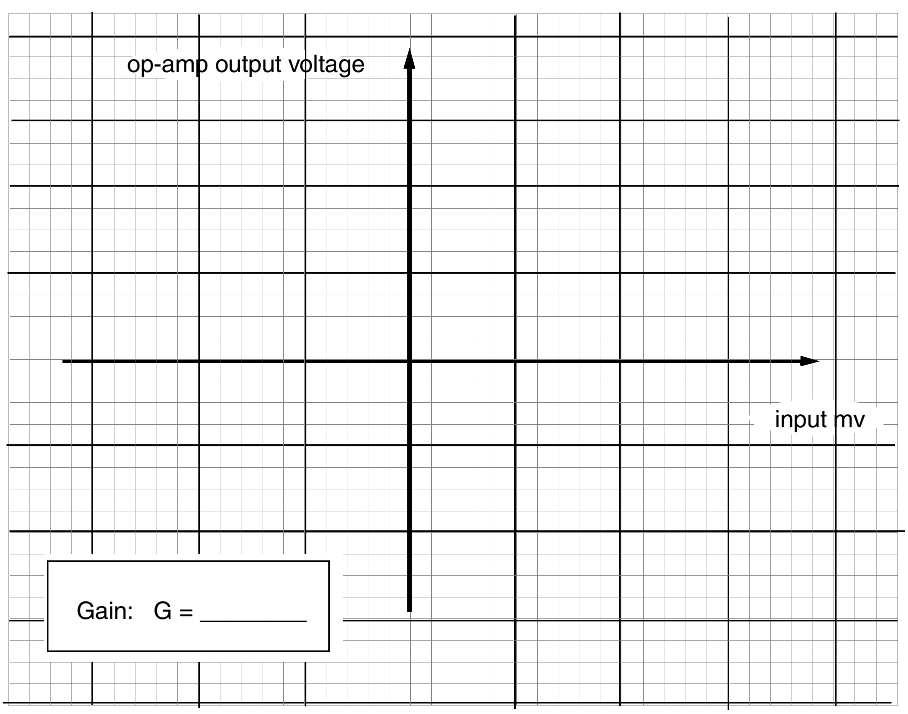


The placement of the gages in the Wheatstone bridge is different from their placement in the tension test. There, in last week's test, the two gages were both in tension. Now, with the loading shown, the gage at the bottom of the beam will be in tension, the top gage in compression. The gages should be located as shown in the figure at the left in order to maximize the differential voltage, e_1-e_2. See appendix B for analysis of this circuit.

Follow the same procedure you used last week to determine the gain of the op-amp, replacing one of the gages with the 2kohm trim pot in parallel with a 360 ohm resistor.

Op-amp Gain

input = e_1-e_2	output
mv	volts
[+/-??]=	[+/-??]=



When you are finished estimating the gain, disconnect the 2 kilo-ohm trim pot and replace the strain gage you had removed from the bridge at the outset.

The bending test

With all four gages now connected in the bridge circuit, adjust the 10 kohm pot to bring the output to zero, within a few millivolts or tenths of millivolts. Make sure you have positioned the dial gage indicator prior to this step since the dial gage will tend to load down the beam.

Attach the light weight yoke to the strings hanging down, through the table. You will add the small blue or better yet, the brown weights, to increment the load. **Do not overload**: You need to estimate how much weight you can add and yet stay below 20% of the yield stress. (See Appendix A)

You are to take two sets of readings with each of the two specimens. For each specimen, set the supports at two different span lengths, e.g, 10 inches, and 8 inches.

Use the set of forms below to take data and present your numerical results. (Or append a spread sheet).

TEST 1. Beam Bending Data ***h*** **= 1/16 in;** ***L*** **= ____________; c = _________.**

Load	op-amp output	Dial gage ind.	Displacement	Stress
[units]	[units]	[units]	[units]	[units]
+/-	+/-	+/-	+/-	+/-
Column 1	Column 2	Column 3	Column 4	Column 5

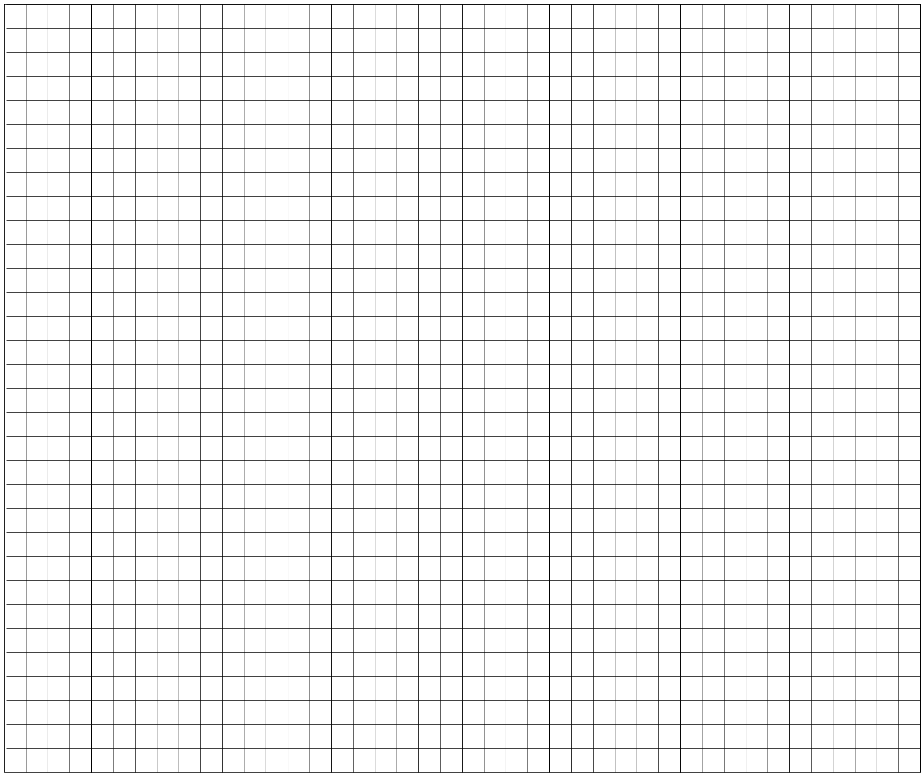
Columns 1 - 3 are measured data. Column 1 is the load; Column 2 is the op-amp output; Column 3 is the dial gage indicator reading.

Column 4, is the vertical displacement at the at the dial gage (positive downwards). Columns 5, is the stress at the top (compression) and at the bottom (tension) of the beam, computed from the product of the strain and the elastic modulus, *E*. The strain, is obtained from the sequence of relationships that relate the strain to the fractional change in resistance to the output of the bridge to the output of the op-amp as in last week's tension test.

Make two plots: One showing how the load (ordinate axis) varies with the displacement (abscissa); a second, showing how the stress varies with the load (abscissa). [Generally, this is the way the results are plotted]. Show the results of engineering beam theory on these same plots.

TEST 1. *h = 1/16   L =   c =*

Load P



Mid-span Displacement

Stress σ

Load P

Make a second test, same specimen, but move the supporting rollers inward to reduce the length of span *L*. keeping the distance *c* the same as in the first test.

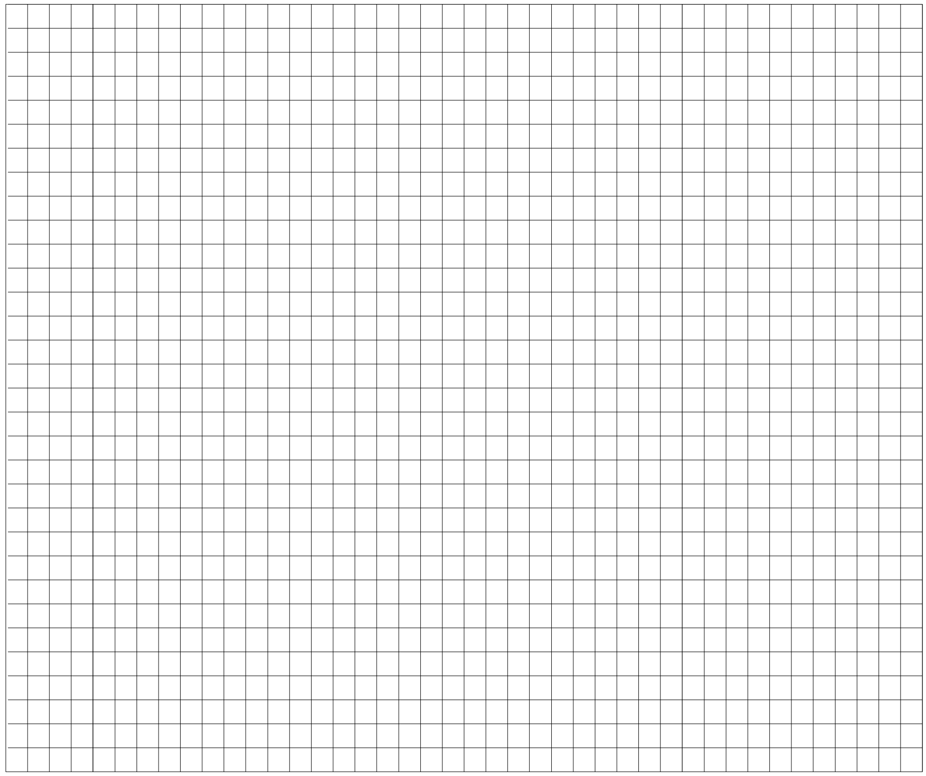
TEST 2. Beam Bending Data thickness *h* = 1/16 in; *L* = ____________; c = _________.

Load	op-amp output	Dial gage ind.	Displacement	Stress
[units]	[units]	[units]	[units]	[units]
+/-	+/-	+/-	+/-	+/-
Column 1	Column 2	Column 3	Column 4	Column 5

Again plot the results and compare with engineering beam theory.

TEST 2. *h = 1/16* *L =* *c =*

Load P



Mid-span Displacement

Stress σ

Load P

Now subject the **thicker specimen** to bending in the same way, starting with a span length of something less than 10 inches.

TEST 3. Beam Bending Data thickness h = 1/8 in; L = ____________; c = _________.

Load	op-amp output	Dial gage ind.	Displacement	Stress
[units]	[units]	[units]	[units]	[units]
+/-	+/-	+/-	+/-	+/-
Column 1	Column 2	Column 3	Column 4	Column 5

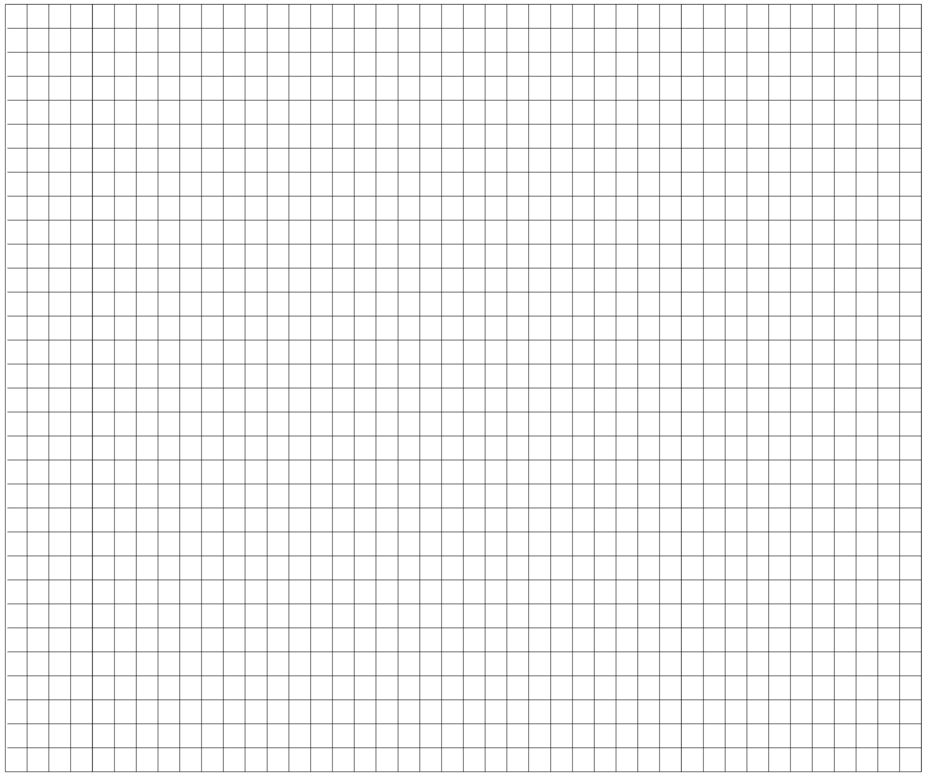
Again plot the results and compare with engineering beam theory. For this, you will need to compute the second moment of area of this thicker specimen;

b = ____________. ***h* =** _______________

$I = bh^3/12$ = ________________ *(units)*

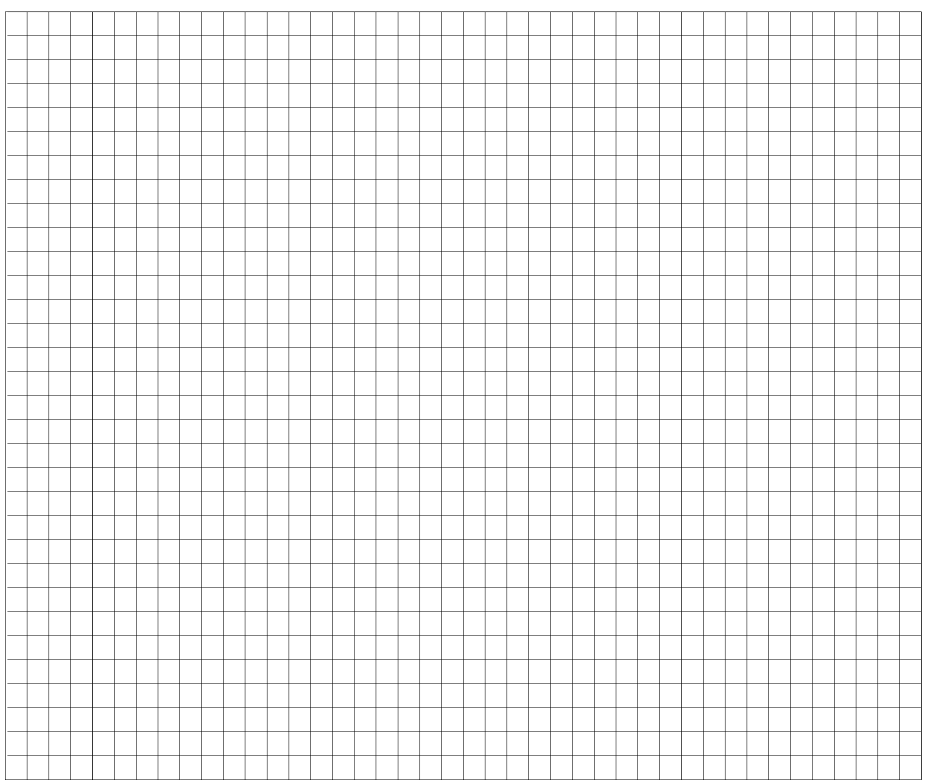
Test 3. *h = 1/8 in*;   *L*=_______________   *c* =

Load P



Mid-span Displacement

Stress σ



Load P

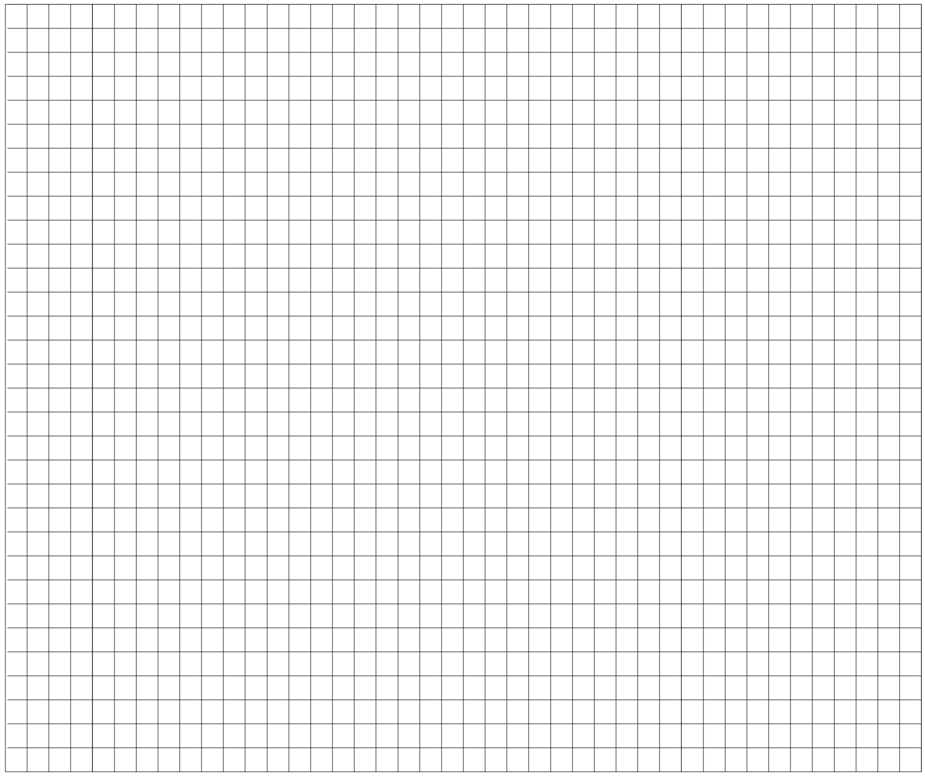
For a final test, reduce the span length.

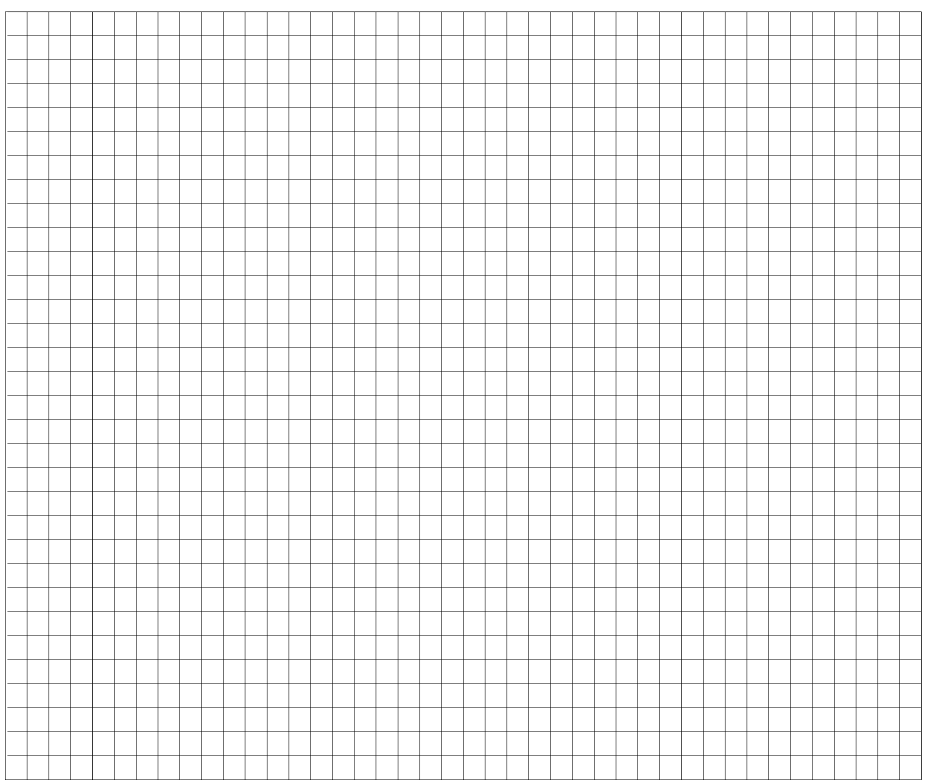
TEST 4. Beam Bending Data **thickness *h* = 1/8 in; *L* = ____________; *c* = _________.**

Load	op-amp output	Dial gage ind.	Displacement	Stress
[units]	[units]	[units]	[units]	[units]
+/-	+/-	+/-	+/-	+/-
Column 1	Column 2	Column 3	Column 4	Column 5

Again plot the results and compare with engineering beam theory.

Test 4. *h = 1/8 in;* *L=*_______________ c =

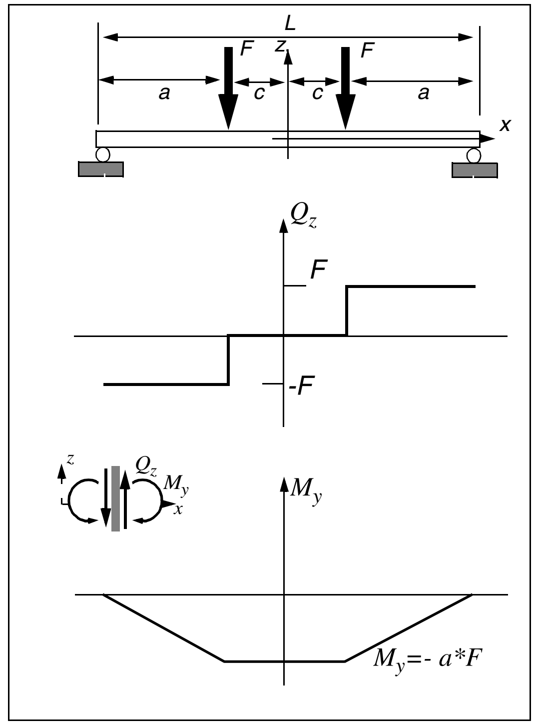




Appendix A

Engineering Beam Theory

For a beam loaded symmetrically with respect to mid-span as shown, the applied loads engender a “bending moment” distribution, $M_y(x)$ and a “shear force” distribution, $Q_z(x)$, over the length of the beam. The convention for positive force and moment, in accord with 1.050, is shown in the figure. Not shown are the two vertical reactions at the points of simply-support. Each is equal in magnitude to *F*.



Note that the shear force and bending moment distributions satisfy the equilibrium requirements:

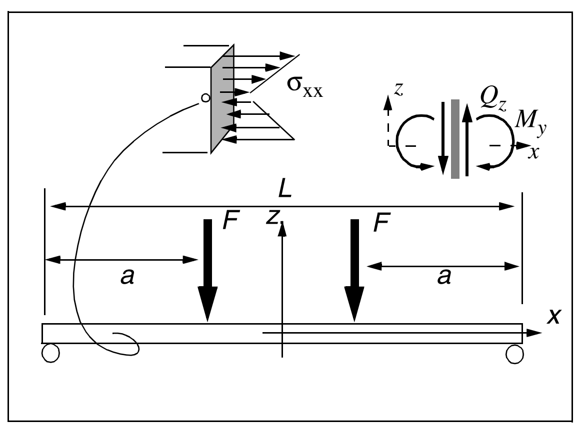
$$\frac{dQ_z}{dx} = 0$$

$$\frac{dM_y}{dx} - Q_z = 0$$

In writing the first of these, which guarantees force equilibrium in the vertical direction, we have neglected the (uniformly distributed) weight of the beam itself.

Note that over the mid-section of the beam, the shear force is zero and the bending moment constant. We say this section is in “pure bending”. The bending moment within the mid-section -c< x <+c, is the product of the distance *a* and the force *F* (as shown in class).

Engineering beam theory shows that the most significant stress is the normal stress component on an “x face”; σ_{xx} in the example at the right. It is related to the bending moment by

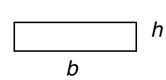


$$\sigma_{xx} = \frac{M_y \cdot z}{I}$$

In this equation, *z* is the distance from the “neutral axis” which, for a doubly symmetric beam, is at the center of the cross-section and I is the “moment of inertia” of the cross section

$$I = \int_A z^2 dA$$

For a rectangular cross-section of width *b* and height *h*, this is ***I = bh³/12***



The shear force engenders a shear stress, σ_{zx} which distributes over the face, directed in the vertical direction (not shown) but this is of a smaller order of magnitude. We will also neglect its effect on the displacement of the beam.

The extensional strain component, ε_{xx}, under the same set of assumptions, is simply proportional to the normal stress component, σ_{xx}.

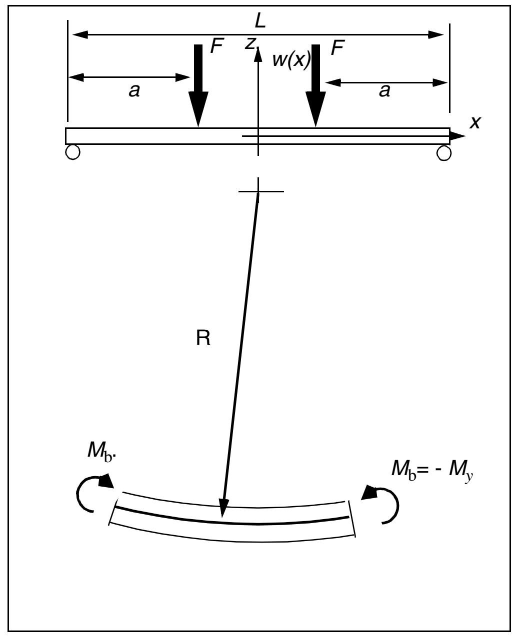
$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} = \frac{M_y \cdot z}{EI}$$

For our beam, with its doubly symmetric cross-section, the (compressive) strain at the top of the beam is equal in magnitude to the (tensile) strain at the bottom of the beam. This justifies our placement of the strain gages in the Wheatstone bridge.

From the geometry of deformation, the extensional strain is given by

$$\varepsilon_{xx} = -z/R$$

where R is the radius of curvature of the axis. $(1/R)$ is the curvature. The negative sign is necessay since we take the curvature, *1/R*, to be positive when the beam bends concave upwards.



From the last two relationships we see that the bending moment is related to the radius of curvature at any point along the span by

$$-M_y = M_b = (EI) \cdot \left(\frac{1}{R}\right)$$

(Please excuse the introduction of M_b).

The curvature, **for small deflections,** is related to the vertical displacement of the axis of the beam by, $\boldsymbol{(1/R) = d^2w/dx^2}$.

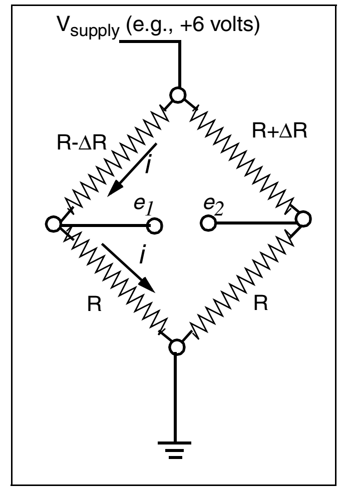
Finally, an integration of the differential equation obtained from the moment curvature relation gives, taking account of the boundary conditions that the displacement at the supports must vanish, for the mid-span deflection

$$w|_{midspan} = -\left(\frac{Fa}{24EI}\right) \cdot (3L^2 - 4a^2)$$

NOTE: *F* is one half of the load added to the chain below the table surface.

Appendix B. - The bridge circuit.

The “Wheatstone bridge” produces an output voltage proportional to the change in resistance of the active strain gages. This voltage signal, in turn, is input into an operational amplifier, which boosts the (dc) signal by a factor, the Gain factor. We analyze the bridge circuit anew since the placement of the gages in the bridge circuit differs from last week.



The two strain gages fixed to the specimen subject to loading are positioned as shown. The figure indicates that the active gage located top left is the gage fastened to the top surface of the beam. It is subject to compression, hence its resistance decreases. The active gage located top right in the figure is fastened to the bottom surface of the beam and experiences tension. Note: **It does not matter if you switch the location of the two active gages in the bridge circuit. This will just change the polarity of the voltage difference between e_1 and e_2.**

As before, we use two “in-active” gages for temperature compensation. We apply the circuit laws to determine the output voltage e_1 - e_2 as a function of ΔR/R.

Consider the current flow through the two resistors on the left side of the bridge. From the usual circuit laws, we have.

$$V_{supply} - e_1 = i \cdot (R - \Delta R)$$

which yields

$$e_1 - 0 = i \cdot (R)$$

$$e_1 = \frac{V_{supply}}{(2 - \Delta R / R)}$$

The same kind of analysis gives, for the current flow through the two resistors on the right:

$$e_2 = \frac{V_{supply}}{(2 + \Delta R / R)}$$

The output of the bridge, the voltage difference e_1 - e_2 is then

$$e_1 - e_2 = V_{supply} \cdot \left[\frac{(2 + \Delta R / R) - (2 - \Delta R / R)}{(2 + \Delta R / R) \cdot (2 - \Delta R / R)}\right] = \frac{2 \cdot (\Delta R / R)}{[4 - (\Delta R / R)^2]} V_{supply}$$

But the ratio $(\Delta R/R)^2$ is going to be much less than 1.0, so we neglect this ratio with respect to 4.0 in the denominator and obtain the same relationship as in our tension test:

$$\boxed{e_1 - e_2 = \frac{V_{supply}}{2}(\Delta R / R)}$$

The change in resistance is proportional to the strain, that is $\Delta R / R = F_{gage} \varepsilon$ where the gage factor is 2.07 So knowing the output voltage (difference) e_1 - e_2 we can use these last two equations to determine the strain, ε. The voltage (difference) e_1 - e_2 is obtained from the output of the op-amp, reduced by the Gain factor, i.e., e_1 - e_2 = (Op-amp output)/(Op-amp Gain)

Appendix C. - Notes on the use of a spreadsheet.

You are encouraged to make use of a spreadsheet in processing your data and presentation of your results. However, if you choose to do so, you must make sure you remain in control of your presentation and make clear all steps in your data reduction and analysis. Be careful to:

- Include only the number of significant figures you can justify;
- Label all columns;
- Make explicit on the spreadsheet itself, or in text that accompanies the spreadsheet, the relationships that relate one column to another.
- On a graph generated by the spreadsheet, label all axes, showing units as well as values at points along the axes that will enable a reader to easily read the plot;
- Do not rely upon color to differentiate among different plots on the same graph. (Unless you want to go to the expense of printing out the graph in color);
- In most cases, the spreadsheet reduction and analysis of data belongs in an appendix. The graph of results, on the other hand, goes in the results section of the main body of a report[1].

1. In the structure labs 1 and 2, you are not required to write a full report but only “fill in the blanks” of the handout. If you use a spreadsheet you may insert the spreadsheet at the appropriate place in the handout.Your design task will be documented differently - to be explained in class.