Structures - Experiment 3B
1.101 Sophomore Design - Fall 2006
Linear elastic behavior of a beam.

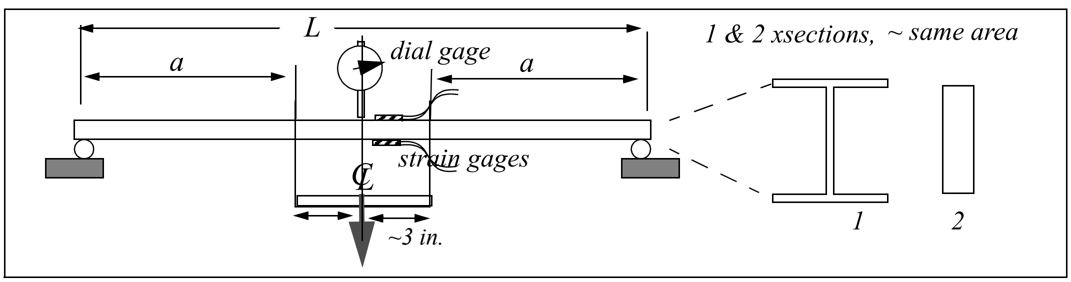
The objectives of this experiment are to experimentally study the linear elastic behavior of beams under four point bending - in particular.

- To compare the stiffness of the beam with the results of engineering beam theory.
- To measure the extensional strain in the top and bottom fibers of the beam and compare the strain/stress with the prediction of engineering beam theory.

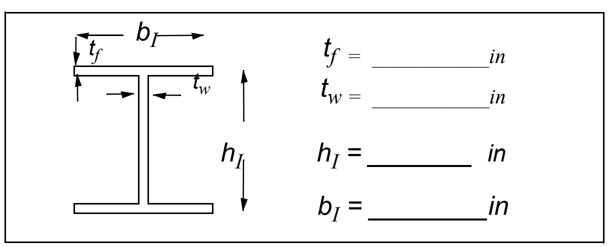
The beam specimen, made of Aluminum and 24 inches long, will be *simply supported* near its ends and loaded at two points symmetrically disposed with respect to midspan, This "four point bending" produces "pure bending" (no shear) over the midspan section. Two strain gages are mounted on the top and on the bottom near midspan, within the range of pure bending. A dial gage, at center span, is employed to measure deflection. The beam will be loaded using dead weights. You will compare bending strain/stress and beam stiffness obtained experimentally to engineering beam theory predictions.

Beam Specimen - Geometry

The figure below shows a beam specimen under four point bending. To the right we show the cross sections of the two Aluminum specimens. The material is 6061-T6.



You will test the "I" beam in the orientation, as indicated in the figure, "1". You will *not* test the beam with rectangular cross section .



Wait until after you have taken the load date before you measure all cross-section dimensions of the beam specimen. Fill in the blanks above. From these dimensions, we obtain the areas:

$Area_{Ibeam}$ = ______________ in^2

and the bending moment of inertia, the second moment of area about an axis into/out of the paper, are (See Appendix on beam bending).

I_{Ibeam} = ______________ in^4

Now mark off two points, three inches to the left and right of the no-lead end of the strain gages. Mark both the top and bottom flanges of the beam at these load points. The three inches need not be "exact" but the two load points should be symmetrically disposed with respect to the point you choose as center of the beam. This center point should not lie atop a strain gage but an eighth of an inch or so off the gage.

Position the dial gage probe on center, midway between the two load points without resting on the strain gage but as close as possible to the end of the gage away from the leads. Now adjust the end supports so that this point is truly midspan.

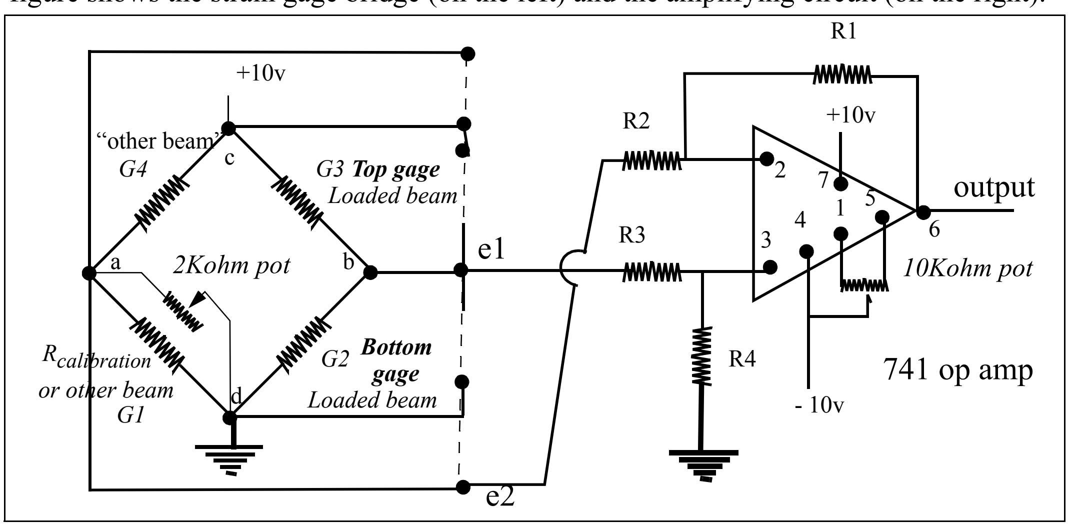
Loop the thin cables with eye hook ends over the beam at the two symmetrically disposed loading points, three inches off center. Connect the cables to the horizontal bar below the test bed with s hooks. Add the empty bucket. Let this be your "no load" condition. (Our interest is in the linear, behavior of the beam so what we designate as "no load" condition is arbitrary - within limits).

Measure the lengths, *a* and *L*.

L = ___________*in* *a* = ___________*in.*

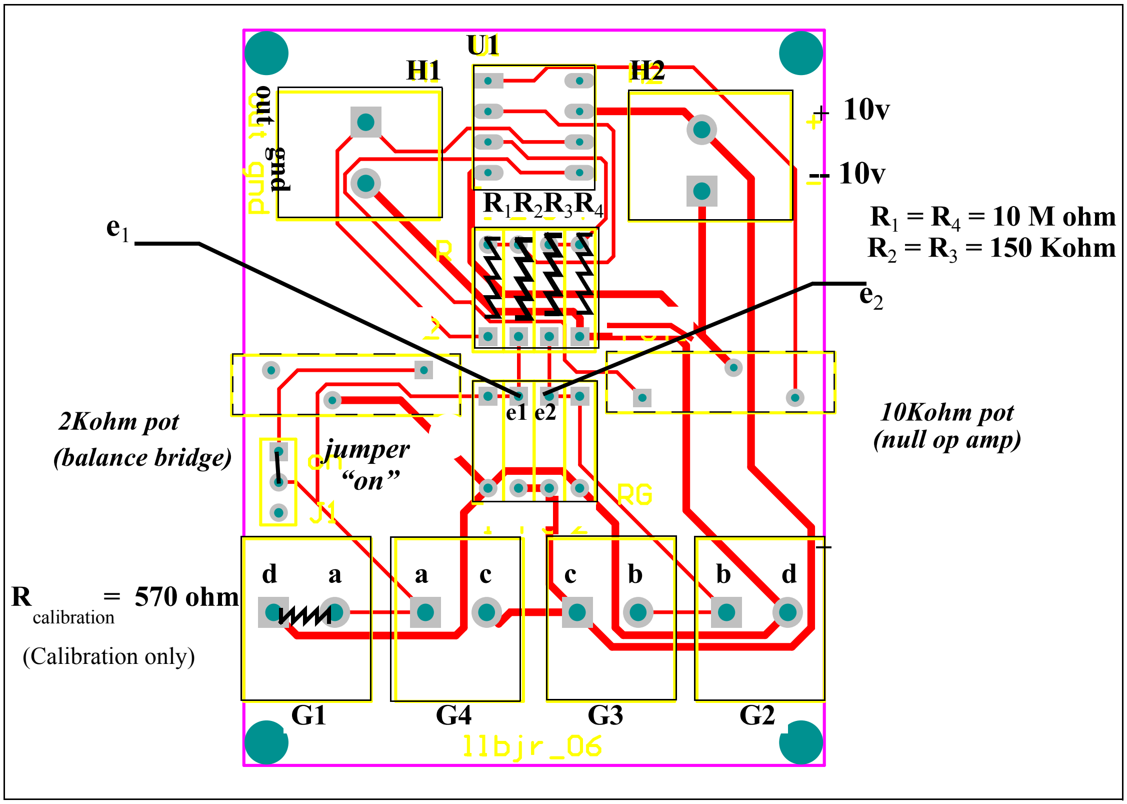
Connecting the signal conditioning circuit.

The figure shows the strain gage bridge (on the left) and the amplifying circuit (on the right).



The figure below is the corresponding printed circuit board layout.

Connect the ***top and bottom strain gages of the beam to be loaded*** to terminals *G3 and G2*. Connect o***ne of the gages of the other beam***, to terminal *G4*. Connect the ***calibration resistor*** to the terminal *G1*. Make sure the jumper is in the proper position so that the ***2 Kohm pot is connected*** across nodes *a* and *d*. (See circuit diagram).



With the power supply **disconnected** from the circuit board, set the +20v supply to +10v and verify that the - 20v supply is set to -10v (with the full tracking nob turned all the way clockwise). **Turn off the power supply**.

Connect the ***+10v*** and the ***-10v*** supply leads to terminal block *H2* as indicated in the figure. Make sure the positive goes to the positive terminal. Connect the ***common*** to the terminal block *H1* at the terminal marked "gnd", also as indicated in the figure.

After having an instructor check your connections, **turn on the power supply** and, with the Digital Multimeter, check the voltages at the strain gage input terminals nodes a, b, c and d on the terminal blocks *G1, G2, G3*, and *G4*. One should read +10v, node *c,* two others approximately +5v, nodes *a* and *b*, and the fourth should read ~0v, (ground) node *d*.

Now connect the leads of the voltmeter to measure the voltage difference of ***e1*** and ***e2***. You should see two bare wires sticking up from about the middle of the circuit board. (See diagram, last page). The polarity - which lead you attach to which wire - doesn't matter.

You should see some positive or negative voltage difference, different from zero - indicating that the bridge is “unbalanced”. Adjust the ***2 Kohm potentiometer*** setting using the small screwdriver. If you can not bring the difference of ***e1*** and ***e2*** to zero, see your instructor.

Now attach the 2nd voltmeter to measure the output of the op-amp, i.e., to the terminal block H1. This voltage will probably be different from zero. Proceed as follows.

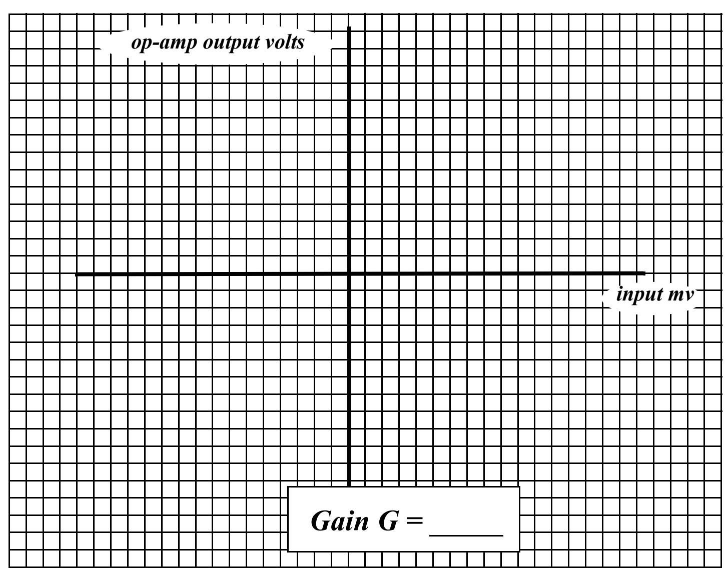
Measuring the op-amp Gain.

Reset the bridge balancing pot, the 2Kohm pot, so that the output of the bridge (which is the input to the amplifier) is on the order of 10 mv, then measure the output of the amplifier to obtain the gain.

Repeat this for a op-amp input, the difference of ***e1*** and ***e2,*** of approximately 20mv. Repeat for op-amp inputs of the same magnitude but of opposite sign. Record all of this data, 4, or 5 points, in the table and fit a straight line. The slope of this line is the amplification, the ***Gain***.

input *e1-e2* mv	**op-amp output volts**

NOTE: Plot output volts versus input millivolts. But the Gain should be the ratio of volts output to volts input (or millivolts to millivolts).



The bending test.

Press down lightly on the beam to verify setup. Op amp output should be on the order of 10 mv.

Now load the beam in increments of 6 pounds (the “big” weights), placing the weights carefully into the bucket. **Do not exceed 48 pounds.** Take data during unloading as well.

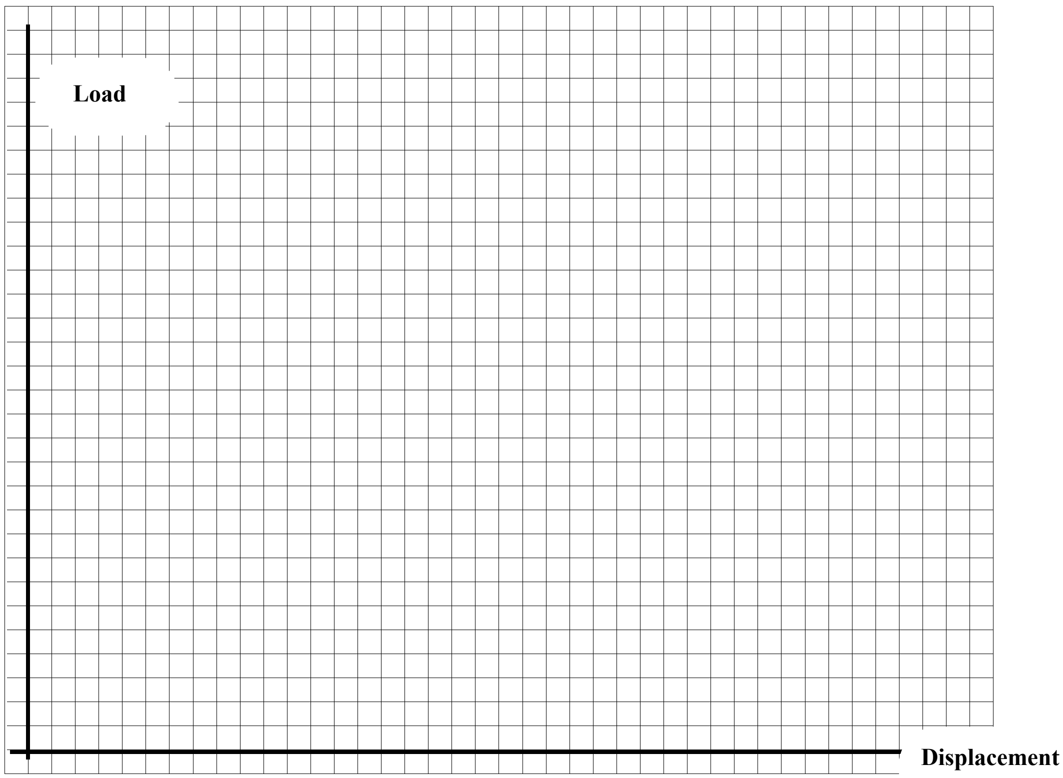
I Beam Test Data

Load lbs.	**op-amp output units =**	**Dial gage units =**	**Displacement in.**	**Stress psi**
Column 1	**Column 2**	**Column 3**	**Column 4**	**Column 5**

Column 1 is the applied load in pounds. Column 2 is the measured op-amp output in volts or millivots; the latter the preferred units. Column 3 is the dial gage reading. Note. The gage will run “backwards” if located on top of the beam. Make sure you develop a good sense of what the readings mean, how you will use them to deduce the vertical displacement before you start the test. But record the reading shown by the gage at each load value.

Columns 4 and 5 are computed from your input values. (If you measured displacement from zero, column 4 will be the same as column 3). The stress at the top of the beam is compressive, that at the bottom, tensile and they are equal for a doubly symmetric cross-section as we have here, according to engineering beam theory. Both values are computed from the product of the strains at top and bottom - also equal - and the elastic modulus, E. See the appendix on the strain gage bridge to see how to compute strain value from the op-amp output.

Make a plot showing how the load (ordinate axis) varies with displacement (abscissa). Fit a straight line through the points (by eye will suffice). What is the “stiffness” of the beam (for this loading condition and geometry) - the slope of the line. On this same plot, show the result from engineering beam theory, using the analysis in the appendix.



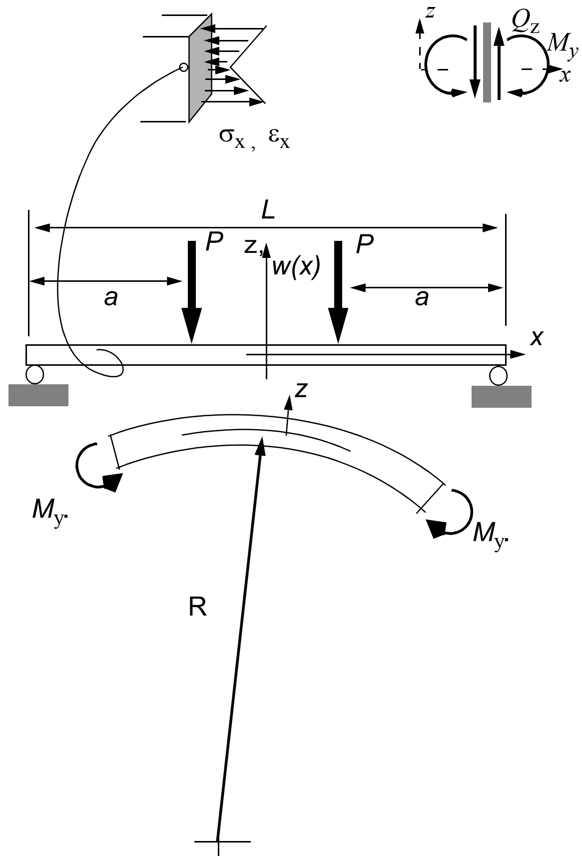
Compare the bending stress at maximum loading with what engineering beam theory gives. Again, see the Appendix.

At Load = ____________ $\sigma|_{test}$ = ___________ *psi* $\sigma|_{theory}$ = ____________ in^2
Difference = ____________%

L = ___________ *in.* a = ___________ *in*

Discussion of results:

Appendix 1: Engineering Beam Theory



Engineering beam theory shows that the most significant stress is the normal stress component on an "x face"; σ_x in the example at the right. It is related to the applied loads by

$$\sigma_x = \frac{M_y \cdot z}{I}$$

where z is the distance from the "neutral axis" which, for a doubly symmetric beam, is at the center of the cross-section and I is the moment of inertia of the cross section.

$$I = \int_A z^2 dA$$

For a rectangular cross-section of width b and height h, this is $I = bh^3/12$

The applied loads come in through the bending moment M_y. The convention for positive shear and bending moment is shown in the figure.

The extensional strain, from the stress/strain relations is just $\varepsilon_x = \sigma_x / E$ where E is the Elastic Modulus. In terms of the geometry of deformation, the extensional strain is given by $\varepsilon_x = z/R$ where R is the radius of curvature of the neutral axis. ($1/R$) is the curvature. The "bending stiffness" is defined as the product ***E I*** as it appears in the "moment-curvature" relationship $M_y = (EI) \cdot \left(\frac{1}{R}\right)$ where the curvature, for small deflections, is related to the vertical displacement of the neutral axis by, ($1/R$) = $-d^2w/dx^2$

or .

$$-M_y/(EI) = \frac{d^2}{dx^2}w(x)$$

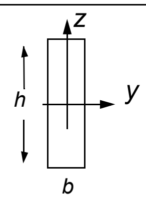
An integration of the differential equation obtained from the moment curvature relation gives, for the case where the beam is loaded as shown, the mid-span deflection

$$w|_{midspan} = -\left(\frac{Pa}{24EI}\right) \cdot (3L^2 - 4a^2)$$

For moments of inertia of cross-section,

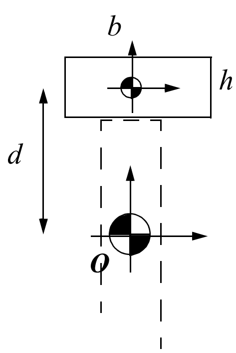
$$I \equiv \int_{Area} z^2 \cdot dA$$

For the rectangular section:



$$I \equiv \int_{Area} z^2 \cdot dA = \int_{z=-\frac{h}{2}}^{z=\frac{h}{2}} z^2 b dz = \frac{bh^3}{12}$$

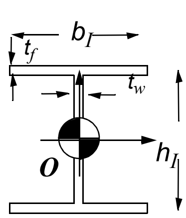
For the "I" section, we use the transfer theorem to obtain the contributions of the top and bottom flanges to the moment of inertia with respect to the centroid of the section. That is, for a segment of area off the axis



$I_{\text{local}} = bh^3/12$ $\qquad A_{\text{local}} = bh$

$$I_{\text{with respect to } O} = I_{\text{local}} + d^2 * A_{\text{local}}$$

For the I section, you must add the contribution of the web to the two terms representing the contribution of the flanges.



t_f = __________ in

t_w = __________ in

h_I = ________ in

b_I = ________ in

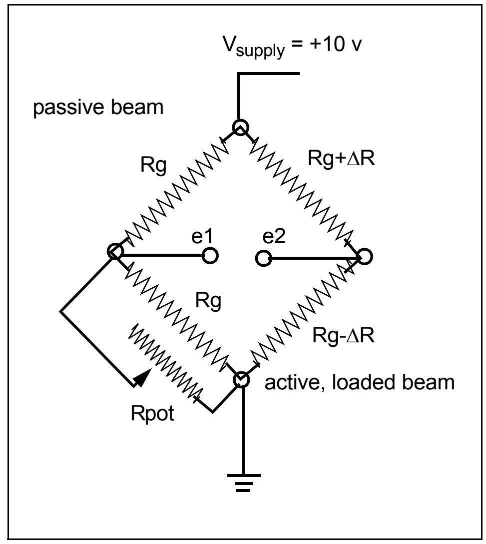
$$I_{\text{with respect to } O} = 2I_{\text{flange}} + bh^3/12$$

***where b =* ____________**

***and h =* ____________**

Appendix 2. Strain-gage Circuit.

The bridge circuit produces an output voltage, $|e_1 - e_2|$ proportional to the change in resistance of the active strain gages. This voltage signal, in turn, is input to an operational amplifier which boosts the (dc) signal by the ***Gain factor, G.***



The two strain gages fixed tothe specimen subject to loading are positioned in the bridge circuit on the right. A positive change in resistance of a gage indicates tension, negative indicates compression. **Note: It does not matter if you switch the location of the two active gages in the bridge circuit. This will simply change the polarity of the voltage difference** $e_1 - e_2$.

The gages on the passive beam, the one not loaded, provide a measure of temperature compensation.

It can be shown (and I probably will) that the output of the bridge is related to the small change in resistance, $\Delta R/R_g$ by:

[1] $$e1 - e2 = (V_{supply}/2)(\Delta R/Rg)$$ where V_{supply} is 10 volts.

The strain as a function of change in resistance is given by

[2] $$\varepsilon = (1/F_{gage})(\Delta R/Rg)$$

where F_{gage} is the "gage factor" stated by the manufacturer[1] to be

$$F_{gage} = 2.07 \pm 0.5\%$$

With these, you can compute the strain in the member, given the voltage difference e1-e2.

I.E., The voltage difference e1 - e2 is obtained from your measured values at the op-amp output by dividing by the amplifier gain.

Then you compute $(\Delta R/Rg)$ from [1].

Then compute ε, the strain, from [2].

The stress is obtained from the linear stress/strain relationship: $\sigma = E\,\varepsilon$

1. BLH Electronics, Inc.