1.105 Solid Mechanics Laboratory
Fall 2003

Experiment 2

Experiment 2 consists of two separate experiments. In the first experiment, 2.1 you will verify static equilibrium consequences for a weight(s) suspended from a cable.

In the second experiment, 2.2, you will determine the force/deflection relationship for a redundant structure built up of cables under pretension. Your report on this experiment will be due at the beginning of your lab session the week of 1 October.

In both experiments you will be using some standard weights, large ones painted grey, smaller ones painted red. At the outset, measure and record the values of these weights using the balance scale on the cabinet near the entrance to the lab. Do a sampling of values of each weight and include an estimate of uncertainty in your measurement.
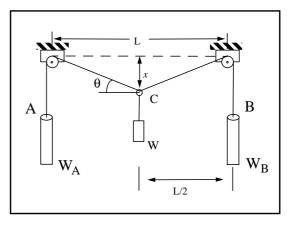
Experiment 2.1a

The figure shows the experimental set-up for the first part (of the first experiment). A cable is supported by two pulleys and loaded by two equal weights at its extremes, A and B. Take $W_A = W_B = W_{big} =$ **One of the big weights**.[1]

A third weight, which you will vary, is suspended from the cable at **midpoint**. The weight W is not quite free to slide along the cable so you will have to ensure it remains at midpoint by adjusting on occasion.

You will apply the load using a light chain and not so light pail hooked to point C. Since the chain and pail have weight, they will deflect the cable some. Hence your first reading for x, the vertical displacement from level, will be with the chain plus pail. Don't forget to measure their weight.

Incrementally increase the weight W, continuing from this point, adding the smaller red weights to the pail. Measure, using the meter stick, the corresponding values of the vertical displacement x. Continue until the latter in nondimensional form, x/L, is in the vicinity of 1/2.

Unload the system incrementally, again measuring and recording the displacement x.
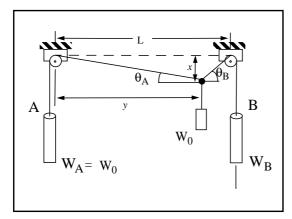
Experiment 2.1b

In this part of the experiment we keep the interior weight (W_0 in the figure) constant, **equal to one of the big weights**. and, in addition, fix the point of application to the cable[2]. The weights at the

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1. Be careful with units. Your rough data in your lab notebook should record measurements in the units of the instrument you are using, e.g., the dial gage indicator reads in increments of .001". Make conversions and report your results using a consistent system, either metric or english.
2. This requires some hardware adjustments which your instructor will take care of.

ends of the cable, as in the prior experiment, are initially the same - hence the initial configuration, all three weights are equal, and each equal to one of the big weights.

Now increment the weight W_B at the right, starting from W_0. using the smaller red weights. (Use the chain hanging from the bottom hook of B and attach the smaller weights to the links of the chain **underneath the grill structure**).

Make measurements of both distances x and y (or $L- y$), at each loading using the meter stick. Make sure you associate an uncertainty with each kind of reading.

Unload incrementally and record x and y again. When finished, remove the middle weight and the chain for the weight at B but leave the two big weights attached to the ends.

Report Contents 2.1

Your report should be in the format:

        Summary (One or two paragraphs)
        Introduction
        Experimental Method
        Results (including comparison with theory)
        Conclusion
        Appendix

In your results for 2.1a, you are to plot W/W_{big} versus x/L (i.e., in nondimensional form). Compare to theory. (The tensions in the cable on either side of midpoint, assuming frictionless pulleys, are equal to W_{big}. Static equilibrium of an isolation of the midpoint where the load is applied then gives a relationship between the tensions in the cable and the applied load. But the angle theta necessarily enters into this equation. Theta, in turn is a function of x - a relationship derived from the geometry of the deflected configuration. In this way you can construct a theoretical relationship between x/L and W/W_{big}, (the tensions being known)

For part 2.1b, make two plots of the data obtained, one for W_B/W_0 versus x/L and another for the ratio W_B/w_0 versus y/L. Show what theory predicts for these relationships on the same graphs. In this, make use of the analysis found in Appendix A. One way to proceed is as follows:

i) pick a value for θ_B

ii) from the fact that $F_A=W_A=W_0$ and the relationship $F_A = W_o \cdot \cos\theta_B / \sin(\theta_A + \theta_B)$ derived from equilibrium as in the appendix, compute a value for θ_A.

iii) The ratios x/L and y/L can then be found from the geometry of the deflected configuration for these values of θ_B and θ_A.

iv) The other equation derived from equilibrium gives W_B/W_0 (or F_B/W_0).

$$F_B = W_o \cdot \cos\theta_A / \sin(\theta_A + \theta_B)$$

(Note: The analysis of Exercise 2.3 of the textbook, has a notation difference from that which is included in the Appendix. The notation in the Appendix is consistent with the notation used above).

Elaboration of these cryptic instructions will be given in a session of 1.050, the companion subject.
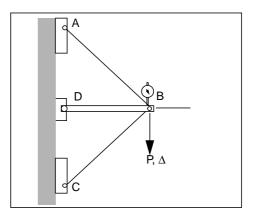
Experiment 2.2

Here, we test the force-deflection behavior of a planar structure which includes two pre-tensioned cables for support. Slots in the support at C will allow you, with the assistance of the lab instructor, to pretension cable BC. Since the support at D is a frictionless pin, or as close to that condition as we can come, this will pretension cable AB as well.

Using a meter stick measure the dimensions between all pins.

Experiment 2.2a

First we will **leave cable BC slack** and incrementally increase the load W at B and measure the corresponding deflection Δ using the dial gage indicator. This will enable you to calculate the "stiffness" at B (in the vertical direction) of this, now statically determinate system.

The dial gage indicator should be "preloaded" to some positive displacement, > .020 in. During loading, this will decrease. Record the actual scale reading. Compute the displacements later.

In loading, use the red weights as increments and **do not exceed ten, (10) pounds.** Use the chain and pail to hold the loads. (Measure their weight before you start) Although the structure should not fail at this loading, **the person reading the dial gage should wear safety glasses.**

Unload incrementally, recording displacements as you go. Record the zero load displacement.

Repeat the proceedure, taking another data set.

Experiment 2.2b

Now, **with the dial gage still in place**, and all loading removed, adjust the support at C to take up the slack, then pretension BC until the dial gage indicates a vertical displacement of around .025". **Do not exceed .035".**

Now again load the structure, using the chain and pail, at point B. Again use one pound increments but go to 20 pounds. **Do not exceed 20 pounds.** You should see the cable BC go slack before you reach this maximum load.

Unload incrementally, recording displacements as you go. Record the zero load displacement.

Repeat the proceedure, taking another data set.

Finally, release the pretension by loosening the support at C, and read and record the dial gage.

Report Contents 2.2

Follow the same format as in Experiment 2.1.

In your results section, for 2.2a, plot load P versus deflection Δ and determine the stiffness - i.e., the value of K in

$$P = K\Delta$$

by "best" fitting a straight line to the plotted data.

In your results section for 2.2b, the pretensioned, statically indeterminate system, plot load P versus deflection Δ. How does this plot differ from that of experiment 2.2a?

At maximum loading conditions, how much has the angle the cable AC makes with the horizontal member changed?

Appendix A

Exercise 2.3– *Show that* the forces in cable AC and CB are given by
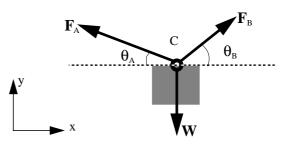
$$F_B = W_o \cos\theta_A / \sin(\theta_A + \theta_B)$$

and

$$F_A = W_o \cos\theta_B / \sin(\theta_A + \theta_B)$$

We first isolate the system, making it a particle. Point *C*, where the *line of action* of the weight vector intersects with the lines of action of the tensions in the cables becomes our particle.

The three force vectors, $\mathbf{F_A}$, $\mathbf{F_B}$ and $\mathbf{W_o}$ then must sum to zero for static equilibrium. Or again, the *resultant force on the isolated particle must vanish*. We meet this condition on the *vector sum* by insisting that two *scalar sums* – the sum of the horizontal (or *x*) components and the sum of the vertical (or *y*) components – vanish independently. For the sum of the *x* components we have, taking positive *x* as positive:

$$-F_A \cos\theta_A + F_B \cos\theta_B = 0$$

and for the sum of the *y* components,
$$F_A \sin\theta_A + F_B \sin\theta_B - W_o = 0$$

A bit of conventional syntax is illustrated here in setting the sums to zero rather than doing otherwise, i.e., in the second equation, setting the sum of the two vertical components of the forces in the cables equal to the weight. Ignoring this apparently trivial convention can lead to disastrous results, at least early on in learning one's way in Engineering Mechanics. **The convention brings to the fore the necessity of isolating a particle before applying the equilibrium requirement.**

We see that what we *need to know* to determine the force in cable AC and in cable CB are the angles θ_A and θ_B and the weight of the block, *Wo*. These are the *givens*; the *magnitudes* of the two forces, F_A and F_B are our two *scalar unknowns*. We read the above then as two scalar equations in two scalar unknowns. We have reduced the problem... *show that*... to a task in elementary algebra. To proceed requires a certain versatility in this more rarefied language.

There are various ways to proceed at this point. I can multiply the first equation by $\sin\theta_A$, the second by $\cos\theta_A$ and add the two to obtain

$$F_B \cdot (\sin\theta_A \cdot \cos\theta_B + \sin\theta_B \cdot \cos\theta_A) = W_o \cos\theta_A$$

Making use of an appropriate trigonometric identity, we can write:

$$F_B = W_o \cdot \cos\theta_A / \sin(\theta_A + \theta_B)$$

Similarly, we find:

$$F_A = W_o \cdot \cos\theta_B / \sin(\theta_A + \theta_B)$$

Appendix B

We want to determine the stiffness *k* of the cable alone. For this statically determinate structure, derive an expression for the force in the cable AB as a function of the structure geometry and the applied load from equilibrium considerations. Derive then the extension, δ, in the cable AB as a function of Δ, the vertical displacement of point B from compatibility of deformation. With these in hand, and using the ***measured values*** for the applied load and Δ, calculate pairs of values for the force in the cable, f_{AB} and its elongation δ and plot f_{AB} versus δ. From this plot, determine the slope, the stiffness, *k,* of the cable.

Note: For small deflections and rotations, the extension of the cable AB is given by the projection of the vertical displacement of point B onto the cable.

That is, compatibility of deformation requires: $\delta = \Delta \sin \phi$

The force in the cable AB, f_{AB}, in terms of the measured, applied load at B, is found from equilibrium of the "pin" at B.

The stiffness, *k*, of the *cable* is then found from the slope of the graph of f_{AB} versus δ.

$$f_{AB} = k\delta$$