

Homework Set #3

Problem 1

Read *Application Example 8* and do Problem 8.1.

Problem 2

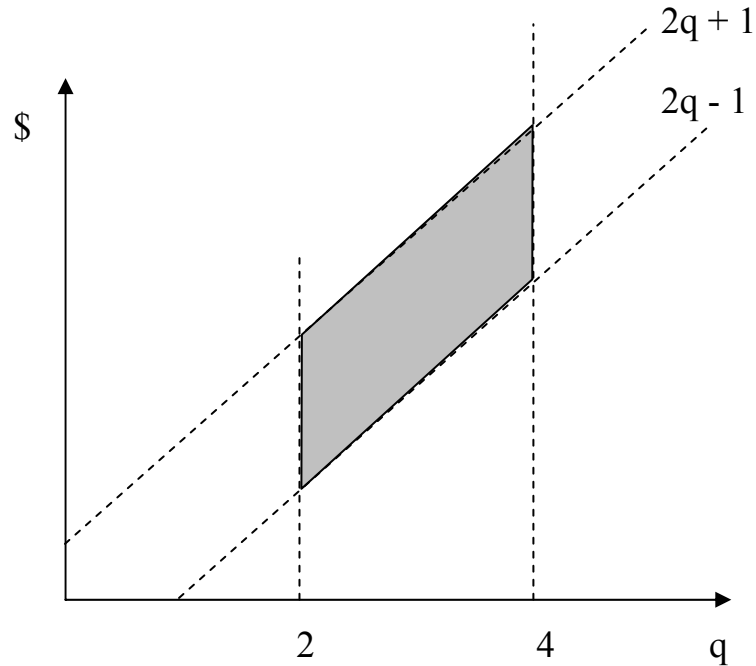
The way MIT admits undergraduate students is exemplified in the following table. Each applicant is rated to a discrete “scholastic index” X (horizontal axis) and a discrete “personal rating index” Y (vertical axis). The top number in each cell (**in bold**) is the number of applicants in a given year with the associated combination. The bottom number in each cell (*in italic*) is the probability of being accepted. (Although this is indeed the way MIT handles applications, all numbers are fictitious).

Personal Rating, Y ↓	← Scholastic Index, X →					
	90-100	80-90	70-80	60-70	50-60	≤ 50
10	20 <i>1.0</i>	40 <i>0.9</i>	52 <i>0.7</i>	32 <i>0.5</i>	10 <i>0.4</i>	6 <i>0.3</i>
9	60 <i>0.9</i>	110 <i>0.7</i>	150 <i>0.5</i>	192 <i>0.4</i>	47 <i>0.3</i>	17 <i>0.2</i>
8	86 <i>0.7</i>	215 <i>0.5</i>	305 <i>0.4</i>	351 <i>0.3</i>	87 <i>0.2</i>	62 <i>0.1</i>
7	39 <i>0.5</i>	173 <i>0.4</i>	250 <i>0.3</i>	192 <i>0.2</i>	102 <i>0.1</i>	53 <i>0.0</i>
6	17 <i>0.4</i>	54 <i>0.3</i>	118 <i>0.2</i>	152 <i>0.1</i>	97 <i>0.0</i>	68 <i>0.0</i>
≤ 5	1 <i>0.3</i>	12 <i>0.2</i>	32 <i>0.1</i>	31 <i>0.0</i>	19 <i>0.0</i>	21 <i>0.0</i>

- (a) Plot the marginal PMF of the two indices.
- (b) Plot the conditional PMFs of $(X|Y = 8)$ and $(X|Y = 6)$.
- (c) Plot the conditional PMF of $(Y|X \leq 50)$.
- (d) What is the probability that an applicant with $Y = 7$ is accepted.
- (e) Are X and Y independent? Why?

Problem 3

In Bounty Town, U.S.A., total precipitation during the crop-growing season, Q , has a uniform distribution between 2 and 4 inches. The total crop value $\$$ depends on Q in such a way that $(\$/Q = q)$ has uniform distribution (in millions of dollars) between $(2q - 1)$ and $(2q + 1)$. Note that the possible values of $(Q, \$)$ are inside the parallelepiped shaded in the figure below:



- What is the joint PDF of Q and $\$$?
- What is the marginal PDF of $\$$?
- What value of $\$$ is exceeded on average every 5 years?

Read *Application Examples 7, 9 and 10* . .