Massachusetts Institute of Technology
Logistical and Transportation Planning Methods

1.203J/6.281J/13.665J/15.073J/16.76J/ESD.216J

Quiz #2 OPEN BOOK

December 5, 2001

Please do Problems 2 and 3 in a booklet separate from Problem 1. Remember to put your name on each booklet! And please explain all of your work! We like to give deserved partial credit. Good luck!

Problem #1. Two Server Hypercube Problem (50 points)

Consider a 1-mile-by-2-mile homogeneous service region served by two mobile patrolling servers as shown in Figure 1. Here are the assumptions of the model:

1. Customer locations are uniformly independently located over the entire rectangular service region.
2. Over time, customers arrive as a homogeneous Poisson process at aggregate rate λ=2 customers per hour.
3. While not busy serving customers, each server patrols its sector (sector 1 or 2, respectively). Under this circumstance, the server's location at a random time is uniformly distributed over its sector. Each sector is a one-mile-by-one-mile square.
4. Travel distance is right-angle ("Manhattan metric"), with speed equal to 1000 mi./hr.
5. The on-scene time to serve a customer is a random variable having a negative exponential probability density function with mean equal to 30 minutes. Upon completion of service of a customer at the scene, the server resumes random patrol of his/her sector.
6. This is the dispatch strategy: For a customer from sector i (i=1,2), dispatch server i if available. Else dispatch the other server if available; Else the customer is lost forever.

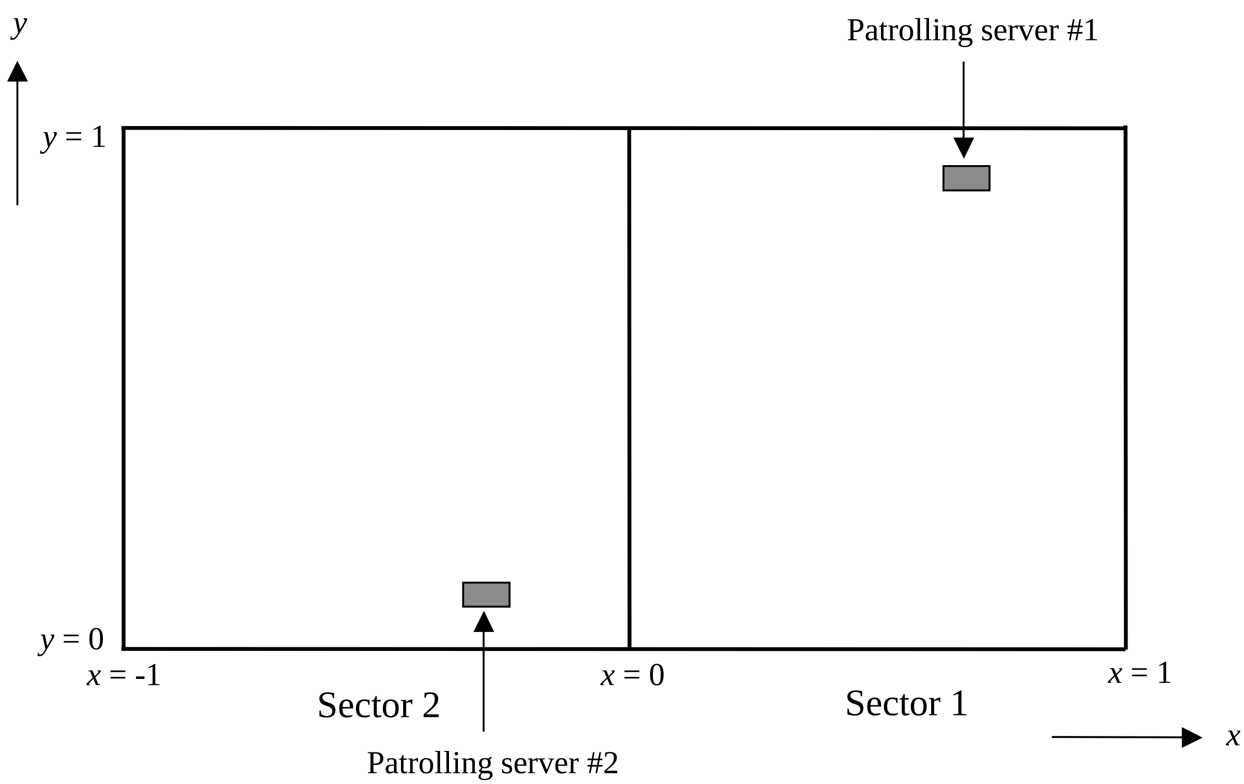

Figure 1

(a) Is it true that the workload (fraction of time busy) of each server is equal to 1/2? If true, briefly explain why. If false, derive the correct figure.
(b) Determine the fraction of dispatches that take server 1 to sector 2.
(c) Determine the mean travel time to a random served customer.

Now consider the situation as shown in Figure 2. Assumptions 1 through 5 above remain correct. However, Assumption 6 is altered as follows:

Case in which both servers are available: For a customer from a part of sector *i* not in Buffer Zone *i*, dispatch server *i*. For a customer in Buffer Zone *i*, dispatch the other server only if that other server is within its own buffer zone and server *i* is not within its buffer zone; else dispatch server *i* to that customer.

Case in which only one server is available: Dispatch that server.

Else the customer is lost forever.

(d) Under this new dispatch policy, determine the fraction of dispatch assignments that send server 1 to sector 2.
(e) Without doing the detailed calculations, describe briefly how you would compute the mean travel time. How would the magnitude of the numerical answer compare to that of part (c)?
(f) Suppose under the simpler dispatch policy #1 above, we find that the workload of Sector #1 is twice the workload of Sector 2, while λ remains the same at $\lambda = 2$. Without doing the calculations, briefly explain how you would find an optimal boundary line separating Sectors 1 and 2, where 'optimal' means minimizing mean travel time. Would it be to the left of right of $x = 0$? Why?

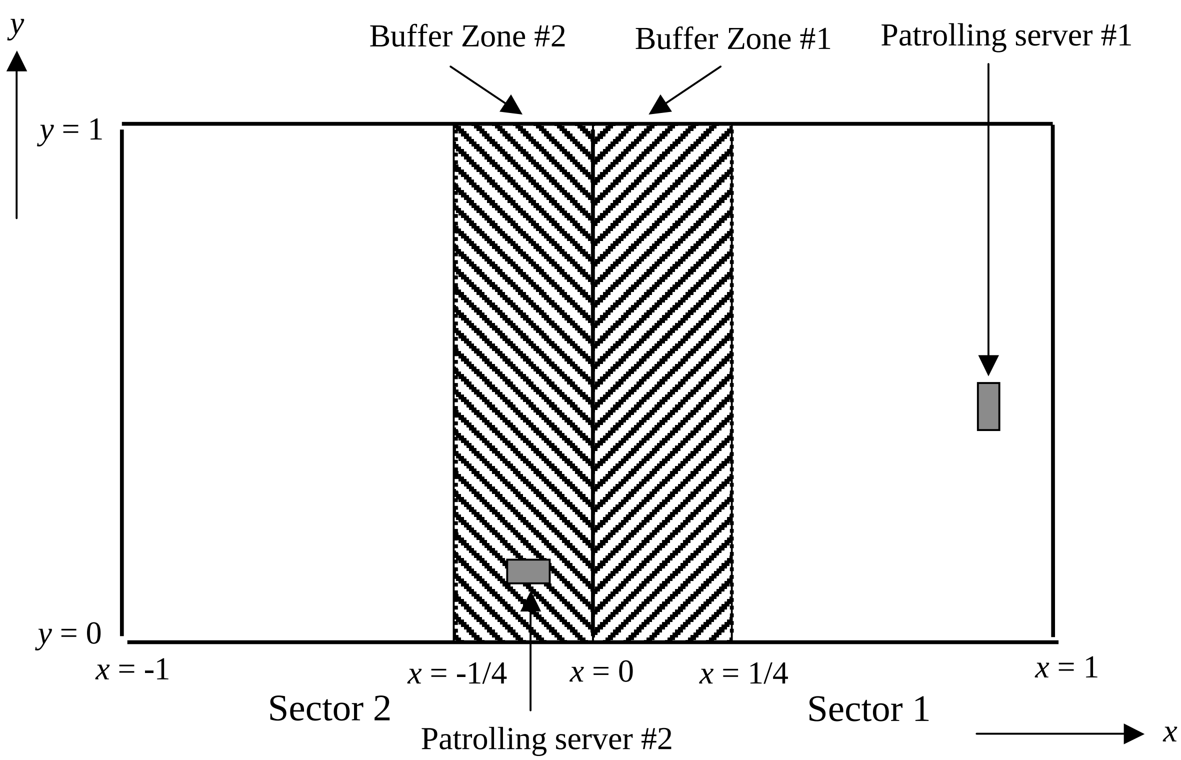

Figure 2

Problem 2: Markov Queues (25 points)

Vincent's barbershop is very modest in size. It consists of two barber's chairs where Vincent and his newly-hired other barber Al serve customers and one other chair for waiting customers. Prospective customers arrive at the barbershop in a Poisson manner at the rate of λ per hour. Those who find the barbershop full (both barber's chairs and the waiting chair are all occupied) go to other barbershops and are lost forever. Those who see an empty chair (barber's or otherwise) will walk into the store and will eventually be served by Vincent or Al.

It takes Vincent $1/\mu_1$ hours, on average, to complete service to a customer while Al takes $1/\mu_2$ hours, on average, per customer. The pdf's for both Al's and Vincent's service times are negative exponential.

The following interesting behavior of barbershop customers has been observed. Because Vincent is considered a great barber and Al is not very popular, any customer who finds both Al and Vincent idle will immediately choose to sit at Vincent's chair. Moreover, customers who upon arriving at the barbershop find Vincent busy and Al idle and the waiting chair empty, will choose to sit at the waiting chair and wait for Vincent, rather than be served immediately by Al. Customers occupying the waiting chair will wait as long as it takes to be served eventually by Vincent. In fact, the only customers that poor Al serves are those who find Vincent busy *and* the waiting chair occupied.

All questions below refer to steady-state conditions.

(a) Carefully draw a state-transition diagram for this queuing system. DO NOT WRITE THE CORRESPONDING EQUATIONS OR SOLVE FOR THE STEADY STATE PROBABILITIES.

(b) How many customers per hour will Vincent actually serve on average in this situation? Your answer should be in terms of λ and/or μ_1 and/or μ_2 and the steady-state probabilities of some of the states you have defined. (Assume you know these steady-state probabilities and use symbols for them, such as P_x.)

(c) We have just observed a customer enter the barbershop and sit in Al's chair (to receive a haircut from Al). What is the probability that the next customer who will enter the shop will have his hair cut by Vincent?

Problem 3: Networks (25 points)

We wish to find a tour that visits every node of the network shown in Figure 3 at least once, starting and ending at node A.

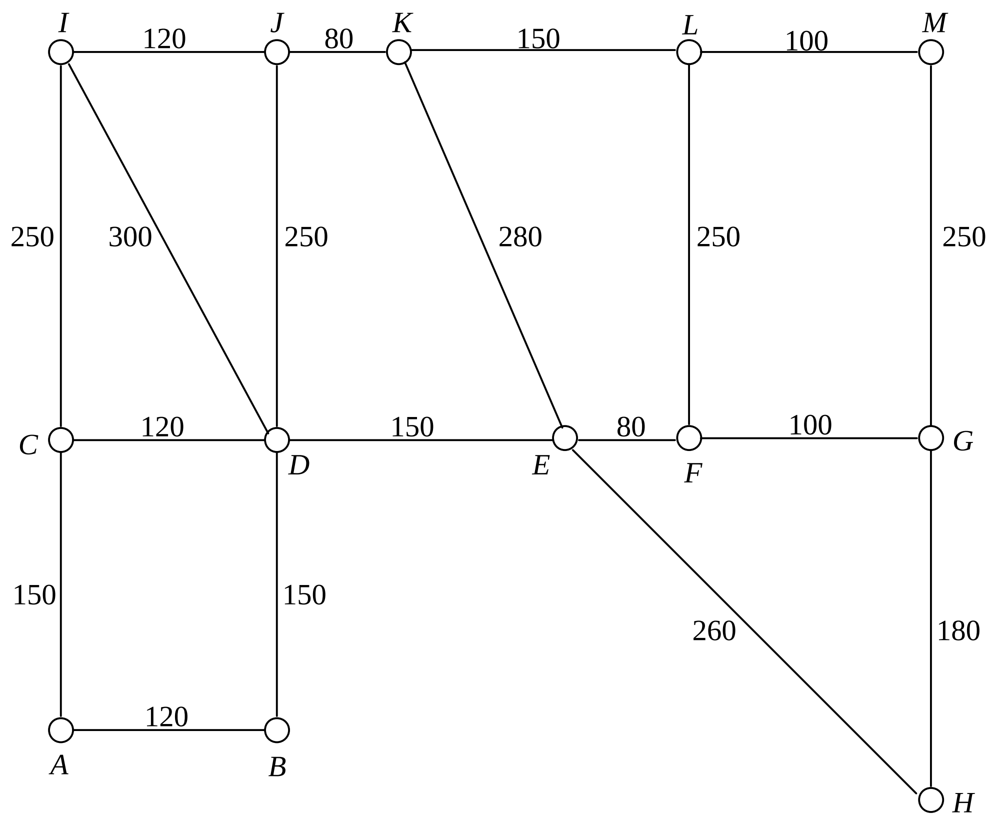

Figure 3

(a) Find such a tour by (i) applying the Christofides heuristic and (ii) improving on the resulting tour by not visiting again, unless you have to, nodes that have already been visited. Give your answer either in the form of a clear sketch or in the form of a listing of the sequence in which nodes are visited in the tour. The list should look like {A, "node name", "node name", …., A}.

(b) With the insight gained from part (a) try to find an optimal solution to the TSP. Non-optimal, but reasonably good solutions will be given partial credit.