**Homework problems on Fluid Dynamics**

(1.63J/2.21J)

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**Channel flow through a geothermal gradient**

In Alaskan oil fields, pipes are driven across the permafrost to a depth of O(1000) m to reach the oil reservoir. Warm oil of $200°F$ is driven out by the given pressure gradient

$$-\frac{\partial p}{\partial z} = \frac{p_R - p_A}{H} \tag{1}$$

where $p_R$ and $p_A$ are the reservoir pressure and atmospheric pressure respectively. Despite the insulation, some heat is lost to the permafrost throught the pipe wall.

Refering to Figure , consider the simplified 2-D problem of hot fluid rising steadily through the vertical channel of width $2a$, i.e., $-a < x < a$. The temperature on the walls are given

$$T_w = T_o - sz/a \tag{2}$$

where $T_o$ and $s$ are positive constants. Assuming no insulation and infinitely long channel, so that the vertical velocity is $v(x)$. Let the temperature in the fluid be $T(x, z) = T_w + \theta(x)$.

Find $v(x)$ and $\theta(x)$ explicitly and discuss the results as functions of the fluid properties and *the geothermal gradient*. Account for buoyancy effects

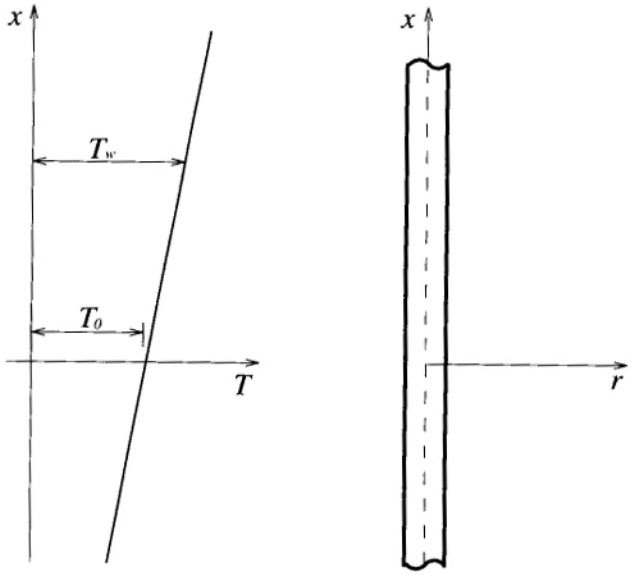


Figure 1: Hot oil rising in a pipe buried in permafrost