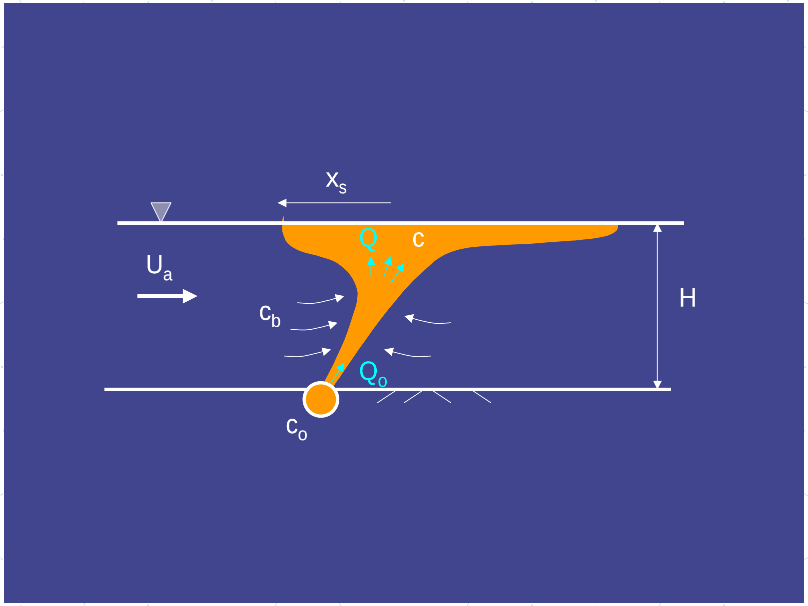
6 Initial Mixing

- Introduction
- Integral Analysis
- Dimensional Analysis
- Multi-port Diffusers
- Gravitational spreading, intrusion & mixing
- Multi-port Diffusers in Shallow Water
- Buoyant Surface Jets
- Combined Near and Far Field Analysis

Submerged Discharge



- Mixing by turbulent entrainment rather than exchange
- Dilution
  - $S = Q/Q_o$
  - $S = (C_o - C_b)/(C - C_b)$
- Mixing zones
  - Hydrodynamic
  - Regulatory

Dilution a solution to pollution?

- Biodegradable contaminant?
- High ambient concentration of contaminant?
- Toxics?

Pure Jet

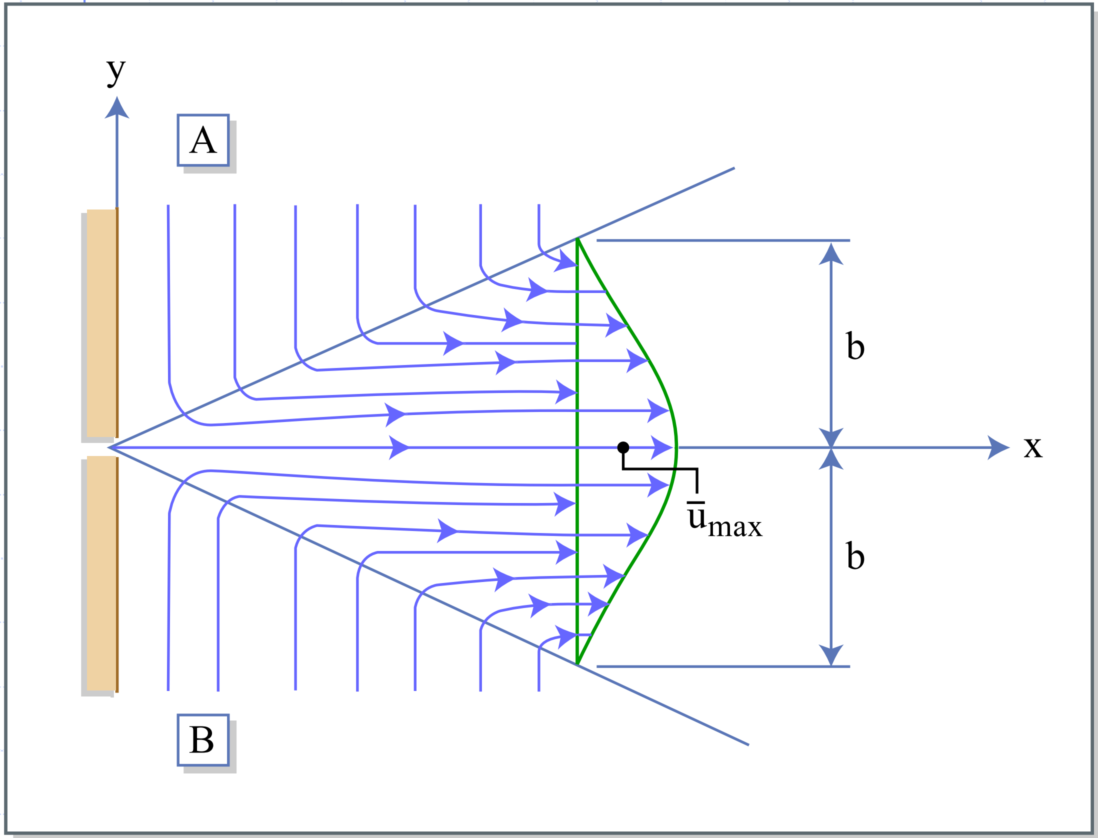


Figure by MIT OCW.

Daily and Harleman, (1966)

- Momentum driven
- Bell-shaped velocity distribution (in jet)
- Irrotational flow (entrainment field)
- Properties
  - $b \sim x$
  - $u \sim x^{-1}$
  - $Q \sim ub^2 \sim x$

Buoyant Jet

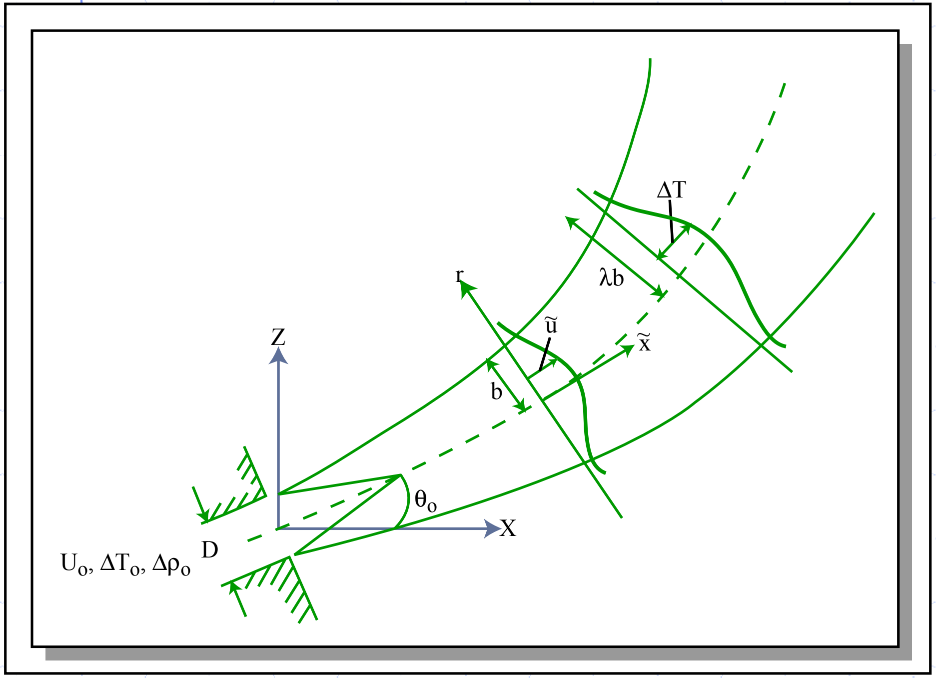


Figure by MIT OCW.

- Buoyancy driven
  - Temperature
  - Dissolved/Suspended solids
- Bell-shaped velocity & scalar distributions
- Linear spread
- Finite initial size (ZOFE)

Equation of State (Gill, 1982)

$$\rho = \rho(T) + \Delta\rho(S) + \Delta\rho(TSS)$$

$$\rho(T) = 1000\left[1 - \frac{T + 288.9414}{508929.2(T + 68.12963)}(T - 3.9863)^2\right]$$

$$\Delta\rho(S) = AS + BS^{3/2} + CS^2$$

$$A = 0.824493 - 4.0899x10^{-3}T + 7.6438x10^{-5}T^2 - 8.2467x10^{-7}T^3 + 5.3875x10^{-9}T$$

$$B = -5.72466x10^{-3} + 1.0227x10^{-4}T - 1.6546x10^{-6}T^2$$

$$C = 4.8314x10^{-4}$$

$$\Delta\rho(TSS) = TSS\left[1 - \frac{1}{SG}\right]x10^{-3}$$

ρ = kg/m^3, T in ^{o}C, S in PSU (g/kg), TSS in mg/L

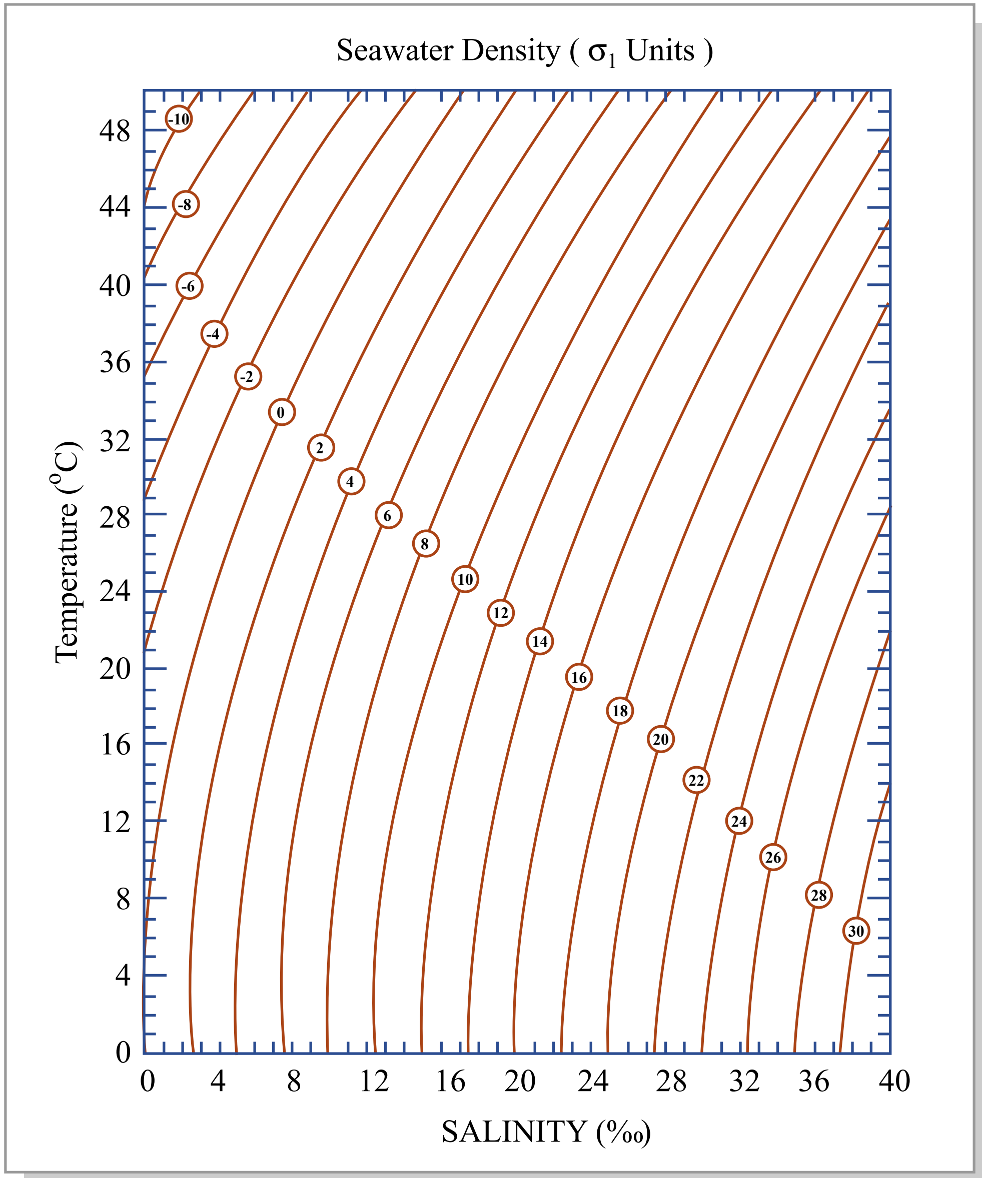


Figure by MIT OCW.

Fischer, et al. (1979)

$$\sigma_t = 1000*(\rho-1)$$

(ρ in g/cm^3)

Model Types

- Computational Fluid Dynamics (3-D)
- Integral Analysis (1-D)
- Dimensional Analysis (0-D)

Integral Analysis: Self-Similarity

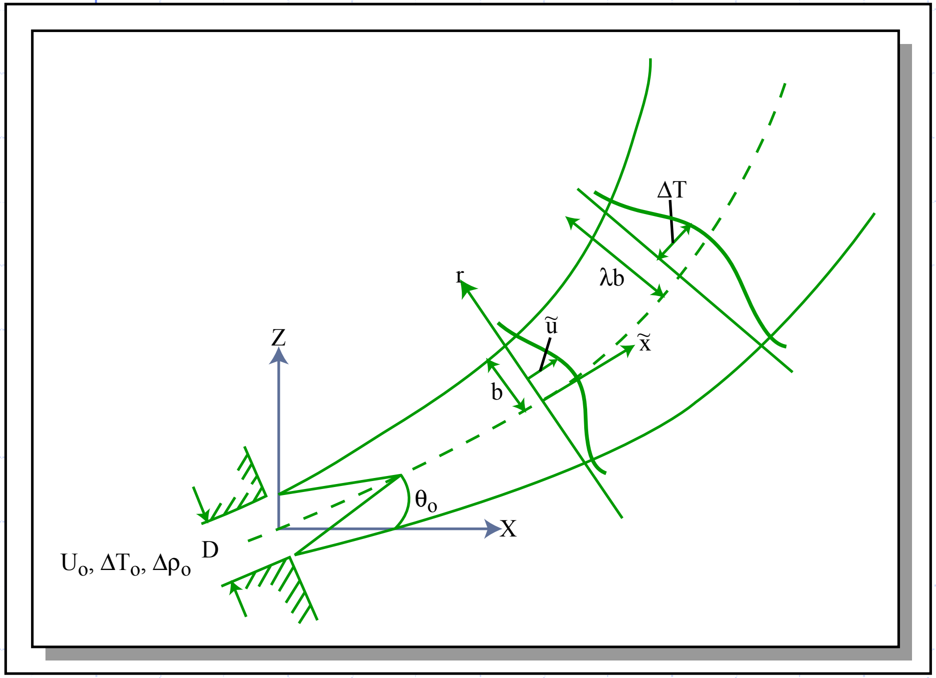


Figure by MIT OCW.

$$\frac{\tilde{u}}{\tilde{u}_c} = f(r/b)$$

$$\frac{\Delta c}{\Delta c_c} = \frac{\Delta T}{\Delta T_c} = \frac{\Delta \rho}{\Delta \rho_c} = g(r/b)$$

$$f(r/b) = e^{-r^2/b^2}$$

$$g(r/b) = e^{-r^2/(\lambda b)^2}$$

Integrated Fluxes

Volume $$Q \cong \int_{-\infty}^{\infty} \tilde{u}\, dA = \int_{0}^{\infty} \tilde{u}_c\, f\, 2\pi r dr = 2\pi I_1 \tilde{u}_c b^2$$

Momentum* $$M \cong \int_{-\infty}^{\infty} \tilde{u}^2 dA = \int_{0}^{\infty} \tilde{u}_c{}^2 f^2\, 2\pi r dr = 2\pi I_2 \tilde{u}_c{}^2 b^2$$

Mass $$J \cong \int_{-\infty}^{\infty} \tilde{u} \Delta c\, dA = \int_{0}^{\infty} \tilde{u}_c \Delta c\, f g\, 2\pi r dr = 2\pi I 3 \tilde{u}_c \Delta_c b^2$$

*Neglects turbulent momentum fluxes

Conservation Statements

Continuity

$$\frac{dQ}{d\tilde{x}} = 2\pi b\left|v_e\right| = 2\pi b\alpha\tilde{u}_c$$

Longitudinal Momentum

$$\frac{dM}{d\tilde{x}} = 2\pi\int_0^\infty \Delta\rho g\sin\theta r dr = 2\pi I_4\Delta\rho g b^2\sin\theta$$

Horizontal Momentum

$$\frac{d(M\cos\theta)}{d\tilde{x}} = 0$$

Contaminant mass

$$\frac{dJ}{d\tilde{x}} = 0$$

Geometry 1

$$\frac{dx}{d\tilde{x}} = \cos\theta$$

Geometry 2

$$\frac{dy}{d\tilde{x}} = \sin\theta$$

Solution Technique

- Initial Value Problem
- 6 equations in 6 unknowns

$$\left[\quad A \quad \right] \begin{Bmatrix} \dot{u}_c \\ \Delta\dot{\rho}_c \\ \dot{b} \\ \dot{\theta} \\ \dot{x} \\ \dot{y} \end{Bmatrix} = \left[\; C \; \right]$$

Results

- Output as function of

Densimetric Froude Number

$$F_o = \frac{u_o}{\sqrt{g(\Delta\rho_o/\rho)D_o}}$$

Dimensionless Distance, Height

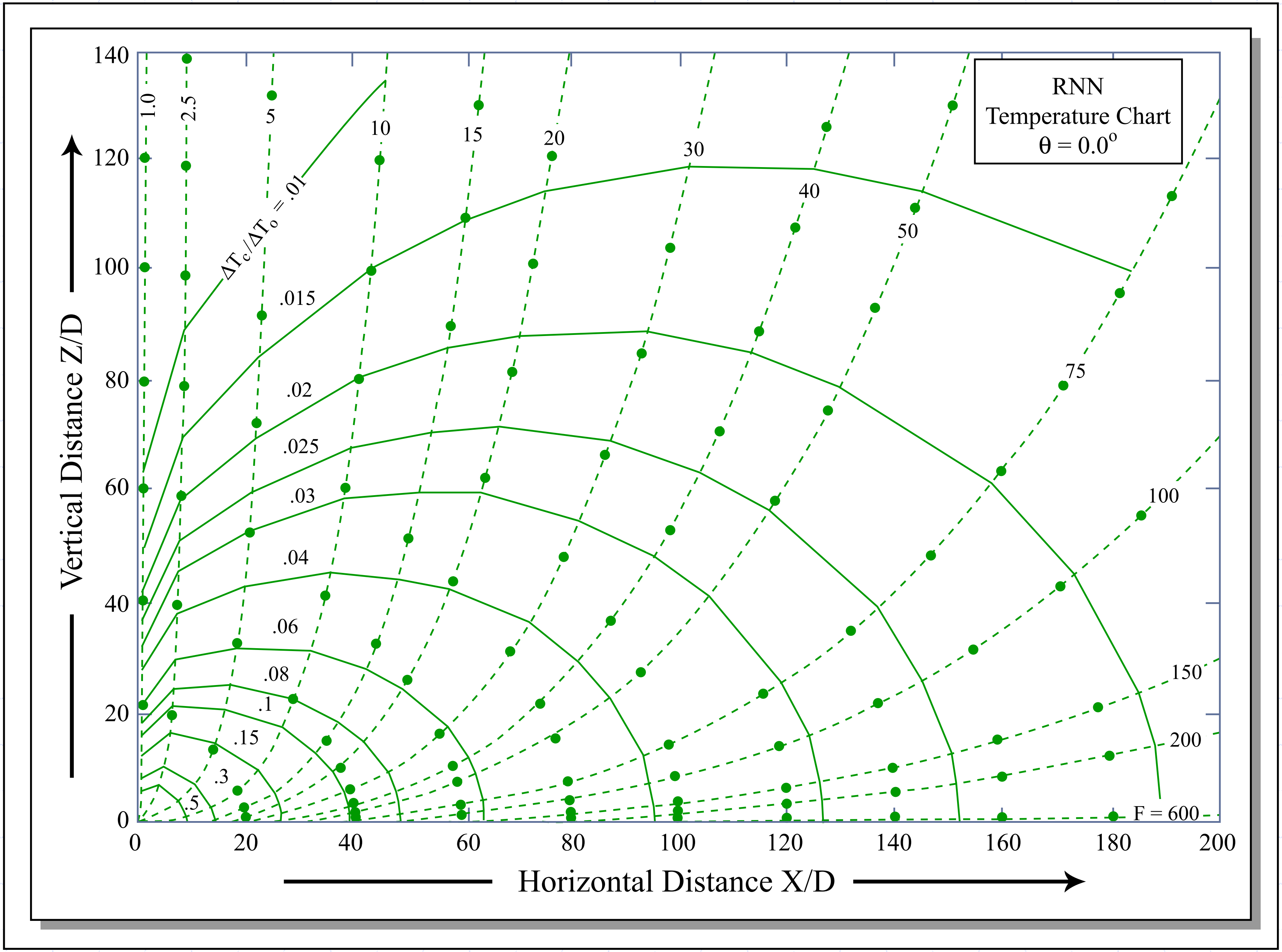
$$x/D_o$$

$$z/D_o$$

Limiting Conditions

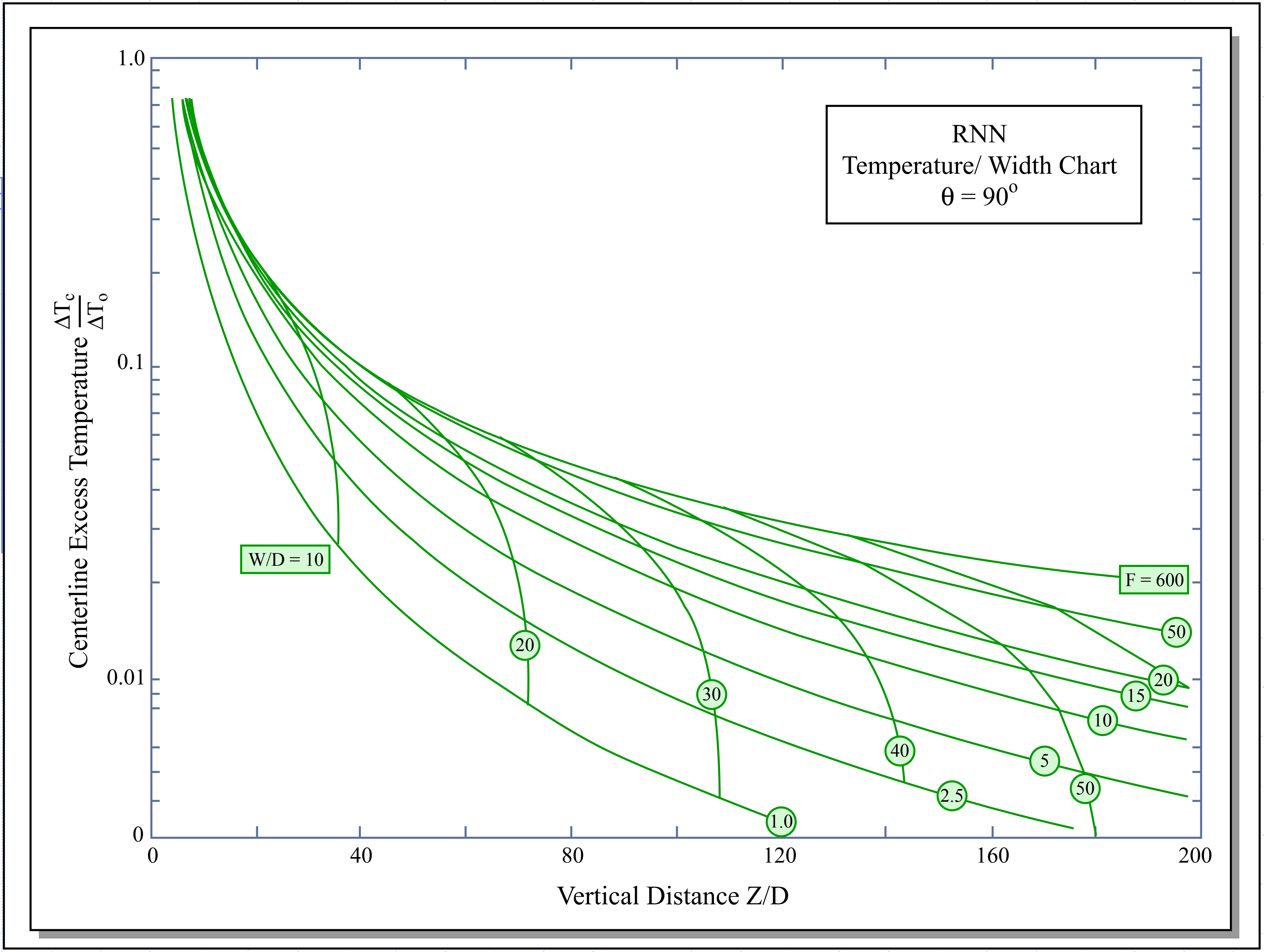
$F_o = \infty$ Pure jet

$F_o = 1$ Pure plume



Shirazi & Davis, 1974

Figure by MIT OCW.

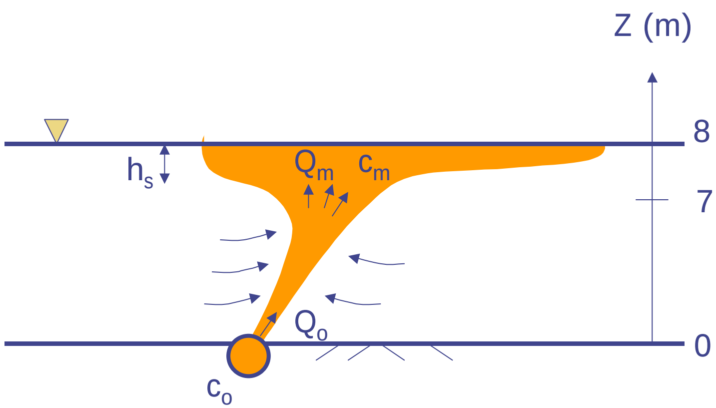


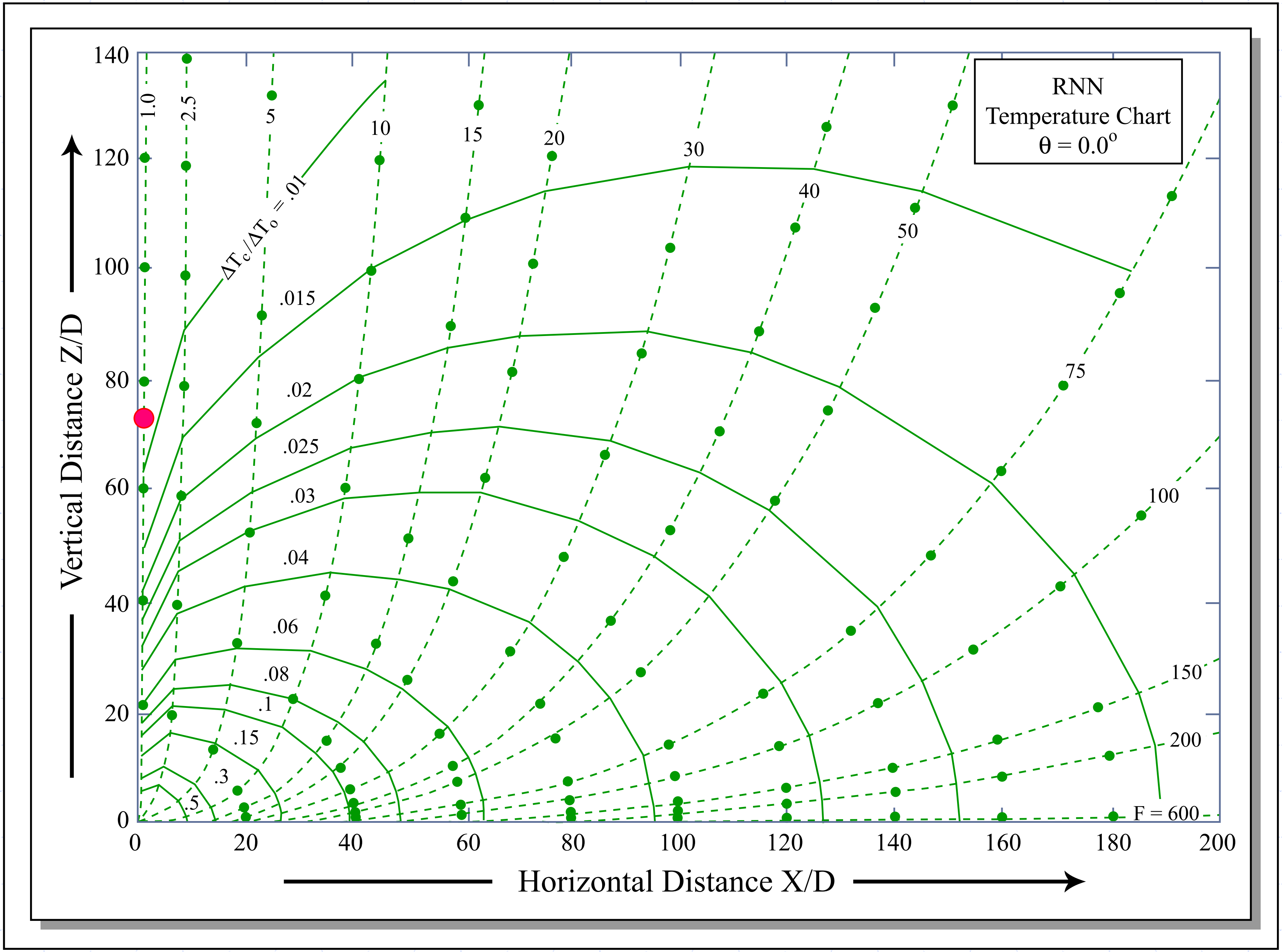
Shirazi & Davis, 1974

Figure by MIT OCW.

Example Calculations (WE 6-1)

- $Q_o = 0.00125\ m^3/s$
- $D_o = 0.1\ m$
- $\Delta\rho_o/\rho = 0.025$
- $u = Q_o/(\pi D^2/4) = 0.16 m/s$
- $F_o = u_o/(\Delta\rho_o g/\rho D)^{0.5} = 1$
- $z/D_o = 70$
- $c/c_o = 0.008$





Shirazi & Davis, 1974

Figure by MIT OCW.

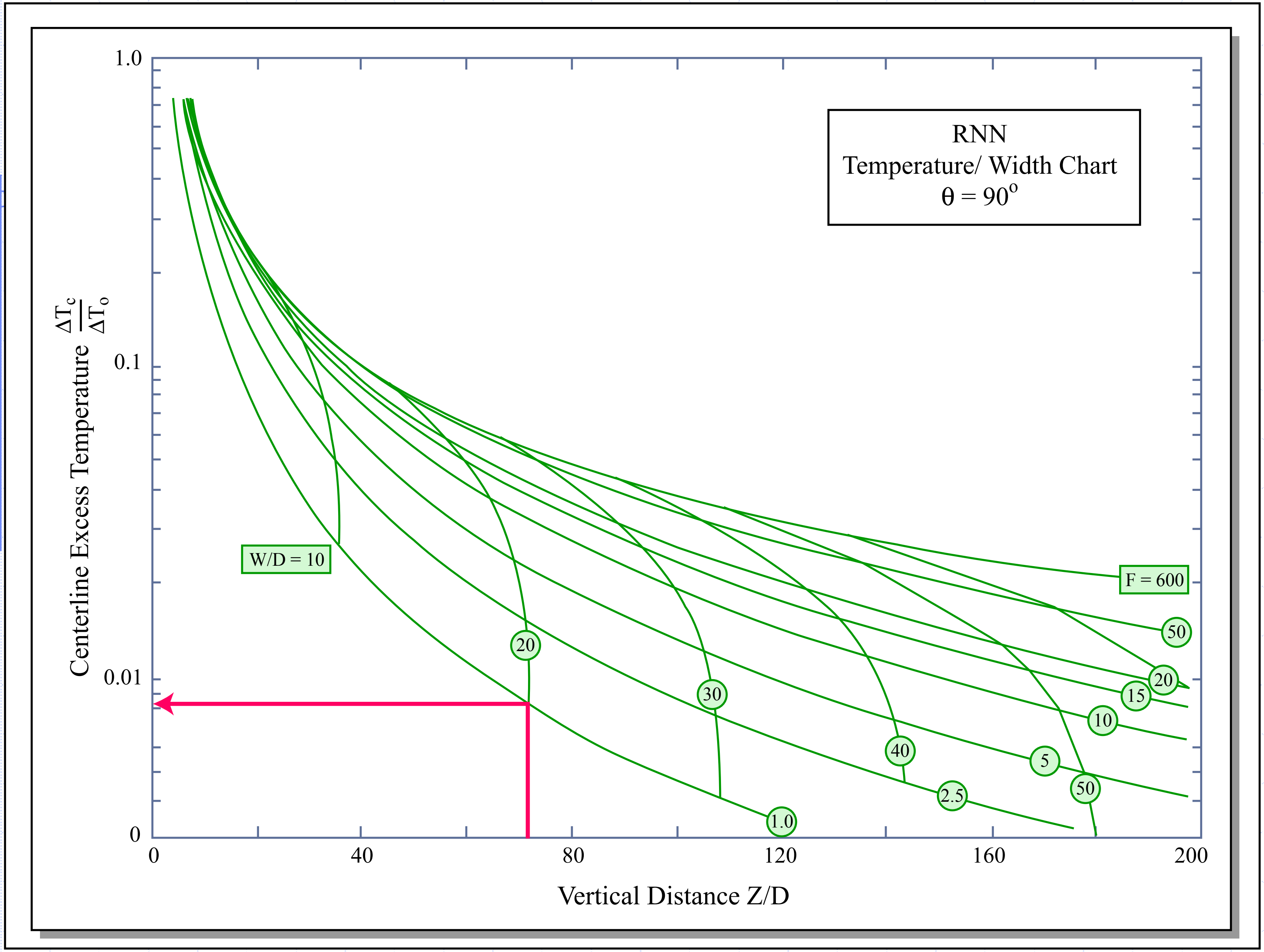


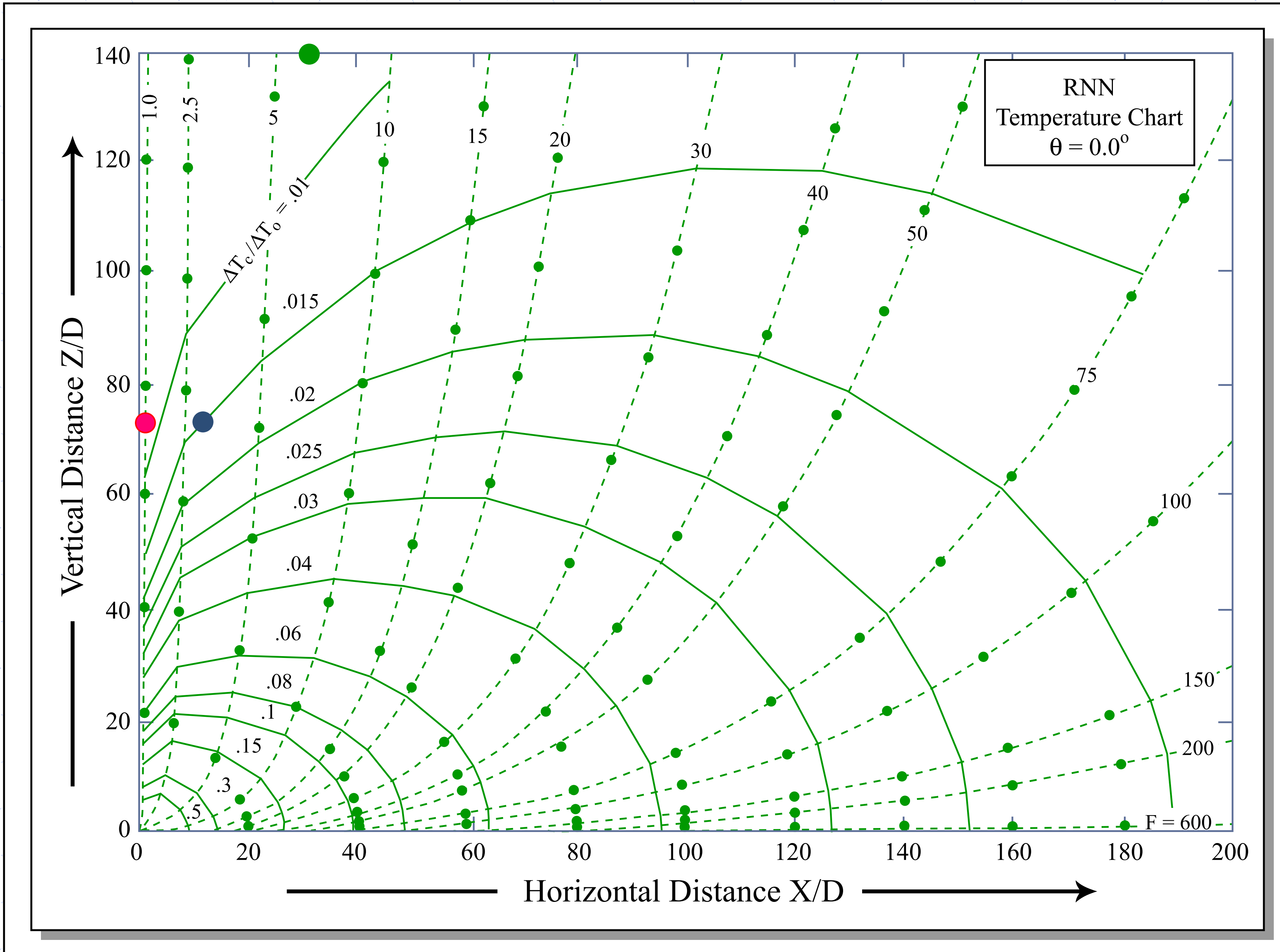
Figure by MIT OCW.

Shirazi & Davis, 1974

Example Calculations (cont'd)

	Base Case	Increased Momentum	Increased Flow
D_o	0.1	0.05	0.1
Q_o	0.00125	0.00125	0.0025
u_o	0.16	0.64	0.032
$\Delta\rho_o/\rho$	0.025	0.025	0.0125
F_o	1	5.7	2.8
z/D_o	70	140	70
c/c_o	0.008	0.008	0.016

In deep water behavior depends mainly on buoyancy—not momentum, flow rate, port size or orientation



Shirazi & Davis, 1974

Figure by MIT OCW.

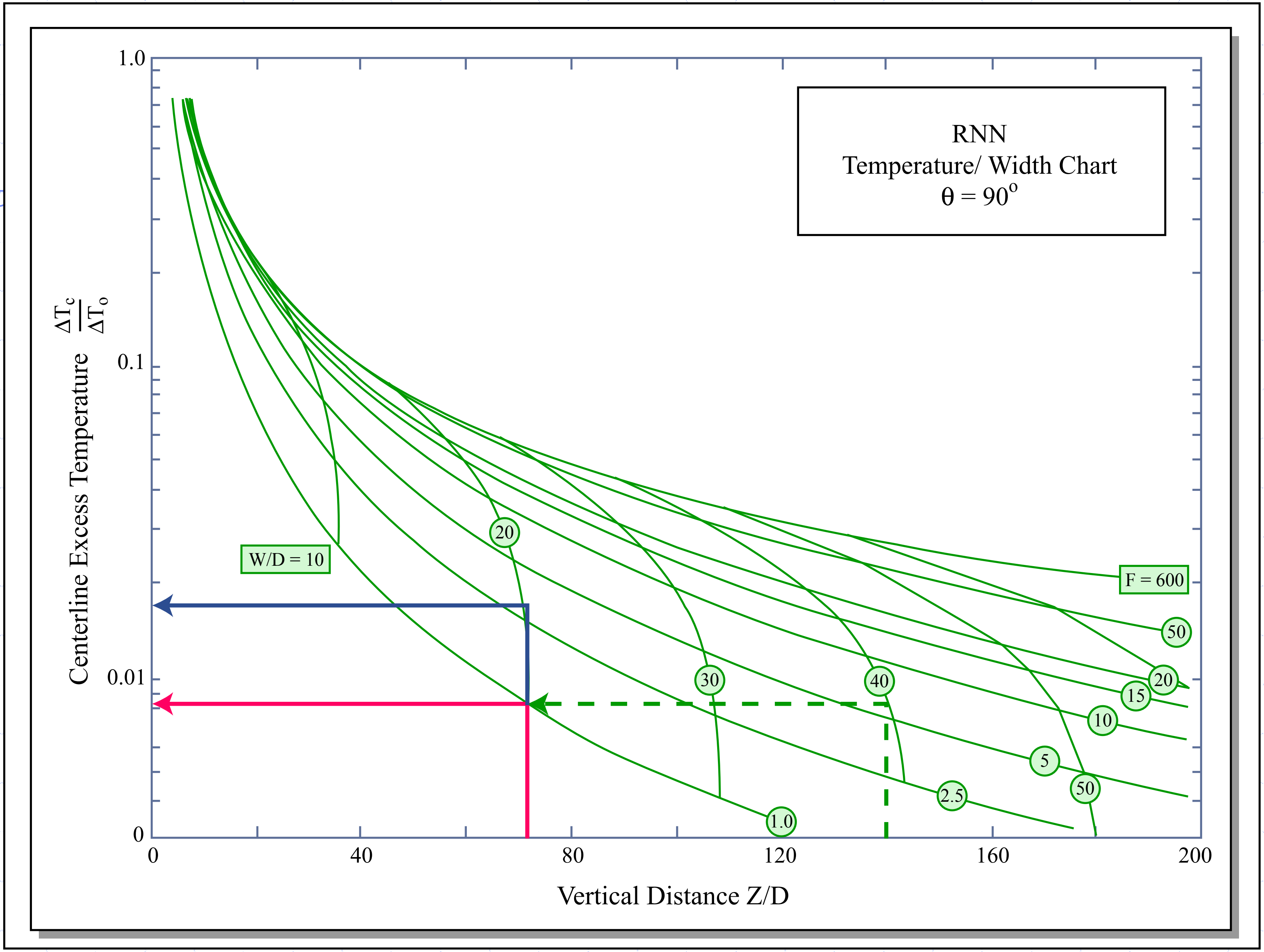


Figure by MIT OCW.

Shirazi & Davis, 1974

Dimensional Analysis

- Identify important independent and dependent variables
- Arrange in dimensionally consistent manner
- Determine coefficients empirically

Buckingham Π Theorem

- Number of dimensionless parameters equals number of independent plus dependent variables minus number of dimensions used to describe these variables
- Example: $D = \frac{1}{2} g t^2$
  - 3 variables (g, t, D)
  - 2 dimensions (length, time)
  - 1 dimensionless variable (D/gt^2)
- "Empirical" coefficient (1/2)

Axi-symmetric Plume

- Neglect ambient current, stratification
- Assume deep water (initial momentum, flow rate, nozzle size, discharge angle less important than buoyancy)
- Kinematic buoyancy flux
  - $B_o = Q_o\, g\Delta\rho_o/\rho \quad [L^4T^{-3}]$

Axi-symmetric Plume (cont'd)

- $Q = f(B, z)$
- 3 variables – 2 dimensions = 1 non-dimensional parameter (c_1)

$$c_1 = \frac{Q}{B_o^{\alpha} z^{\beta}}$$

$$Q = c_1 B_o^{\alpha} z^{\beta}$$

Axi-symmetric Plume (cont'd)

$$Q \sim B_o^{\ \alpha} z^{\beta}$$

$$\frac{L^3}{T} = \frac{L^{4\alpha}}{T^{3\alpha}} L^{\beta}$$

$$3 = 4\alpha + \beta$$

$$1 = 3\alpha$$

$$\therefore \alpha = 1/3, \beta = 5/3$$

$$S = Q / Q_o$$

$$S_c = \frac{c_1 B_o^{\ 1/3} z^{5/3}}{Q_o}$$

$$c_1 \cong 0.1$$

Integral vs Dim Anal (WE 6-2)

- Input variables
  - $Q_o = 0.00125\ m^3/s$
  - $D_o = 0.1\ m$
  - $z = 7\ m$
  - $\Delta\rho_o/\rho = 0.025$ (salt water-fresh water)
- Derived variables
  - $B_o = Q_o\, g\, \Delta\rho_o/\rho = 0.00031\ m^4/s^3$
  - $F_o = u_o/(g\, \Delta\rho_o/\rho\ Do)^{0.5} = 1$
  - $z/D_o = 70$

Integral vs Dim Anal (cont'd)

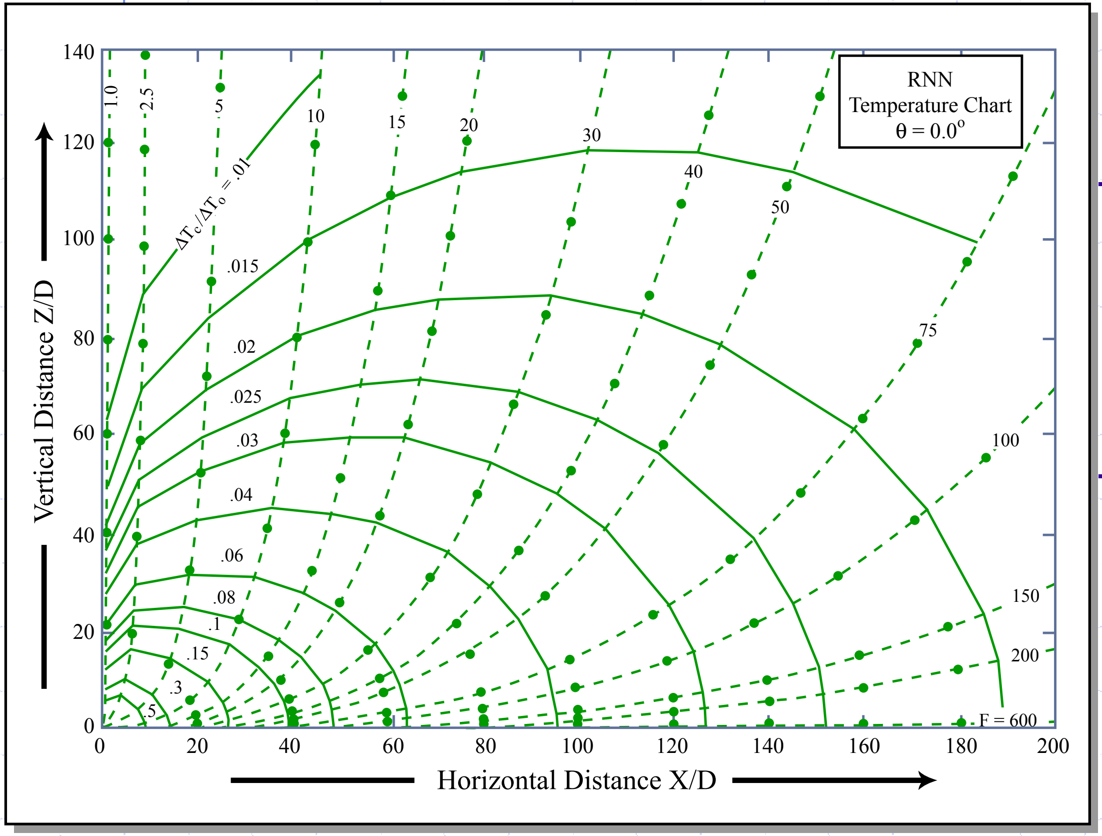
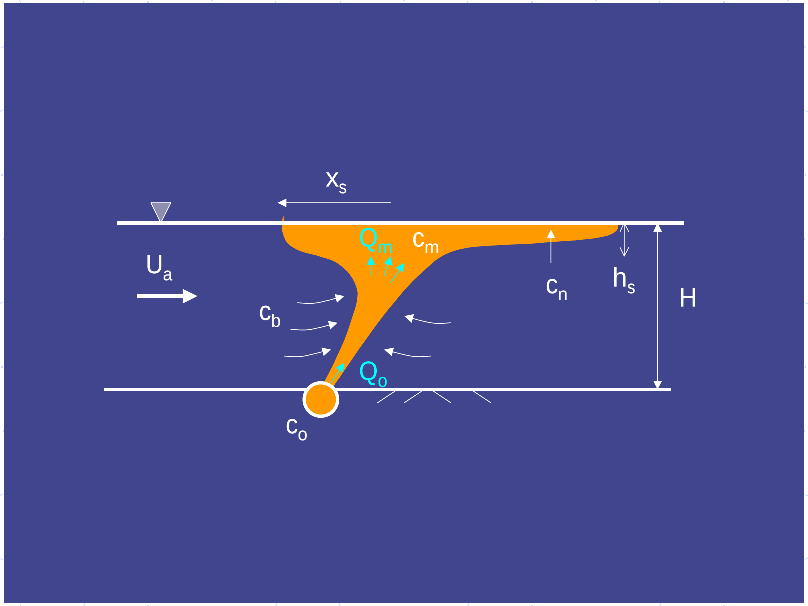


Figure by MIT OCW.

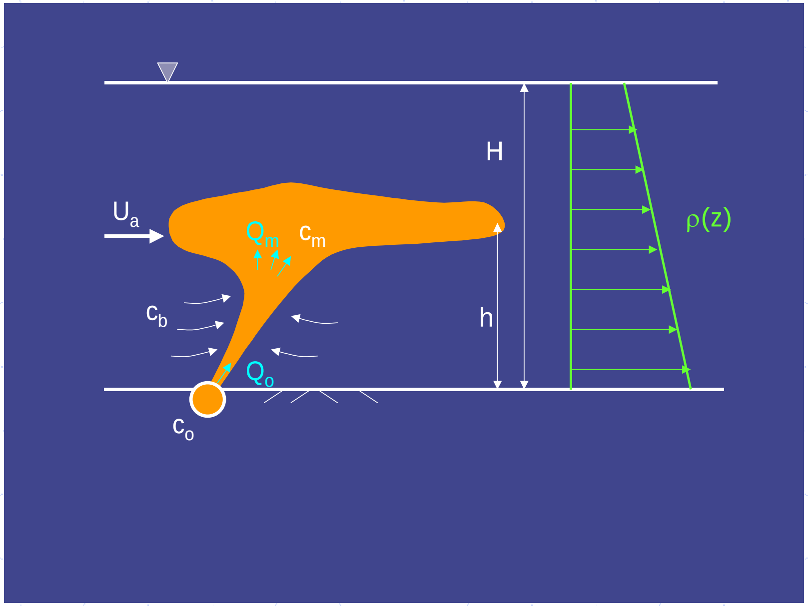
- Integral Analysis
  - $\Delta c_c/\Delta c_o = 0.008$
  - $S_c = 125$
- Dimensional Analysis
  - $S_c = 0.1 B_o^{1/3} z^{5/3} / Q_o = 138$

Blockage at surface (or trap elevation)



- Prevents entrainment of ambient water near top of trajectory
- Mixing & extra entrainment as jet "turns the corner"
- "Near field" dilution
  - $S_n = 0.26 B_o^{1/3} H^{5/3} / Q_o$
  - $X_n/H = 2.8$
- $h_s/H \sim 0.11$ (horizontal discharge)

Ambient Stratification

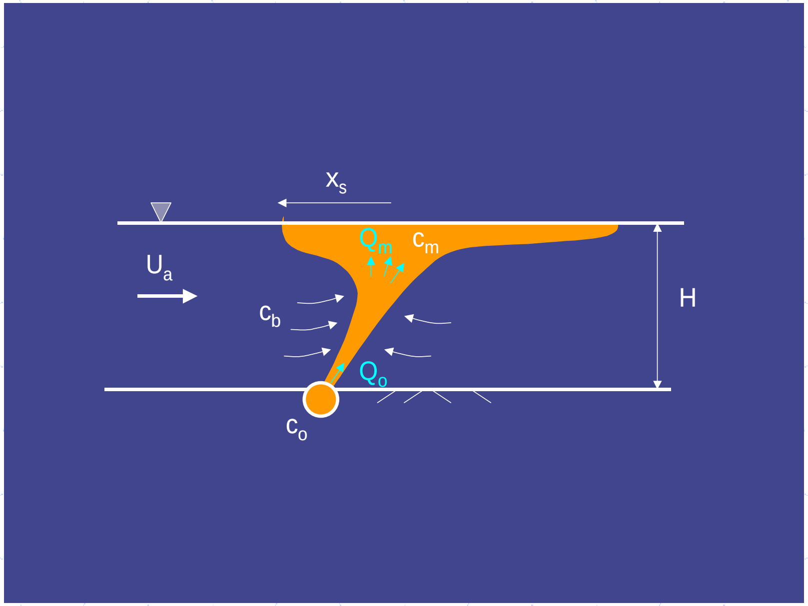


- Stratification frequency N

$$N^2 = \left| \frac{g \partial \rho}{\rho \partial z} \right|$$

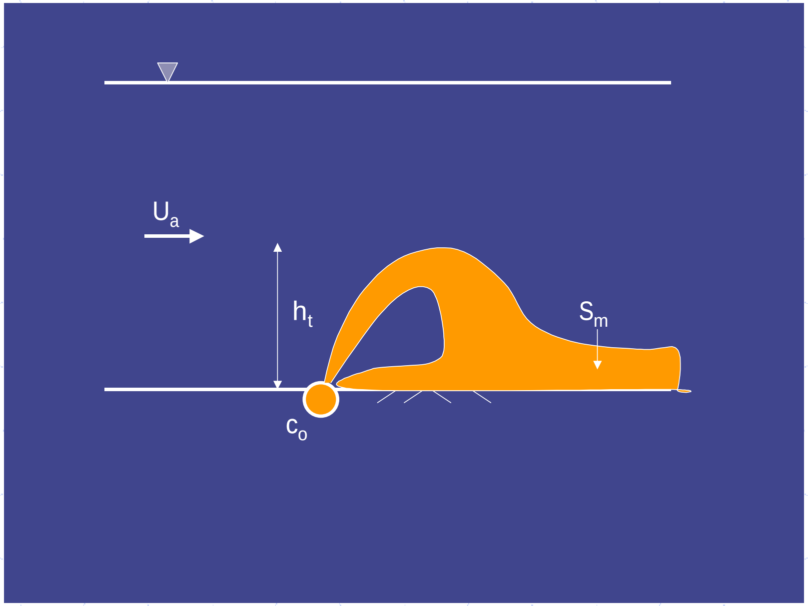
- Plume traps at level of neutral buoyancy with reduced dilution
- $h_t = 2.8 B_o^{1/4}/N^{3/4}$
- $S_m = 0.9\ B_o^{3/4}/Q_o N^{5/4}$

Ambient Current



- Deflects plume downstream
- Augments dilution if strong
- $S_m = 0.32 u_a H^2 / Q_o$
- $x_s = 0.3 Q_o (g \Delta\rho_o / \rho) / u_a^3$

Dense plumes



- Typical applications:
  - Cold water from LNG terminals
  - Brine from desal plants, sol'n mining of salt domes
- $h_t = 2.3 M_o^{3/4} / B_o^{1/2}$
- $S_m = 2.8 M_o^{5/4} / Q_o B_o^{1/2}$

Example: solution mining of salt domes

- Strategic Petroleum Reserve
  - Dates from 1970's
  - ~700×10^6 bbl stored in 4 domes in LA & TX
  - Salinity gradients in GoM confuse shrimp
- Also used for
  - Salt production
  - Compressed gas storage
  - Waste isolation

Multi-phase Plumes

- Bubble plumes
  - Reservoir destratification
  - Aeration
  - Ice prevention
  - Pollutant containment
- Droplet plumes
  - Deep oil spills
- Sediment plumes
  - Dredged mat'l disposal
  - CO_2 ocean storage

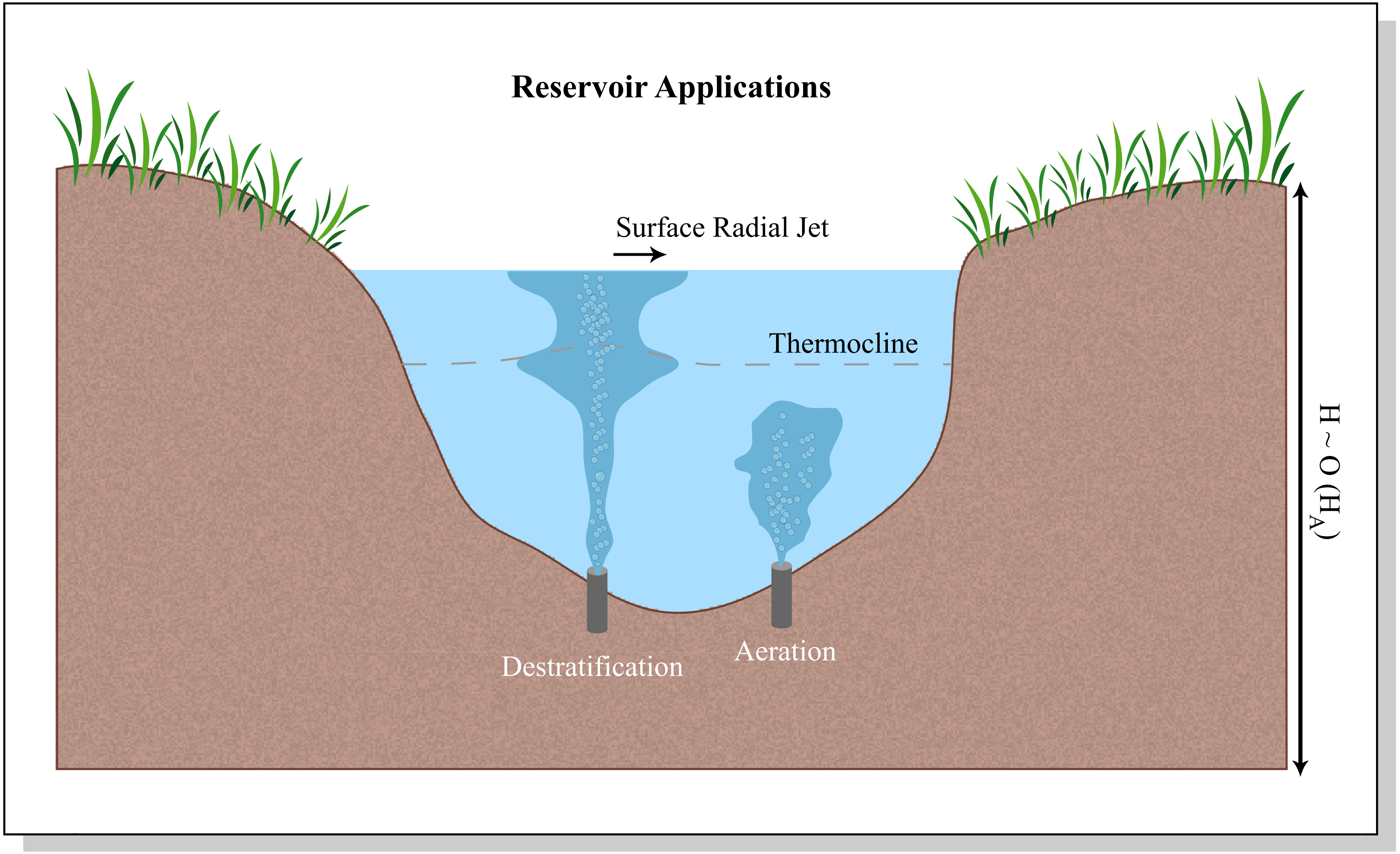


Figure by MIT OCW.

Deep Oil-well Blowout

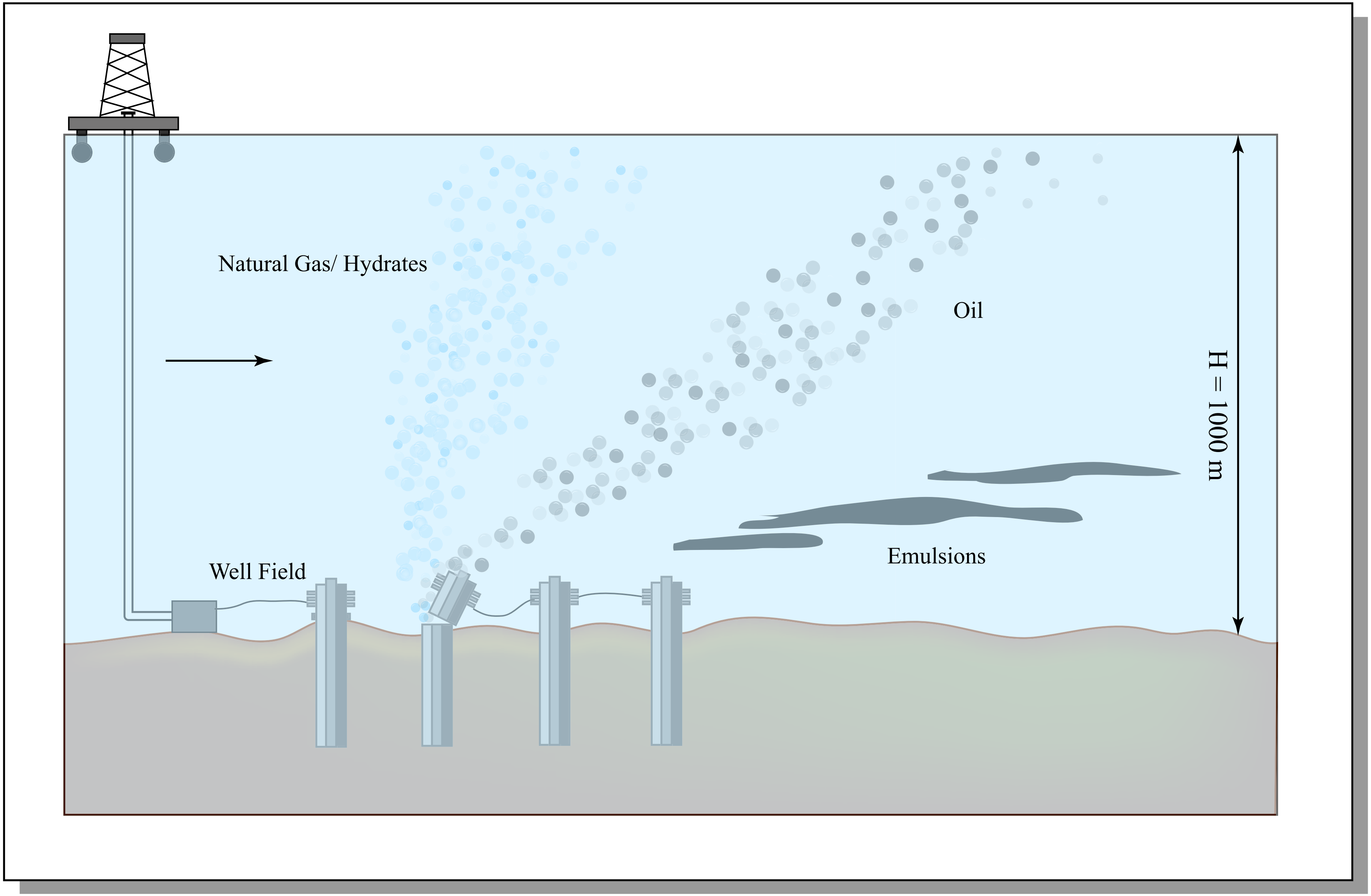


Figure by MIT OCW.

CO_2 Sequestration

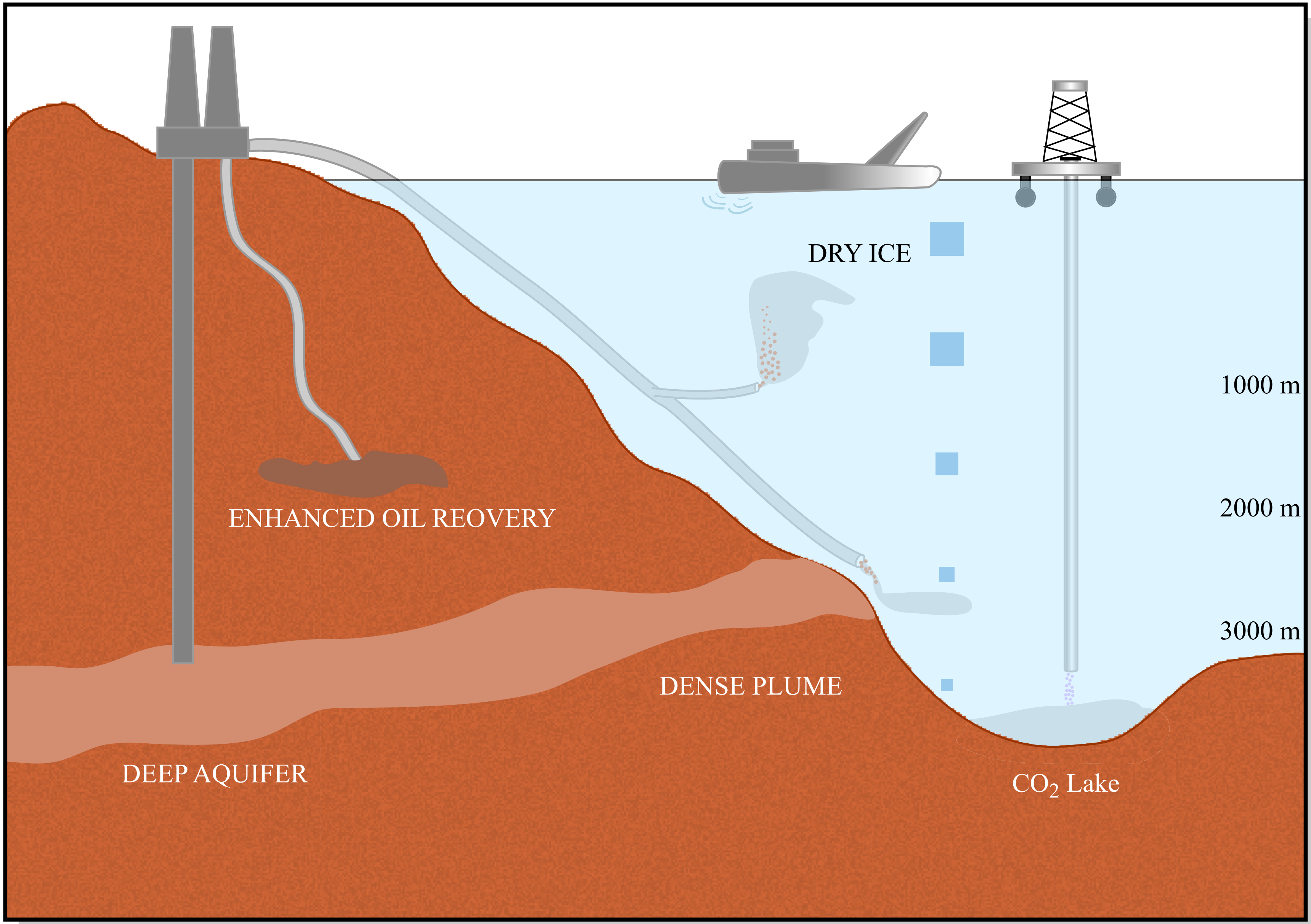
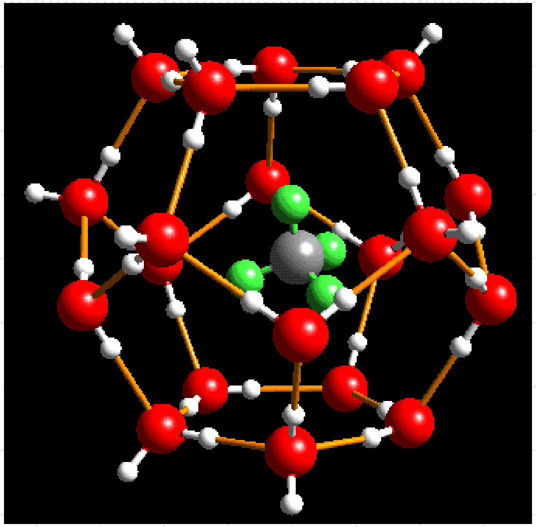


Figure by MIT OCW.

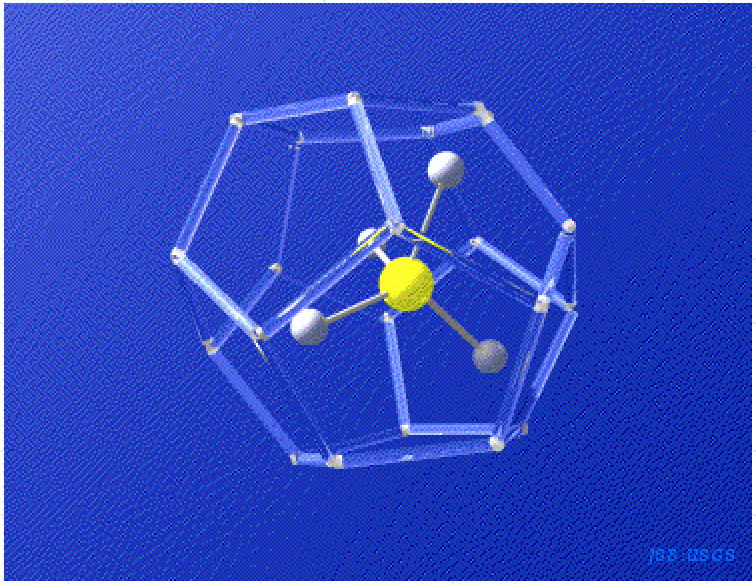
Adapted from Heroz et al. (2000).

What are gas hydrates?



“Filled ice”

Example: methane hydrate



Cage structures of gas hydrates

$$CO_2 + nH_2O \xleftrightarrow[T,P]{} CO_2 \cdot nH_2O$$

$$n \approx 5.75$$

$$\rho_h = 1100 - 1140\ \mathrm{kg/m^3}$$

CO_2/seawater phase diagram

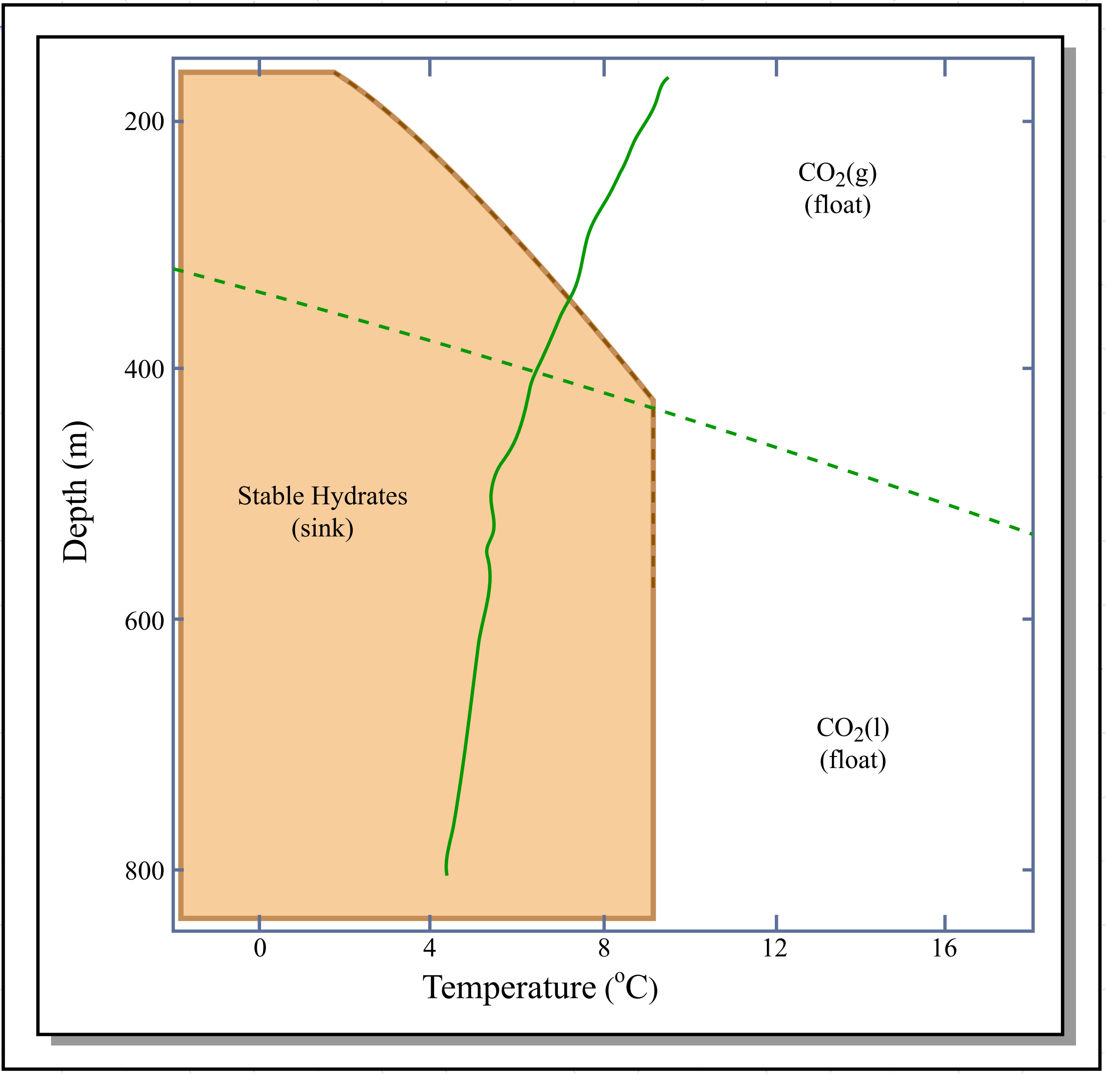
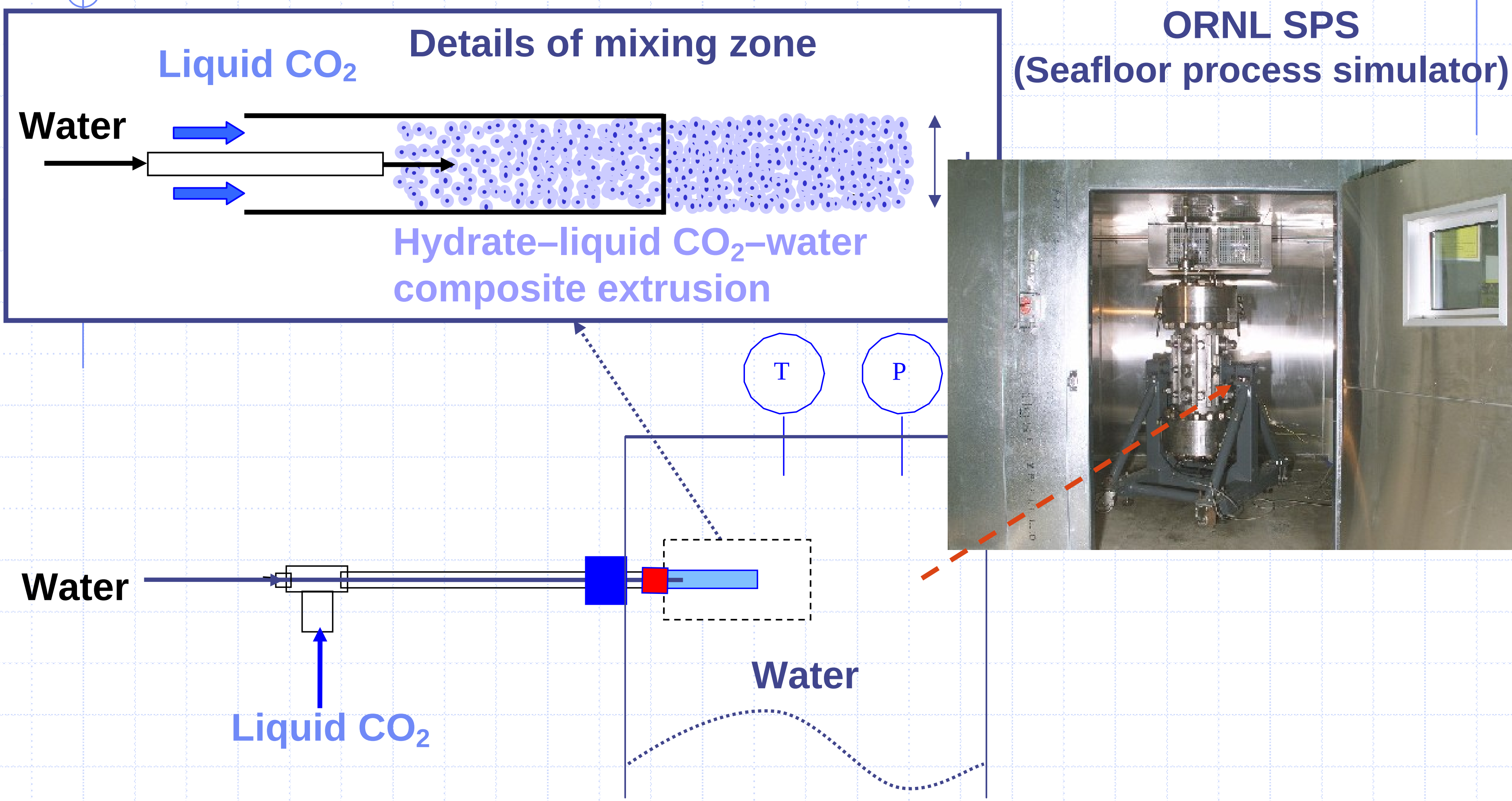


Figure by MIT OCW.

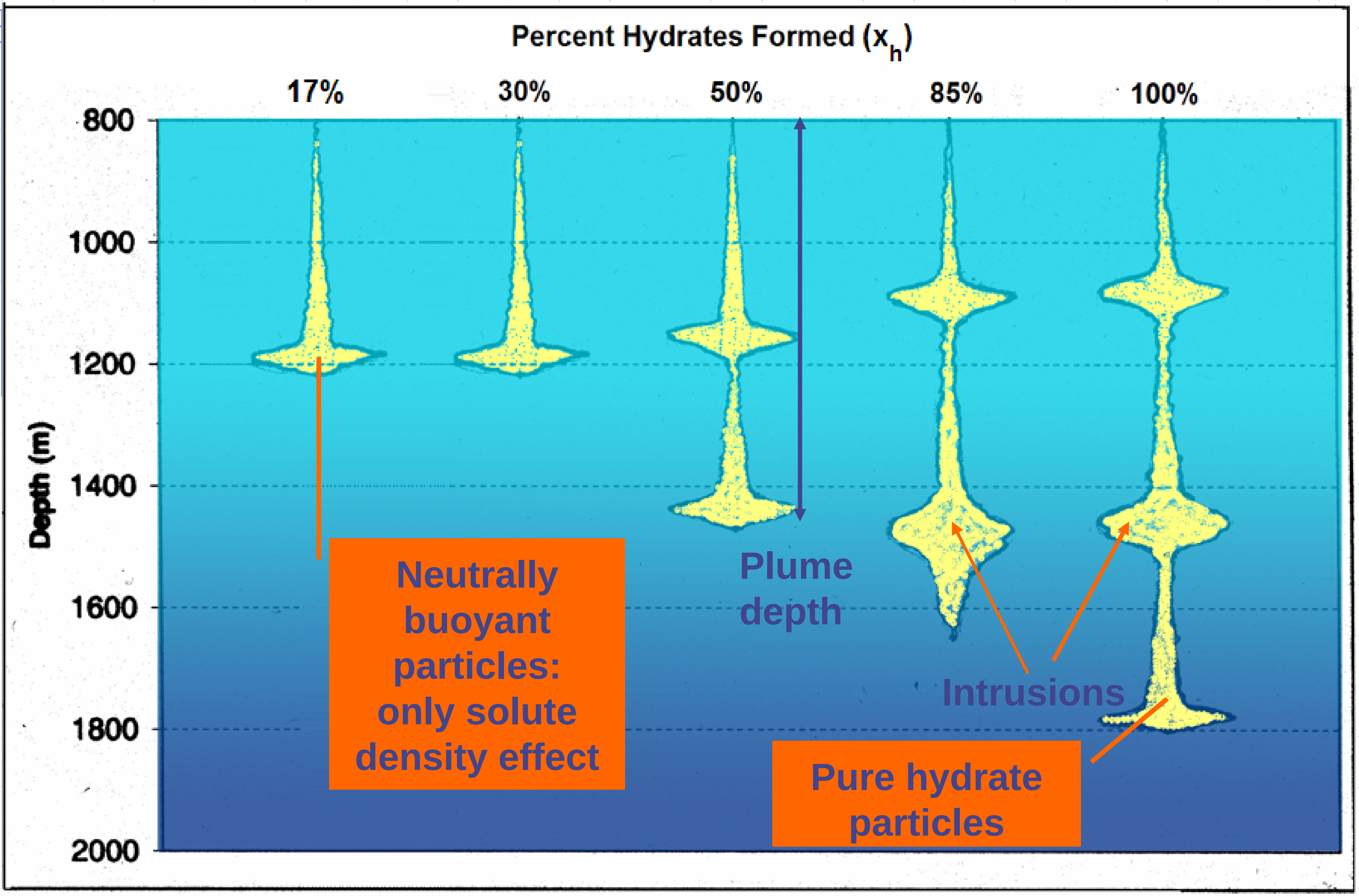
Laboratory studies (Oak Ridge National Lab)



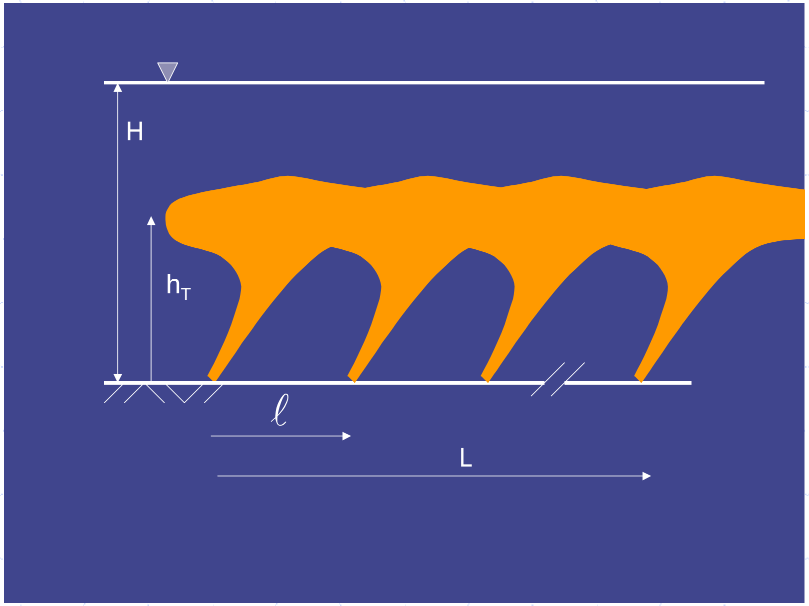
West et al., 2003; Lee et al 2003

Two-phase plume model

(100 kg/s CO_2, 1 cm diameter spheres, release depth 800 m, $Q_c/Q_w = \lambda = 0.49$)



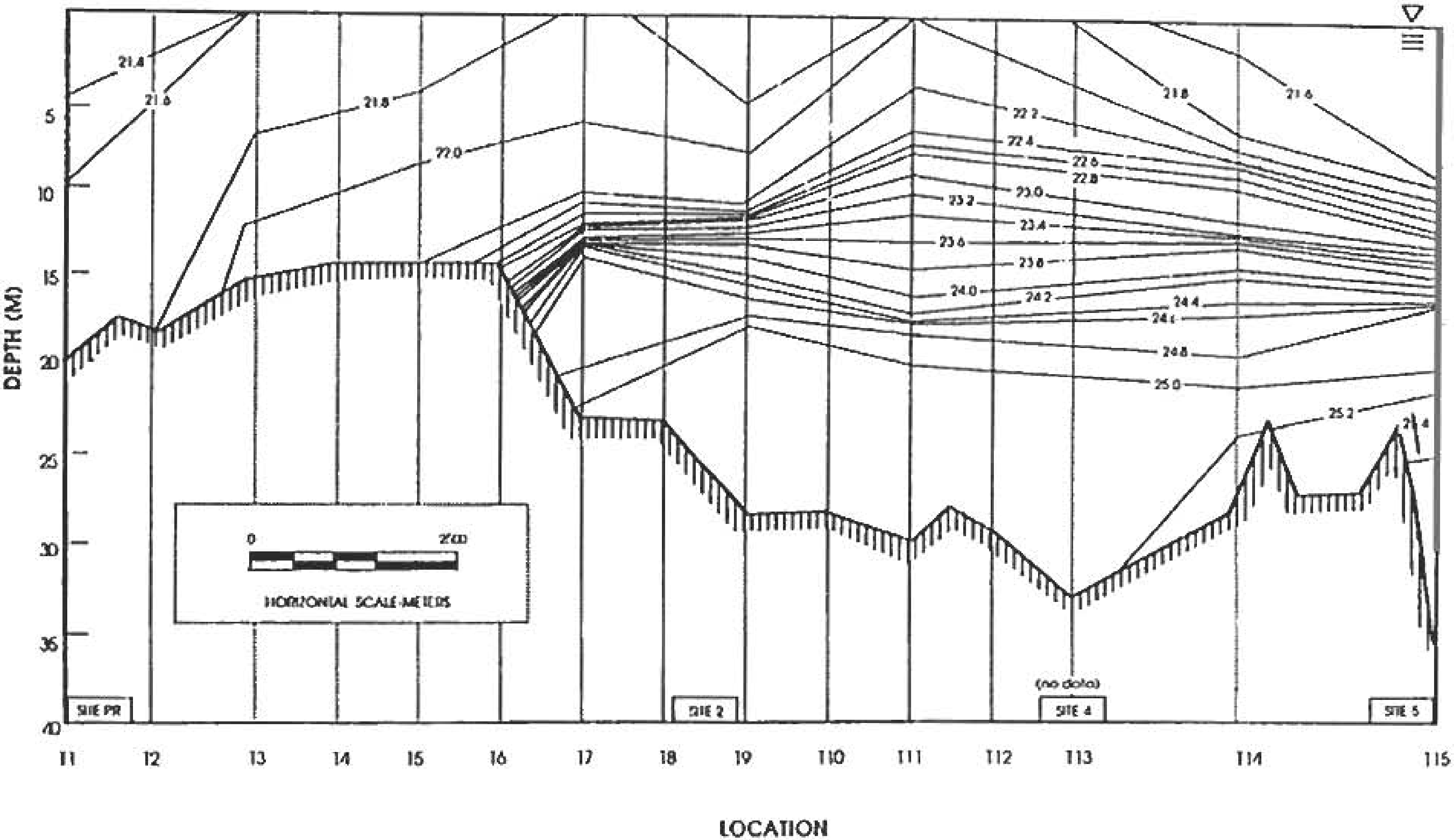
Multi-port diffusers



- Construction:
  - Cut and cover
  - Bored tunnel
- Ports
  - $\ell \sim 0.3H$ (or 0.3 h)
  - Often 2 or more per riser
- Line source approx.
  - $q_o = Q_o/L$, $b_o = B_o/L$
- No current; no strat
  - $S_m = 0.42 H b_o^{1/3}/q_o$
- No current; strat
  - $S_m = 0.97 b_o^{2/3}/q_o N$
- Current; strat
  - $S_m = 2.2 u_a^{1/2} b_o^{1/2}/N q_o$

Single vs Multiport (WE 6-3)

- Boston Outfall
  - Diffuser Length L = 2000 m
  - No ports N_p = 440
  - Flow rate Q_o = 20 m^3/s
  - Water depth H = 30 m
  - Stratification frequency $N^2 = \left|\frac{g\partial\rho}{\rho\partial z}\right|$
    - $N^2 = (9.8)(25-22)/(1025)(30) = 0.001\ s^{-2}$



MASSACHUSETTS WATER RESOURCES AUTHORITY

FIGURE 1

CROSS-SECTION OF SEAWATER DENSITY ALONG NORTHERN TRANSECT (Units of Sigma-t): 8/12/87 AM

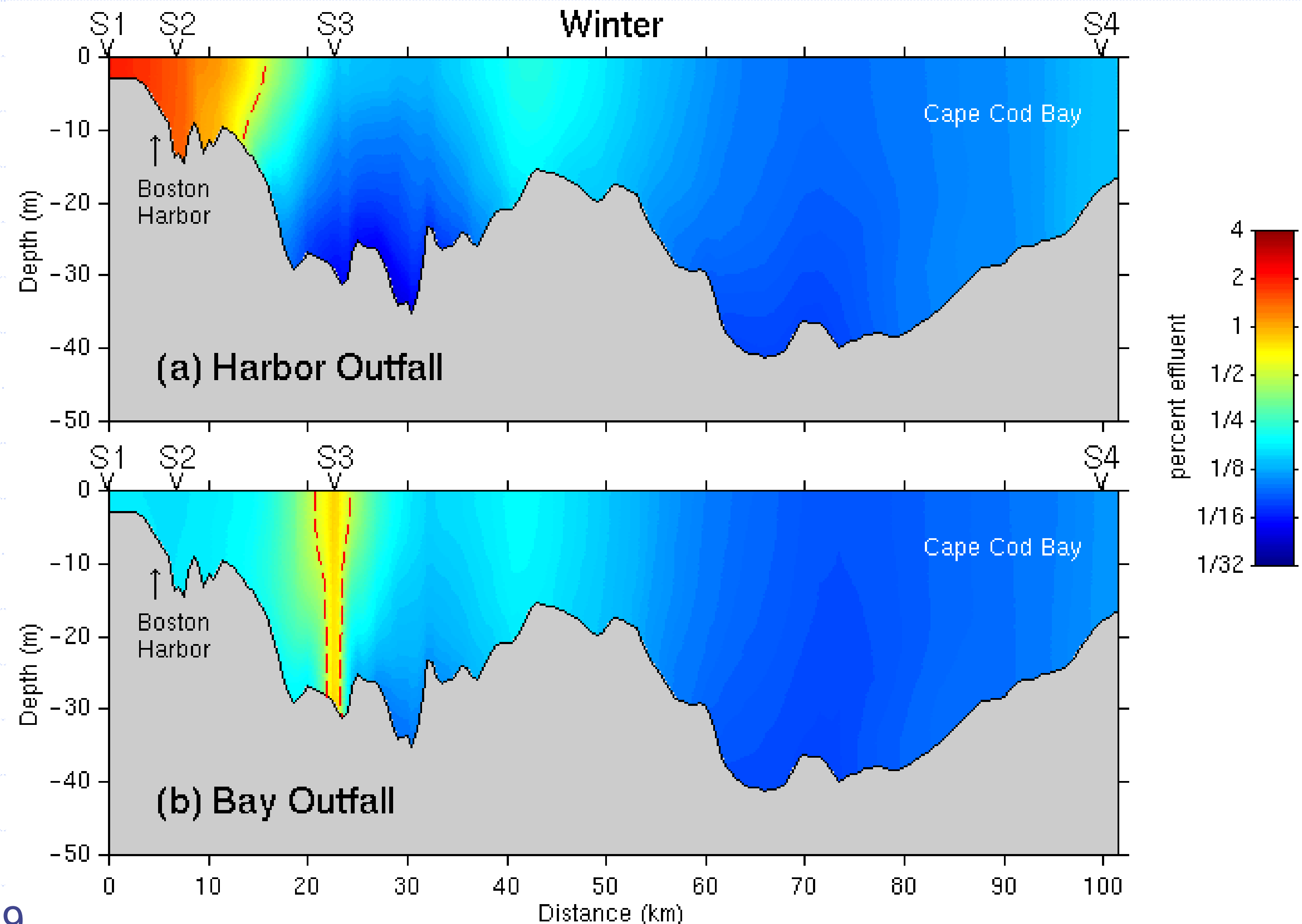
Single vs Multiport (cont'd)

- As single port
  - $Q_o = 20/440 = 0.045\ m^3/s$
  - $B_o = 0.045*9.8*0.025 = 0.011\ m^4/s^3$
  - $h_t = 2.8B_o^{1/4}/N^{3/4} = 12\ m$
  - $\ell = L/N_p = 2000/440 = 4.5\ m$
    - $\ell > 0.3\ h_t$ => no merging
  - $S_m = 0.9\ B_o^{3/4}/Q_oN^{5/4} =$ $0.9(0.011)^{3/4}/(0.045)(0.0013)^{5/8} = 51$

Single vs Multi-port (cont'd)

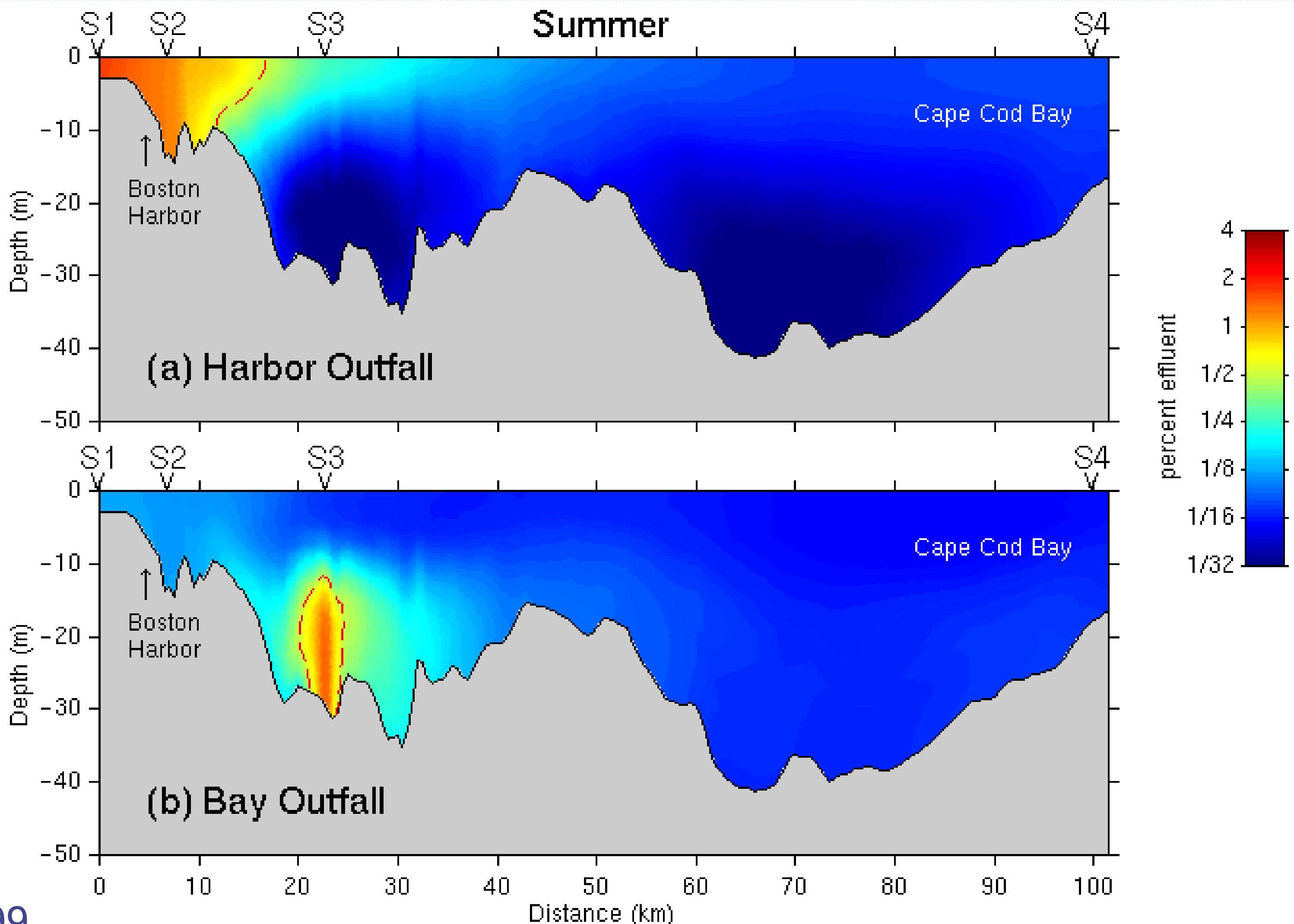
- As multi-port diffuser (line source of buoyancy)
  - $q_o = 20/2000 = 0.01\ m^2/s$
  - $b_o = 0.01*0.025*9.8 = 0.0025\ m^3/s^3$
  - $h_t = 2b_o^{1/3}/N = 2(0.0025)^{1/3}/(0.001)^{½} = 9\ m$
  - $S_m = 0.97b_o^{2/3}/q_oN = 0.97(0.0025)^{2/3}/(0.01)(0.001)^{½} = 56$

Numerical modeling of sewage outfalls?



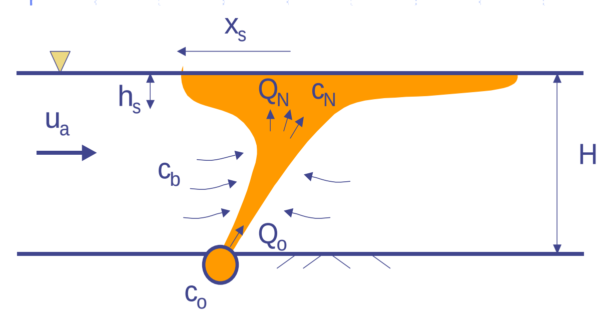
MWRA, 1999

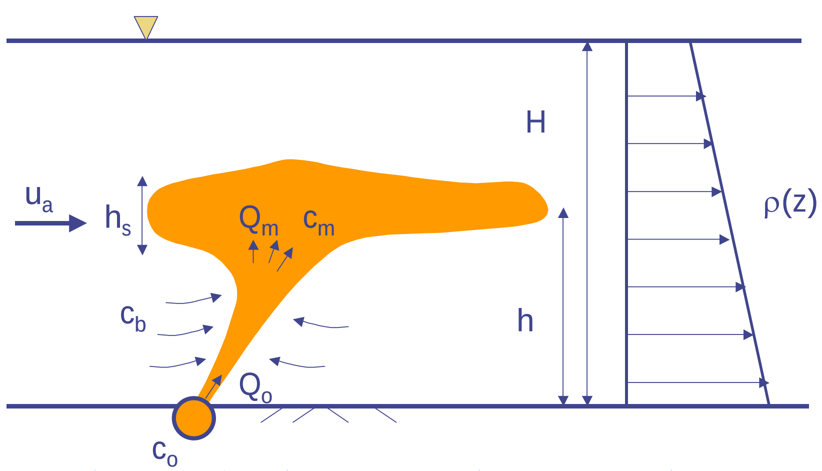
Numerical modeling of sewage outfalls?



MWRA, 1999

Gravitational spreading, intrusion, mixing





- Surface spreading layer

$$h_s = u_a^2 / g_N'$$

$$x_s = 0.3 Q_N g_N' / u_a^3$$

$$b_s = 0.8 Q_N g_N' / u_a^3$$

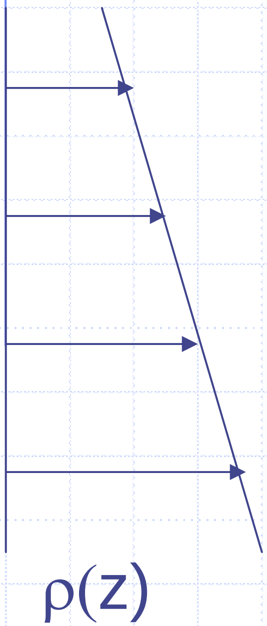
- Internal spreading layer

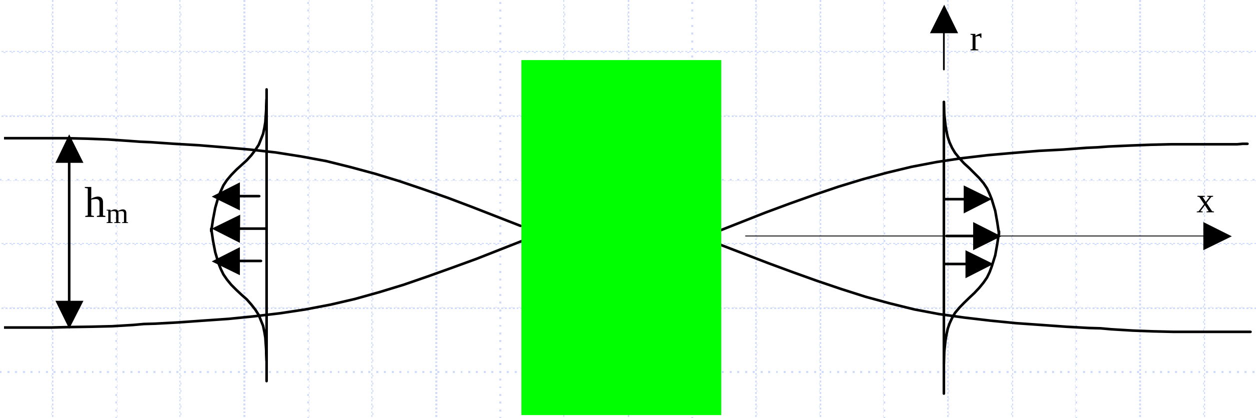
$$h_s = 1.2 u_a / N$$

$$x_s = 0.25 Q_N N / u_a^2$$

$$b_s = 0.65 Q_N N / u_a^2$$

Neutrally buoyant jet in stratification



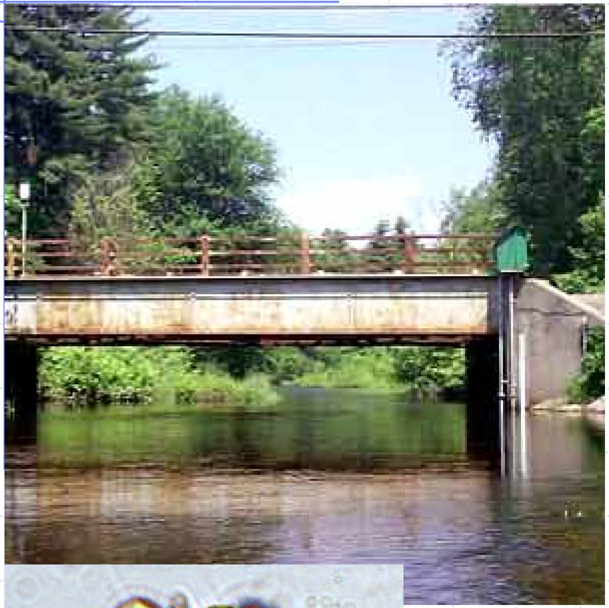


$$x_m = 3.5 M_o^{1/4} / N^{1/2}$$

$$S_m = 0.63 M_o^{3/4} / N^{1/2} Q_o$$

$$h_m = 0.95 M_o^{1/4} / N^{1/2}$$

Wachusetts Reservoir Algae



- Occasional taste and odor problems
  - Synura (left)
  - Chrysosophaerella
- Algal locations
  - Hypolimnion
  - Metalimnion
  - Under ice
- Conventional treatment (surface algae) with $CuSO_4$ from boat
- How to efficiently treat (place algaecide in proper stratum) under ice & at depth?

Layout of Treatment System
(potential system being discussed)

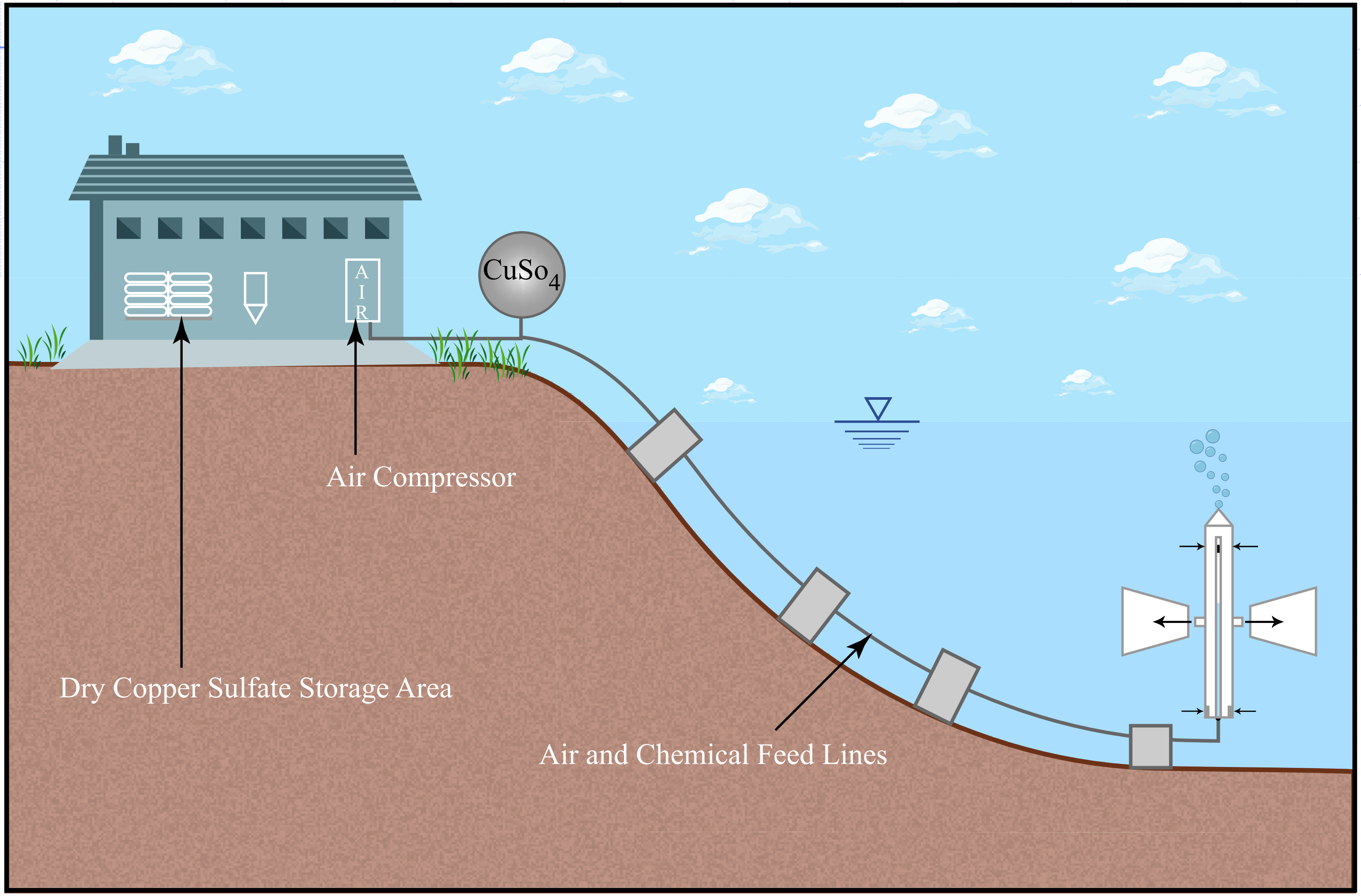
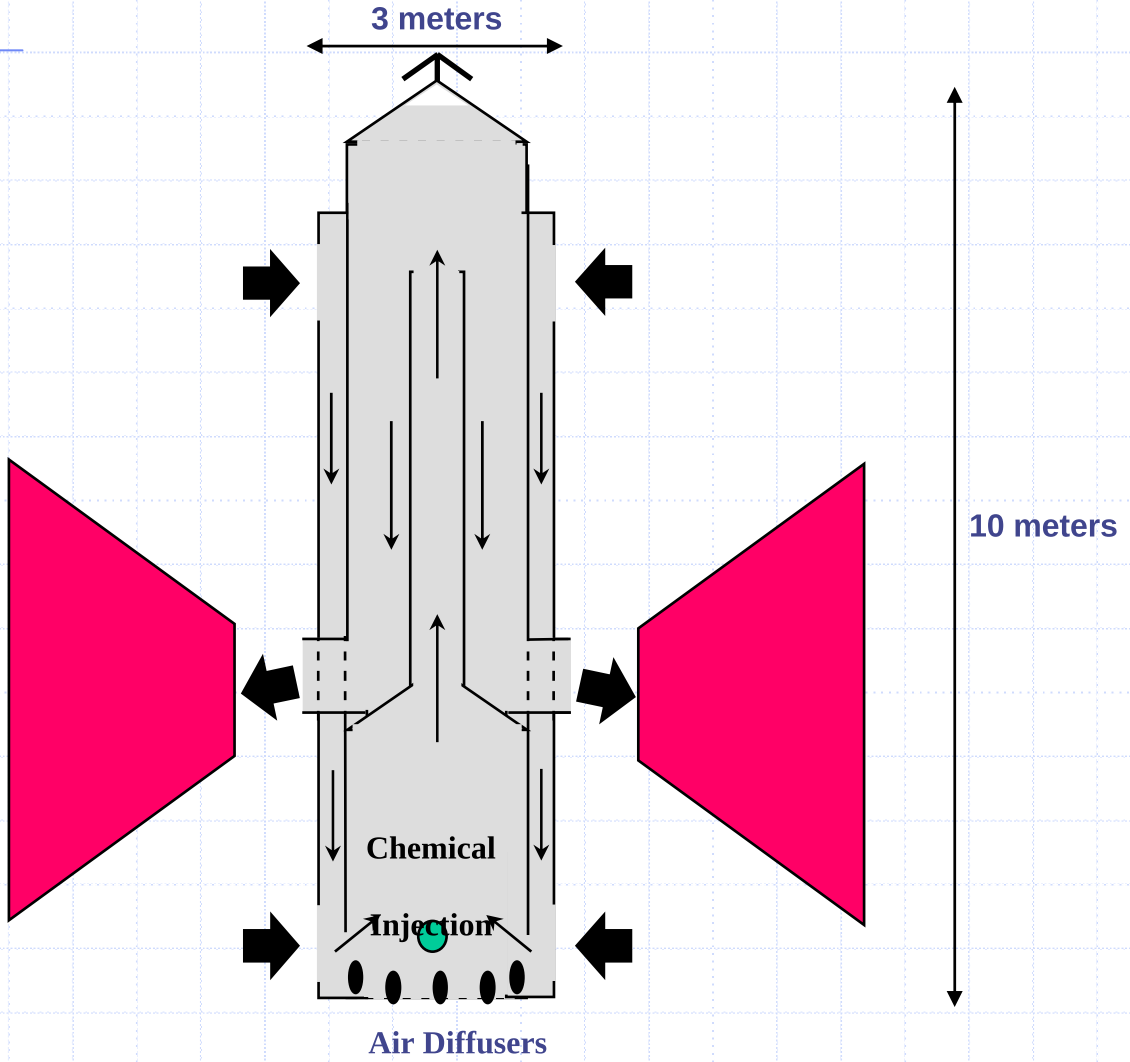


Figure by MIT OCW.

CDM, 2005

Mid-Depth Air Driven Circulator



Application at Depth

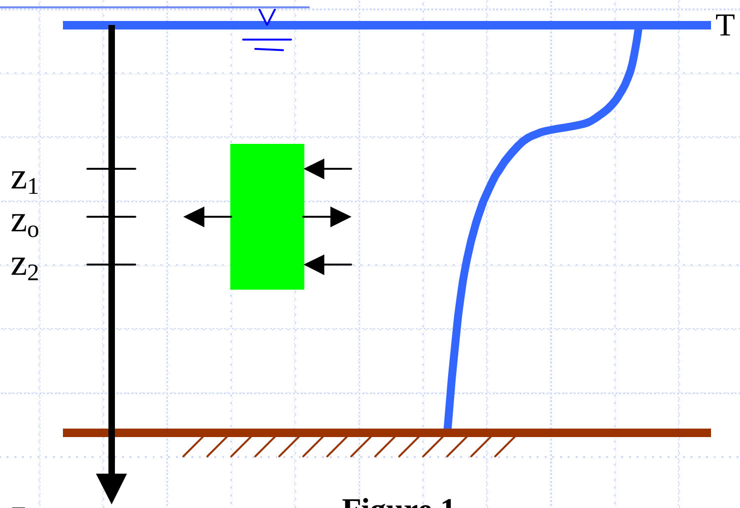
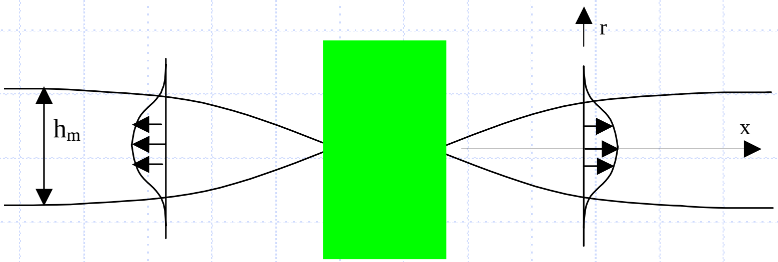
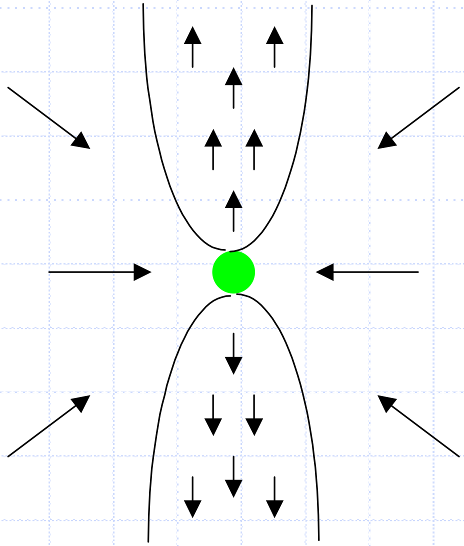


Figure 1

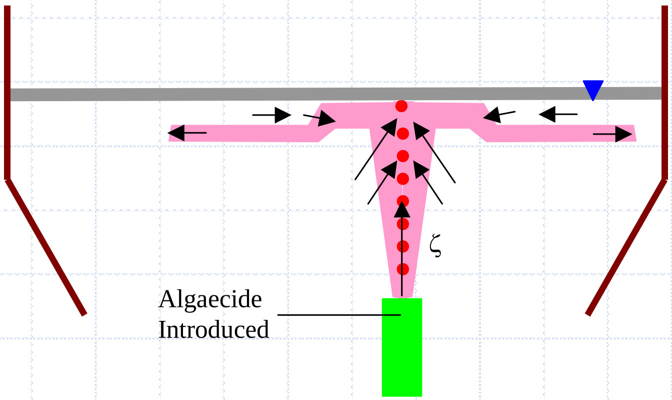


Elevation view


Plan view

Length, thickness and dilution (hence required operation time) depend on reservoir stratification and discharge momentum

Application under Ice



Relies on bubble plume to transport algaecide to surface

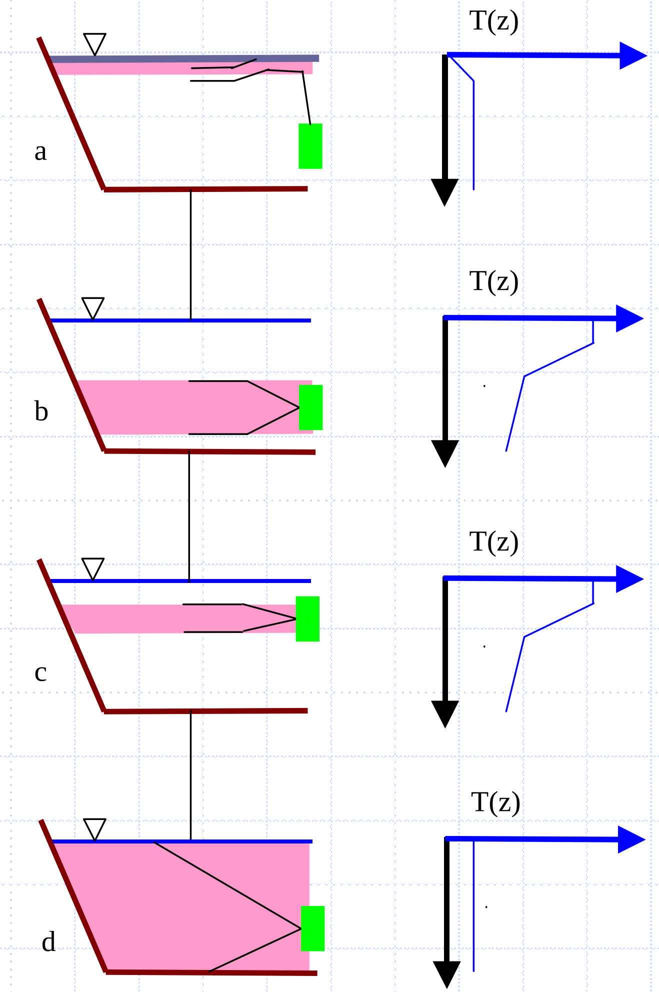


Figure 5

Multi-port diffusers in shallow water

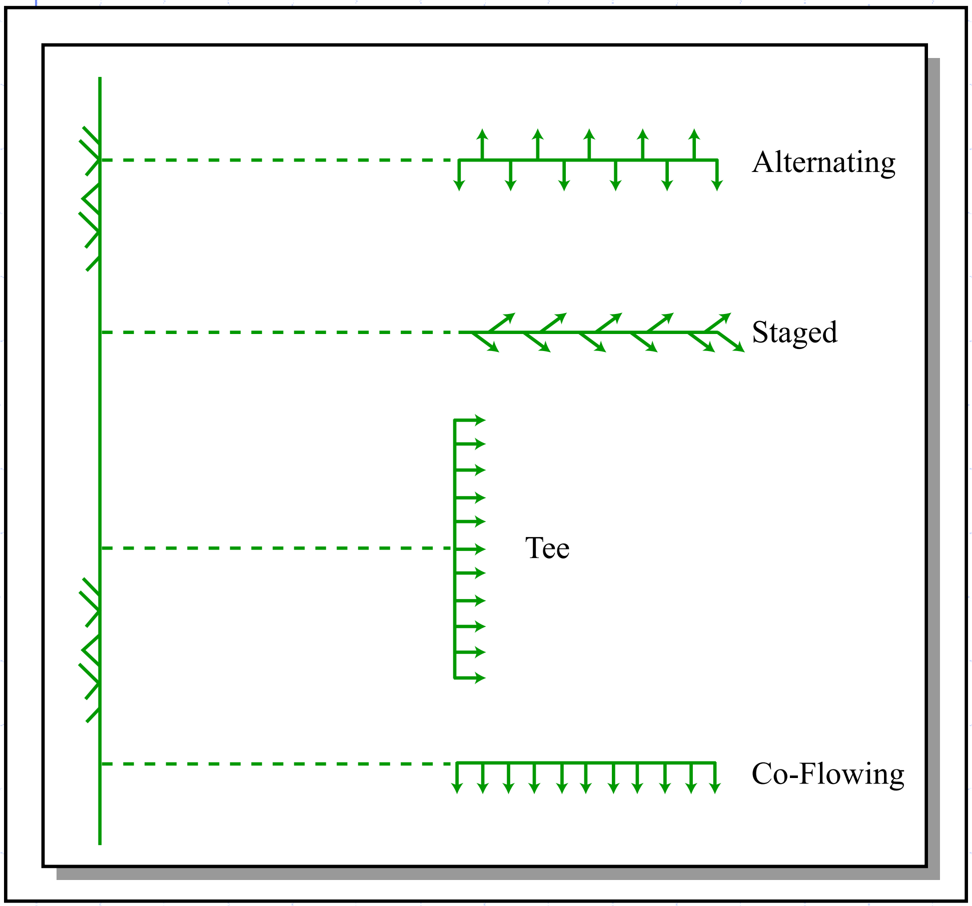


Figure by MIT OCW.

- Typical for power plant (thermal) discharges

$$S_a = \sqrt{\frac{0.26 g'^{2/3} H^2 L^{4/3}}{Q_o^{4/3}} + \left(\frac{u_a HL}{Q_o}\right)^2}$$

$$S_s = \frac{0.5 u_a HL}{Q_o} + \sqrt{\left(\frac{0.5 u_a HL}{Q_o}\right)^2 + \frac{0.19 HL u_o}{Q_o}}$$

$$S_t = \sqrt{\frac{HL u_a^2}{2 Q_o u_o + 10 u_a^2 HL}}$$

$$S_c = \frac{0.5 u_a HL}{Q_o} + \sqrt{\left(\frac{0.5 u_a HL}{Q_o}\right)^2 + \frac{0.5 HL u_o}{Q_o}}$$

Buoyant surface discharges

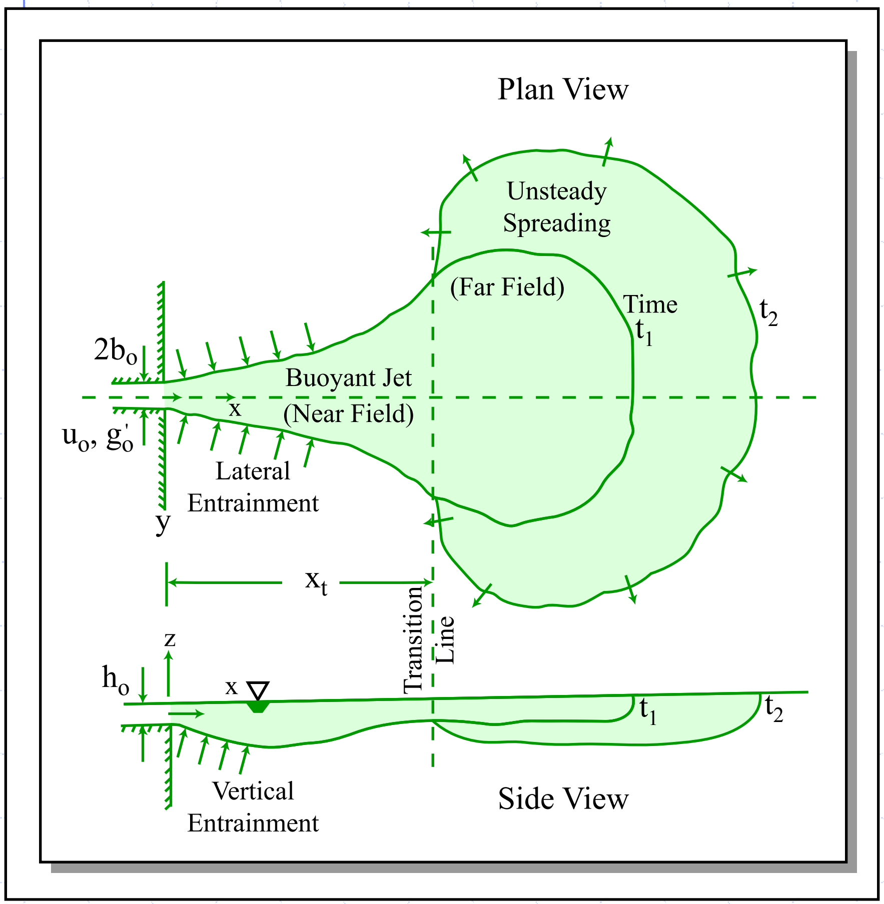


Figure by MIT OCW.

- Thermal plumes and river discharges
- Independent variables

$$F_o' = \frac{u_o}{\sqrt{g(\Delta\rho_o/\rho)\ell_o}}$$

$$\ell_o = \sqrt{h_o b_o}$$

- Dependent variables
  - $S = 1.4F_o'$
  - Lengths $\sim F_o' \ell_o$

Combined near and far field analysis
(accounting for background build-up)

Far Field Dilution

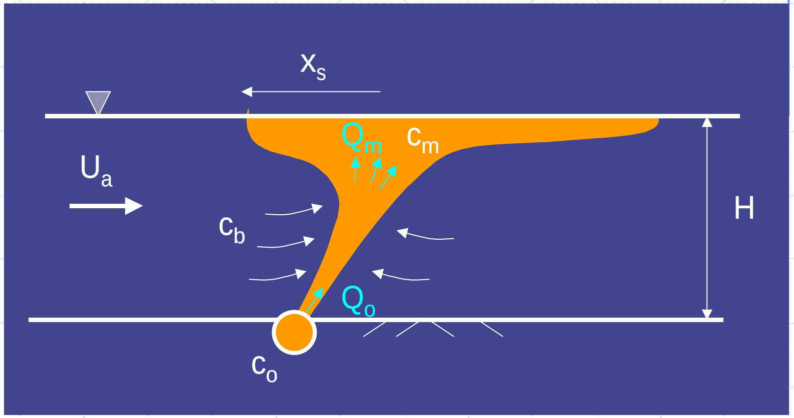
$$S_F = \frac{c_o - c_a}{c_F - c_a}$$

Near Field Dilution

$$S_N = \frac{c_o - c_F}{c_N - c_F}$$

Total Dilution

$$S_T = \frac{c_o - c_a}{c_N - c_a}$$

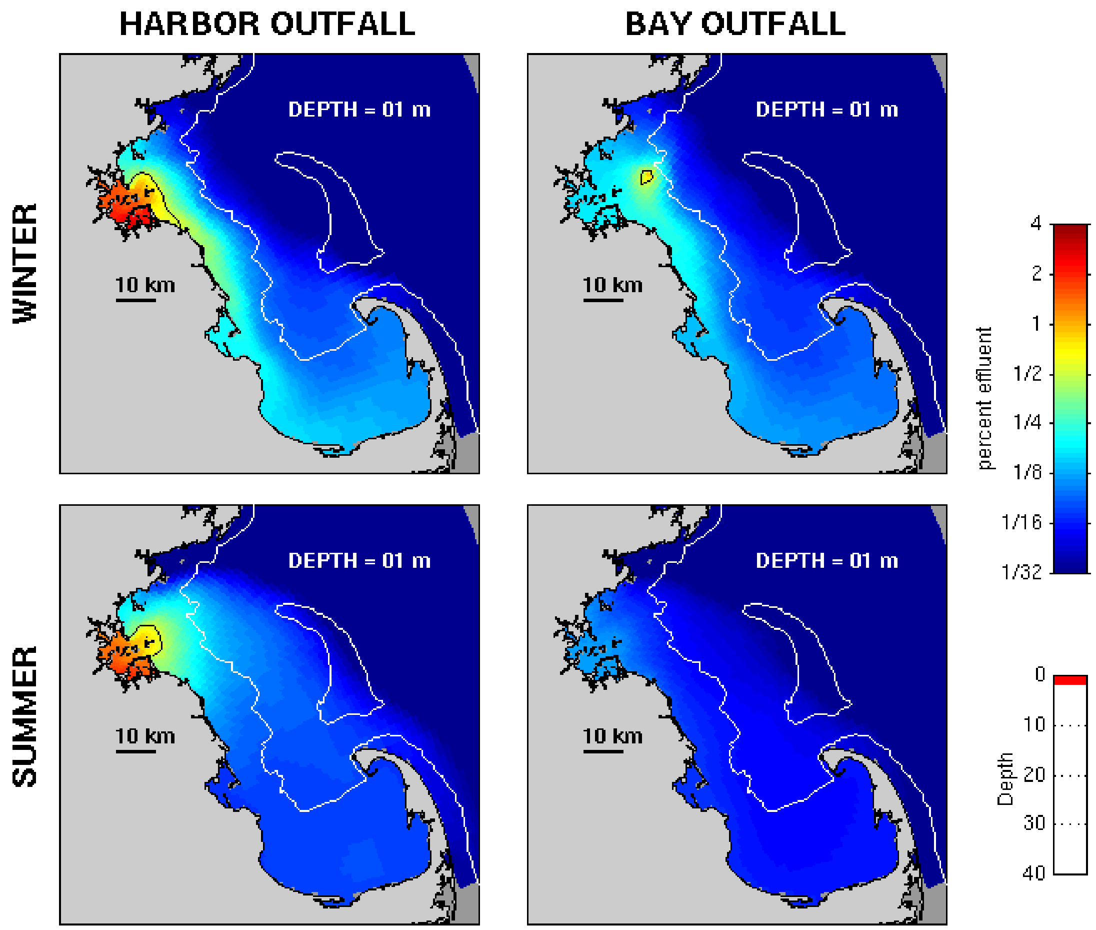


$$\frac{1}{S_T} = \frac{1}{S_N} + \frac{1}{S_F} - \frac{1}{S_N S_F} \cong \frac{1}{S_N} + \frac{1}{S_F}$$

Total dilution less than either near field or far field dilution and controlled by the smaller of the two

Example

- Far field dilution S_F = 50 to 100
- Near Field dilution S_N = 50 to 100
- Total Dilution S_T = 25 to 33 to 50



MWRA, 1999