# CHAPTER 13

# THE SEDIMENT TRANSPORT RATE

## INTRODUCTION

**1** By the ***sediment transport rate***, also called the ***sediment discharge***, I mean the mass of sedimentary material, both particulate and dissolved, that passes across a given flow-transverse cross section of a given flow in unit time. (Sometimes the sediment transport rate is expressed in terms of weight or in terms of volume rather than in terms of mass.) The flow might be a unidirectional flow in a river or a tidal current, but it might also be the net unidirectional component of a combined flow, even one that is oscillation-dominated. Only in a purely oscillatory flow in which the back-and-forth phases of the flow are exactly symmetrical is there no net transport of sediment. Here we focus on the particulate sediment load of the flow, leaving aside the dissolved load, which is important in its own right but outside the scope of these physics-based notes.

**2** Over the past hundred-plus years, much effort has been devoted to accounting for, or predicting, the sediment transport rate. Numerous procedures, usually involving one or more equations or formulas, have been proposed for prediction of the sediment transport rate. These are commonly called "sediment-discharge formulas". (The term "formula" here is in some cases a bit misleading: some of the procedures involve the use of reference graphs in addition to mathematical equations.) No single formula or procedure has gained universal acceptance, and only a few have been in wide use. None of them does anywhere near a perfect job in predicting the sediment transport rate—which is understandable, given the complexity of turbulent two-phase sediment-transporting flow and the wider range of joint size–shape frequency distributions that are common in natural sediments. Prediction of the sediment transport rate is one of the most frustrating endeavors in the entire field of sediment dynamics.

**3** In this brief chapter we focus on the concept of the sediment transport rate more than on the procedures by which it might be predicted. it would take a lot of additional space in these course notes to do justice to the details of even the small number of sediment-discharge formulas that are in common use.

## THE SEDIMENT LOAD AND THE SEDIMENT TRANSPORT RATE

### *The Sediment Load*

**4** First you must be clear on the distinction between the sediment load and the sediment transport rate. Recall from Chapter 10 that the load is all of the sediment that is being moved by the flow at a given time. Figure 13-1 shows how to conceptualize the sediment load. In Figure 13-1, you can imagine somehow freezing a block of the flow that contains both water and particulate sediment, and then melting the block to collect the sediment in the block. That sediment is the load. You can think of the sediment load as the depth-integrated sediment mass above a unit area of the sediment bed:

$$L = \text{sediment load} = \int_0^d c(y)dy$$

where $c$ is the local time-average sediment concentration. Then the average concentration of transported sediment, $C$, is equal to $L/d$.

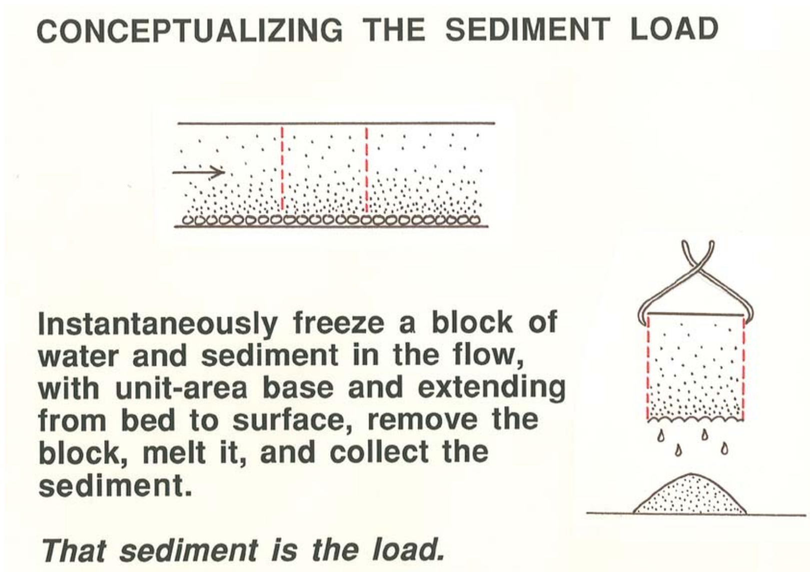


Figure 13-1. Conceptualizing the sediment load.

**5** Just as a review of what was said about the sediment load back in Chapter 10, here are some points or comments about the sediment load:

- There is no fundamental break between the bed load and the suspended load.
- For a given particle that is susceptible to suspension in a given flow, the particle at various times might be traveling as either bed load or as suspended load, or it might temporarily be at rest on the bed surface or within the active layer.

- The ranges of particle size for the bed load and the suspended load in a given flow overlap.
- The suspended bed-material load is not really "suspended"; it is merely traveling, temporarily, in the turbulent flow above the bed.
- The bed-load layer is thin relative to the suspended-load layer.
- The bed-load layer is the lower boundary condition of the suspended-load layer.
- The sediment concentration in the bed-load layer is ordinarily much greater than that in the suspended-load layer.

### *The Sediment Transport Rate*

**6** The sediment transport rate is commonly denoted by $Q_s$. What is more useful, however, and what you are likely to encounter if you have to deal with sediment transport, is the sediment transport rate *per unit width of the flow*. That is called the ***unit sediment transport rate***; it is often denoted by $q_s$. Think in terms of a vertical slice of the flow, with unit width and oriented parallel to the flow. Which you use depends upon whether you are interested in how much sediment the entire flow carries ($Q_s$) or in the inherent intensity of the sediment transport ($q_s$).

**7** Below are descriptions of three ways of conceptualizing the sediment transport rate. Each represents, in principle although not necessarily in practice, a way of measuring the sediment transport rate.

*The magic screen:* Obtain a magic screen, which, when installed across the flow, allows you to measure the mass $m_i$ of each of the $n$ particles that pass across the screen in unit time (Figure 13-2). Then

$$q_s = \left( \sum_1^n m_i \right) \Big/ width\, of\, flow$$

*The magic vacuum suction trap:* Install a slot, across the entire width of the flow, that allows you to remove all of the particles, both bed load and suspended load, that pass across the cross section of the flow above the slot (Figure 13-3). Think in terms of a magic vacuum cleaner that sucks all of the sediment particles out of the flow and into the trap. (In real life, that would not be extraordinarily difficult for the bed load but virtually impossible for the suspended load.) Suppose that you thereby extract a mass $M$ of sediment that would have been transported across the location of the cross section in an interval of time $T$. Then the unit sediment transport rate $q_s$ would be equal to $M/T$ divided by the width of the flow.

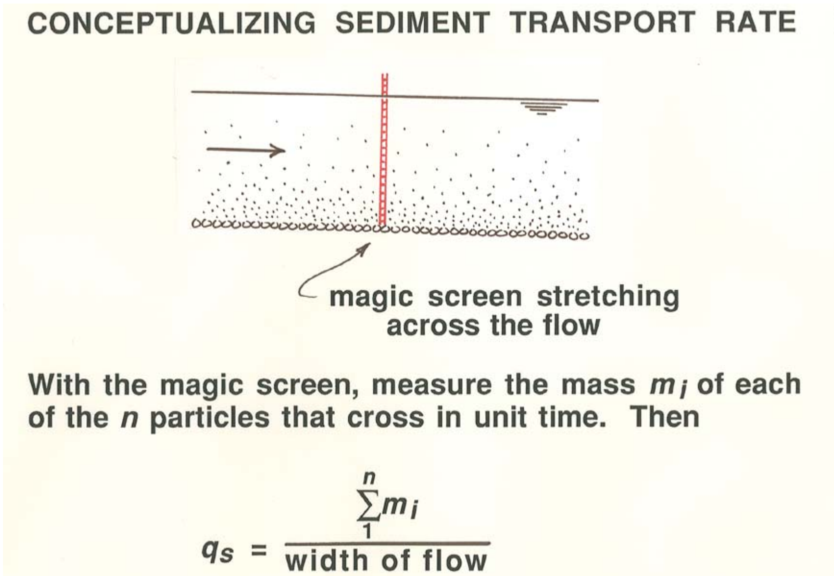


Figure 13-2. Conceptualizing the measurement of the sediment transport rate by use of a magic screen.

*Depth-integrated sampling:* (Figure 13-4) Along a vertical in the flow, measure the downstream component of velocity $v_i$ of all of the particles in a tiny imaginary cube in the flow, with volume $V$, at a given instant. Then multiply $v_i$ by the mass $m_i$ of the particle, and sum over all $n$ particles found. Divide the result by $V$ to obtain the transport rate per unit area, and integrate the result over flow depth on a vertical traverse. That give you $q_s$ for that cross-stream position in the flow.

**8** It is notoriously difficult to measure the sediment transport rate, even in controlled settings in laboratory flumes. In a flume, if only bed load is being transported, you can arrange a sediment trap in the form of a narrow slot extending across the entire channel, transverse to the flow direction. Provided that the width of the trap is at least as great as the longest excursions of bed-load particles, all of the bed load falls into the trap, to be collected and weighed. A warning is in order, however: the sampling time must short enough that the deficit in transport does not propagate, by recirculation of the sediment, back to the trap.

**9** In a small stream you can catch the passing bed load by building a dam across the flow, and catch the load in a basket under the overfall across the dam.

the problem is that in building the dam you are changing the nature of the stream and its sediment transport for some distance upstream of the dam.

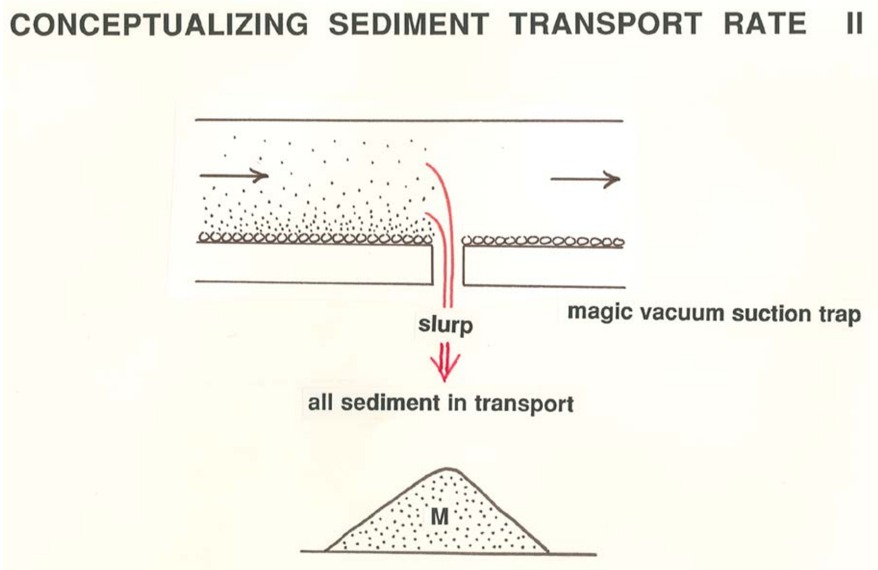


Figure 13-3. Conceptualizing the measurement of the sediment transport rate by use of a magic vacuum suction trap.

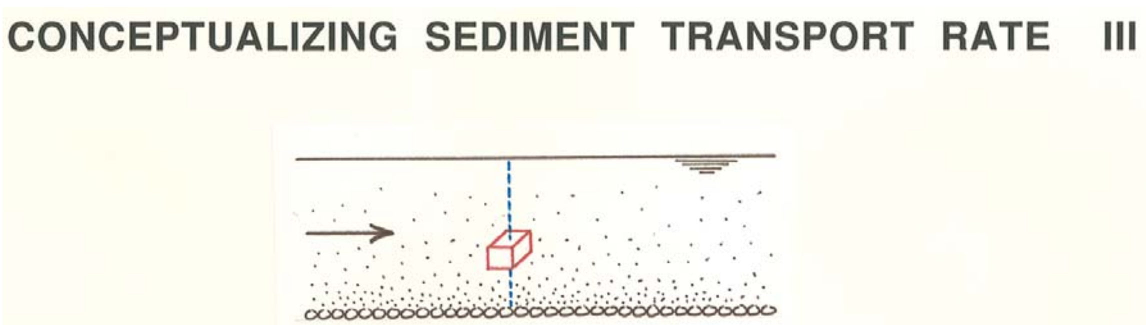


Figure 13-4. Conceptualizing the measurement of the sediment transport rate by use of depth-integrated sampling.

**10** Various kinds of portable bed-load traps have been devised and are in common use. Generally they consist of a receptacle that is open to the flow on the upstream side and screened to pass the water, but catch the sediment, on the downstream side. They are placed on the sediment bed for a time sufficient to catch a measurable quantity of the passing bed load. No matter how well designed, however, such traps distort the flow in their vicinity to a certain extent, and also, if there are rugged bed forms like ripple or dunes on the bed, then the catch depends strongly upon where the trap is placed relative to the crests and troughs of the bed forms.

**11** Measuring the suspended load is a simpler matter, at least in principle. What is commonly done is to trap a volume of passing flow, which contains the suspended load at that particular height above the bed, and combine that with the mean flow velocity at the given level to obtain the proportion of the entire sediment transport rate associated with a narrow interval of the flow depth. If that is done at a large number of heights above the bed, the combined result is a good measure of the suspended-load transport rate. (This is akin to the procedure outlined in the section on depth-integrated sampling, above.)

### *What Is the Relationship between the Sediment Load and the Sediment Transport Rate?*

**12** Here is a question for you to ponder. Look back at Figure 13-1 and think about the particle size distribution you would find in the pile of sediment that you obtained by melting that instantaneously frozen block of the flow. Now look back at Figure 13-3 and think about the particle size distribution you would measure in the pile of sediment you obtained by magically vacuuming out all of the sediment passing by the location of the slot trap. *Would those two size distributions be the same or different?*

**13** Just a moment’s reflection should convince you that the two distributions would be the same if and only if the transport velocities of each of the particle size fractions in the sediment are the same. If they are not the same—and in general they are not the same, because, at least to a certain extent (we will look at that in more detail in Chapter 3), the coarser fractions tend to move more slowly than the finer fractions—then the particle size distributions will be different in the two cases. That highlights the fundamental difference between the sediment load and the sediment transport rate.

## PREDICTING THE SEDIMENT TRANSPORT RATE

### *The Variables That Govern the Unit Sediment Transport Rate*

**14** It should seem natural to you to make a list of all of the important variables and physical effects that govern the sediment transport rate. Once we have such a list, we can frame our consideration of the sediment transport rate by expressing the sediment transport rate, in dimensionless form, in terms of a natural or convenient set of governing dimensionless variables. Even if for no other reason, such a functional relationship should guide your thinking about the various sediment-discharge formulas you might encounter in the literature on sediment transport.

**15** Here is a list of the important physical effects on sediment transport rate, together with the variables associated with those physical effects; see Figure 13-5.

- Fluid forces on bed-surface particles: This is what moves the sediment. These fluid forces involve the bed shear stress $\tau_o$, the fluid properties $\rho$ and $\mu$, and the particle size $D$.
- The submerged weight of the particles is what resists the forces that tend to cause particle movement. It depends on the submerged specific weight of the particles, $\gamma'$, and the particle size $D$.
- The relative inertia of the sediment particles might have an effect on the sediment transport rate. It depends on the density of the sediment, $\rho_s$, and the density of the fluid, $\rho$.
- Turbulent diffusion of particles is important by virtue of its role in distribution sediment in suspension upward in the turbulent flow. It depends on a number of variables (Figure 13-5).
- Fluid forces on particles in motion also depends upon a number of variables (Figure 13-5).
- The presence of bed forms has an important effect on the sediment transport rate. As you saw in Chapter 11, that depends on a long list of variables (Figure 13-5).

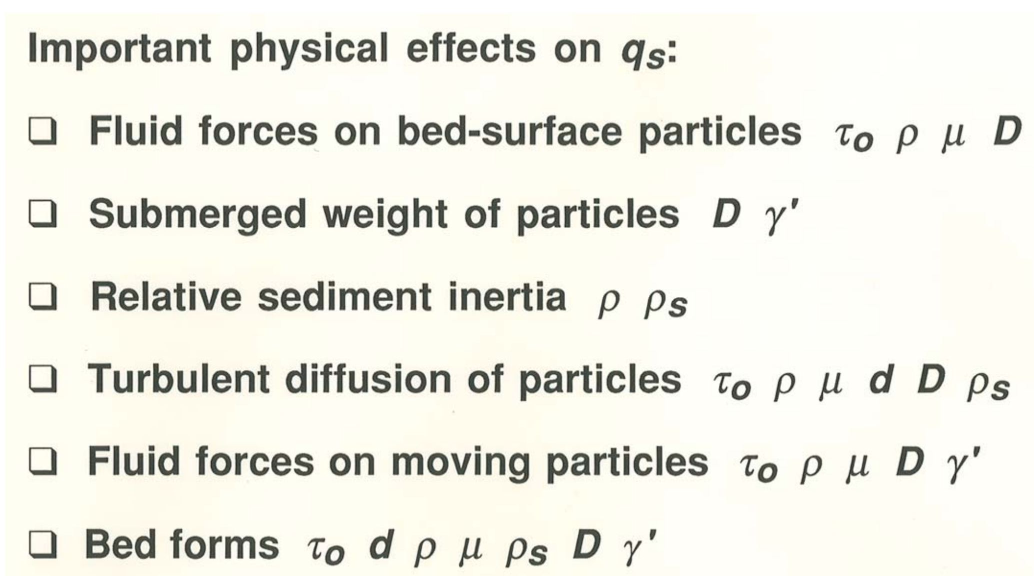


Figure 13-5. Important physical effects governing the unit sediment transport rate $q_s$.

**16** If we collect all of the variables in the above list, we see that each of the seven variables $\tau_o$, $d$, $\rho$, $\mu$, $\rho_s$, $D$, and $\gamma'$ appears somewhere in the list. Additionally, the effects of the joint size–shape–density ("SSD") frequency distribution of the sediment are not taken into account by these seven variables. So we can express the dependence of $q_s$ on these variables as

$$q_s = f(\tau_o, d, \rho, \mu, \rho_s, D, \gamma', \text{SSD distribution}) \qquad (12.1)$$

If we make the simplifying assumption that the SSD distribution is adequately represented by the mean or median size $D$ and the standard deviation $\sigma$, then $q_s$ is a function of no fewer than eight governing variables:

$$q_s = f(\tau_o, d, \rho, \mu, \rho_s, D, \sigma, \gamma') \qquad (12.2)$$

Then, nondimensionalizing in a physically revealing way, an appropriate nondimensionalized sediment transport rate can be expressed as a function of five governing dimensionless variables:

$$\frac{q_s}{(\rho\gamma' D)^{1/2}} = f\left(\frac{\tau_0}{\gamma' D}, \frac{\rho u_* D}{\mu}, \frac{d}{D}, \frac{\sigma}{D} . \frac{\rho_s}{\rho}\right) \qquad (12.3)$$

**17** Here we have chosen to nondimensionalize the unit sediment transport rate by use of $D$ rather than $\tau_o$, although it is more common, in the literature on sediment transport, to do the latter. The most important governing independent dimensionless variable (which might be called the "leading variable", is the first, the Shields parameter; see Chapter 9. The next two, the boundary Reynolds number and the relative roughness, express the turbulent structure of the flow. An alternative nondimensionalization might segregate the boundary shear stress and the median particle size into separate dimensionless variables.

**18** Clearly, the list of governing dimensionless variables is unworkably long. It can be simplified in the following ways. If we restrict consideration to quartz-density sand in water, the density ratio becomes irrelevant, and if we restrict consideration to well-sorted sediments, the dimensionless sorting becomes unimportant. If we consider only flows for which the particle size is much smaller than the flow depth (that leaves out all white-water mountain streams), then we can safely omit the relative roughness from the list. That leaves two important variables, expressing the importance of the boundary shear stress and the median particle size. (Our intuition would have told us, in the first place, that the sediment transport rate should depend mainly on the force that moves the particles, and the size, and thus the weight, of the particles!) Most, if not all, of the various sediment-discharge formulas that have been proposed make use of one or both of the boundary shear stress and the particle size, in one or another form.

### *Sediment-Discharge Formulas*

**19** To an untutored observer, the most natural way of developing a sediment-discharge formula would be to start from the equations of motion (the Navier-Stokes equations) for turbulent sediment-transporting flow. There are two severe problems in that, however: (1) the turbulence closure problem (see Chapter 4) makes it impossible to work from first principles without making certain assumptions, and (2) the complexity of the physics of particle transport in turbulent shear flows means that the physics of the sediment transport cannot be supplied in a fundamental way.

**20** What is commonly done, in the face of these difficulties, is first to attempt to adduce a rational dynamical basis for the sediment-transport process as a kind of framework, which results in one or more equations with certain adjustable parameters, and then use judiciously chosen data sets on measured sediment transport rates, from laboratory or field studies, to fit the equations to the data. (The ideal, of course, would be to have an equation with *no* adjustable parameters; a function with three or more adjustable parameters could be fitted to almost any data set!) The problem is that there is then no guarantee that the given sediment-discharge formula will work particularly well outside the range of data on which it was based.

**21** The first modern attempt to develop a sediment discharge formula dates way back to DuBoys, in 1879. In the course of the twentieth century, a great many sediment-discharge formulas were proposed. Several of those have been widely used. If you go into the literature on sediment-transport rates, you will repeatedly encounter the names of certain workers, mainly hydraulic engineers, whose names are associated with sediment-discharge formulas: Einstein (Hans Albert, not the more famous father, Albert); Meyer–Peter and Müller; Bagnold; Engelund and Hansen.

**22** These course notes are not the place to describe the various widely used sediment-discharge formulas. Vanoni (1975) gives brief descriptions of several such formulas. Here I will concentrate only upon comparisons among those formulas. Three useful comparison studies have appeared in the literature: Vanoni (1975), Gomez and Church (1989), and Nakato (1990).

### *Comparison of the Various Sediment-Discharge Formulas*

**23** First, what do the data on sediment transport rate look like? Figure 13-6 is a plot of dimensionless unit sediment transport rate, nondimensionalized by use of sediment size $D$, fluid density $\rho$, and submerged specific weight of the sediment, $\gamma'$, against a dimensionless measure of the boundary shear stress, the Shields parameter $\tau_o/\gamma' D$. The data are from both laboratory studies and measurements in rivers. In this undistorted log–log plot, you can see clearly the following:

- The sediment transport rate is a very steeply increasing function of the boundary shear stress. From the slope of the best-fit line in the graph, *the unit sediment transport rate goes approximately as the cube of the boundary shear stress*.
- Over five orders of magnitude of the unit sediment transport rate, the data fall along a fairly well-defined trend.
- Nonetheless there is considerable scatter in the data: if you pick one value of the dimensionless boundary shear stress, then, even if you ignore outlying points, there is an approximately order-of magnitude (factor of ten) spread in the data points.

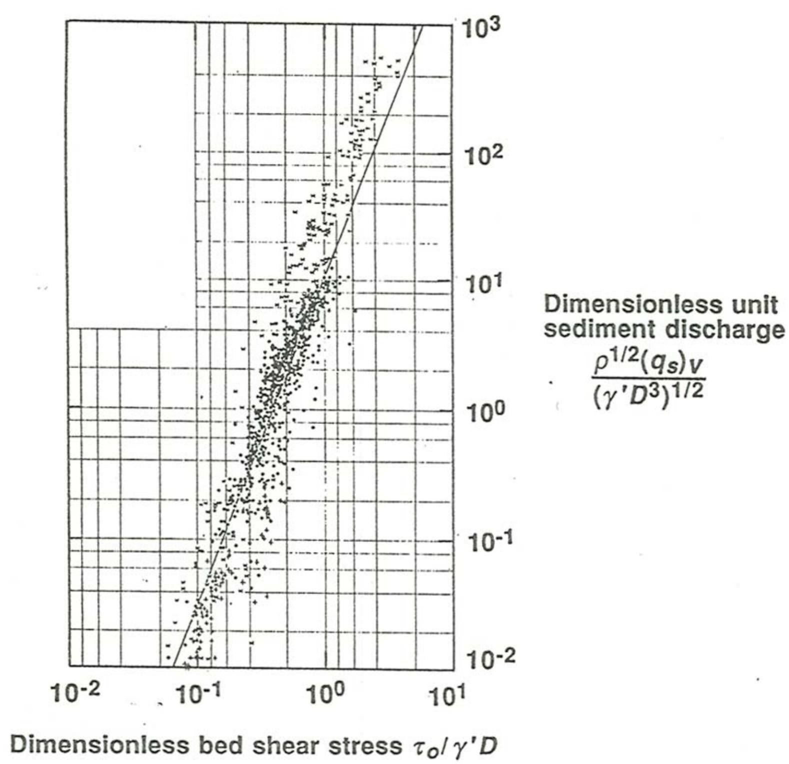


Figure 13-6. Plot of dimensionless unit sediment transport rate (expressed as volume, not mass) against dimensionless boundary shear stress (in the form of the Shields parameter) for various sets of measurement data. (Modified from Vanoni, 1975.)

**24** This large spread in values should not surprise you, when you consider that (1) the effects of particle size distribution and of the variable presence of bed forms is not taken into account, and (2) as you saw above, it is difficult to make accurate measurements of the sediment transport rate.

**25** Figure 13-7 shows a comparison of the performance of several sediment-discharge formulas in accounting for a single high-quality set of measurements of unit sediment discharge. You can see that the various formulas vary greatly in how well they match the actual data. In fairness, however, I could point out that the conditions represented by this data set are far from the conditions for which some of the discharge formulas were derived. For example, the widely used Meyer–Peter formula was derived for coarse sediments, whereas the data set shown in the figure is for sand that falls just barely within the medium-sand range. The lesson here, I suppose, is that you cannot expect any sediment-discharge formula to work well outside the range for which it was conceived.

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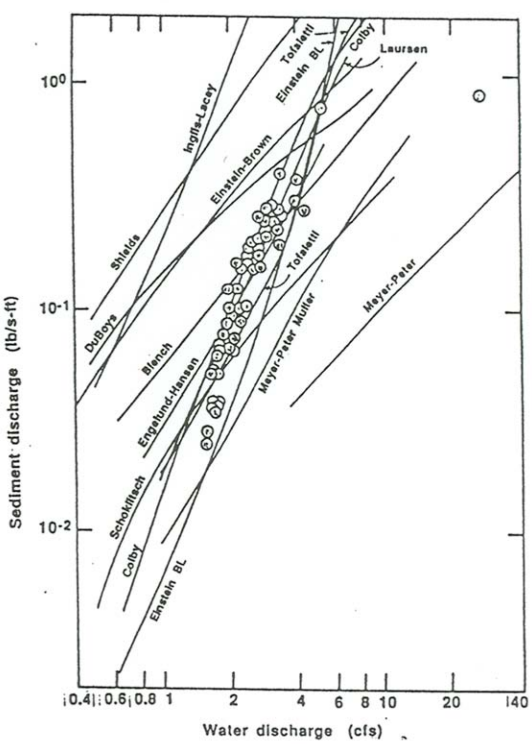


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Figure 13-7. Sediment discharge against water discharge for the Niobrara River near Cody, Wyoming, U.S.A. obtained from observations (data points) and calculations on the basis of several sediment-discharge formulas (solid curves). (From Vanoni, 1975.)

***References cited:***

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Gomez, B., and Church, M., 1989, An assessment of bed load sediment transport: formulae for gravel bed rivers: Water Resources Research, v. 25, p. 1161-1186.

Nakato, T., 1990, Tests of selected sediment-transport formulas: Journal of Hydraulic Engineering, v. 116, p. 362-379.